PLANE AND SPHERICAL TRIGONOMETRY

THE MACMILLAN COMPANY
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PLANE AND SPHERICAL TRIGONOMETRY

WITH TABLES

By **PAUL R. RIDER, Ph.D.**
PROFESSOR OF MATHEMATICS
WASHINGTON UNIVERSITY

THE MACMILLAN COMPANY
NEW YORK

PRINTED IN THE UNITED STATES OF AMERICA

Published February, 1942. Reprinted April, 1942; February, June (twice), November, 1943.

Preface

The primary purpose of this book is to present in a sound pedagogical manner the usual course in trigonometry as offered in colleges and technical schools. Only those methods are employed which have withstood the test of many years of actual classroom use. The arrangement of topics is such as has been found desirable as a result of long experience. Even logical order has at times been sacrificed to make the material more teachable. For example, the special definitions of the trigonometric functions for acute angles are given before the more general definitions. Applications are introduced early, as it has been found that the student's interest in a subject is considerably stimulated if he can see the utility of it. Moreover, the first problems have been made simple from a numerical standpoint in order to enable him to grasp principles and to learn methods without becoming lost in a maze of computations. Formulas are developed as needed, so that there is a certain amount of purposeful alternation between theoretical and practical aspects. On the other hand, the discussion of the more difficult of the theoretical topics is postponed to the latter part of the book. Many students find it easier to solve triangles than to handle some of the analytic phases of trigonometry such as proving identities and solving equations. By solving triangles they acquire confidence, as well as a certain amount of familiarity with the relations among the functions, so that they have a greater chance of success when they tackle the more difficult portions of the subject. Too much analytic work in

the early part of the course has been found to discourage many students and to kill their interest.

A few other features of the book seem worthy of note. An effort has been made to introduce simplifications into the treatment of certain topics, notably logarithms. The use of approximate numbers in computation and the question of significant figures have been stressed. Emphasis has been placed on the orderly arrangement of computations. Sets of carefully chosen and carefully graded exercises are to be found throughout the book. Answers to the odd-numbered exercises are printed at the back, answers to the even-numbered exercises are available in pamphlet form.

The book contains a complete course in plane and spherical trigonometry as these subjects are ordinarily taught. The part on spherical trigonometry has been made rather comprehensive in view of the present interest in subjects requiring a knowledge of this branch of mathematics. The student who has mastered this part will be well equipped to pursue courses in navigation and avigation, astronomy, and other applications. If a shorter course in plane trigonometry is desired, those topics marked with a ⋆ may be omitted. A thorough course in computational trigonometry is provided by the first seven chapters. Although, as stated above, the arrangement of material is that which seemed most desirable, the separate chapters are to a large extent independent, so that the instructor who prefers a different order of presentation should have no difficulty in outlining a course to his taste.

Advice concerning some of the figures and assistance with them were kindly given by my colleagues, Professors W. H. Roever and R. W. Bockhorst, to whom I am very grateful.

My thanks are due to The Macmillan Company for making every effort to give the book a pleasing format, and for the very valuable editorial assistance which they

rendered during its preparation. The manuscript was critically read by five different advisers, and the suggestions of these advisers were given thoughtful consideration during the process of revision. The revised manuscript was then read in great detail by one of these advisers, who even worked all of the exercises. It is hoped that because of its careful preparation the book will be found both clear and teachable, as well as mathematically sound.

P. R. R.

WASHINGTON UNIVERSITY
ST. LOUIS, MISSOURI
January, 1942

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PLANE TRIGONOMETRY

CHAPTER I

Trigonometric Functions of Acute Angles

1. Trigonometry.

The word **trigonometry** is derived from the Greek and means "measurement of triangles." The subject is principally concerned with the measurement of triangles (i.e., their sides and angles), or, more specifically, with the indirect measurement of line segments and angles. For example, it is possible, by trigonometry, to measure the width of a river without crossing it, or the height of a pole or cliff without climbing to the top.

The uses of trigonometry are many. The sciences of physics, mechanics, and astronomy could hardly have developed without it; practical arts, such as engineering, find it indispensable. It is a valuable aid in the study of periodic phenomena such as the tides, or even economic data which seem to be cyclic in their nature. Various specific uses will be illustrated throughout the book, particularly in the examples and exercises.

2. Trigonometric functions of an acute angle.

Let us consider the right triangle ABC, with the right angle at C (Fig. 1). The sides opposite the angles A, B, C will be denoted by the corresponding small letters, a, b, c, respectively. Then, by taking ratios of the sides of the triangle, we define three **trigonometric functions** of the acute angle A as follows:

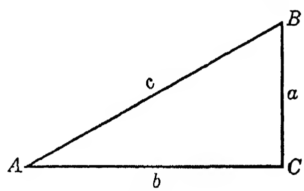


FIG. 1

sine of A (abbreviated **sin** A)

$$= \frac{\text{side opposite } A}{\text{hypotenuse}} = \frac{a}{c}, \tag{1}$$

cosine of A (abbreviated **cos** A)

$$= \frac{\text{side adjacent to } A}{\text{hypotenuse}} = \frac{b}{c}, \tag{2}$$

tangent of A (abbreviated **tan** A)

$$= \frac{\text{side opposite } A}{\text{side adjacent to } A} = \frac{a}{b}. \tag{3}$$

Thus, for example, in a right triangle in which $a = 3$, $b = 4$, $c = 5$ (see Fig. 2), we have

$$\sin A = \frac{3}{5}, \qquad \cos A = \frac{4}{5}, \qquad \tan A = \frac{3}{4}.$$

The values of these functions are completely determined by the angle A. Thus, if we had another right triangle with the same acute angle A, it would be similar to the above triangle and its sides would be in the same proportion. For example, they might all be twice as long, namely, $a = 6$, $b = 8$, $c = 10$. Then we should have $\sin A = 6/10 = 3/5$, as before, and similarly for the other functions. On the other hand, if the size of angle A were changed, the values of these functions would be changed.

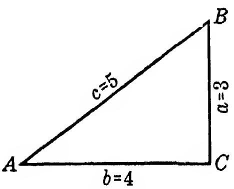


FIG. 2

Three, and only three, other ratios may also be formed from the sides of the triangle ABC. They are

cosecant of A (abbreviated **csc** A)

$$= \frac{\text{hypotenuse}}{\text{side opposite } A} = \frac{c}{a}, \tag{4}$$

secant of A (abbreviated **sec** A)

$$= \frac{\text{hypotenuse}}{\text{side adjacent to } A} = \frac{c}{b}, \tag{5}$$

cotangent of A (abbreviated **cot** A)

$$= \frac{\text{side adjacent to } A}{\text{side opposite } A} = \frac{b}{a}. \qquad (6)$$

It will be noted that these three functions are the reciprocals * of the other three, and we may write

$$\begin{aligned} \csc A &= \frac{1}{\sin A}, & \sin A &= \frac{1}{\csc A}, \\ \sec A &= \frac{1}{\cos A}, & \cos A &= \frac{1}{\sec A}, \\ \cot A &= \frac{1}{\tan A}, & \tan A &= \frac{1}{\cot A} \end{aligned} \qquad (7)$$

NOTE. Three other functions are:

versed sine of A (abbreviated **vers** A) $= 1 - \cos A$,
coversed sine of A (abbreviated **covers** A) $= 1 - \sin A$,
haversine of A (abbreviated **hav** A) $= \frac{1}{2}(1 - \cos A)$.

They will not be used in this book.

EXERCISES I. A

Draw the right triangles whose sides have the following values, and find the six trigonometric functions of the angle A:

1. $a = 4$, $b = 3$, $c = 5$.

2. $a = 5$, $b = 12$, $c = 13$.

3. $a = 2$, $b = 3$, $c = \sqrt{13}$.

4. $a = 1$, $b = 1$, $c = \sqrt{2}$.

5. $a = 2$, $b = \sqrt{5}$, $c = 3$.

6. $a = \sqrt{2}$, $b = \sqrt{3}$, $c = \sqrt{5}$.

7. $a = 8$, $b = 15$.

8. $b = 21$, $c = 29$.

9. $a = 7$, $c = 25$.

10. $a = 5$, $b = 3$.

11. $a = 1$, $b = \sqrt{3}$.

12. $a = 1$, $b = 3$.

13. $a = 1$, $b = \frac{1}{3}$.

14. $a = \frac{1}{2}$, $b = \frac{1}{3}$.

15. A guy wire 15 feet long is fastened to a point 13 feet above the foot of a vertical pole, which stands on level ground. Find the sine of the angle that the wire makes with the horizontal.

* The reciprocal of a number is 1 divided by the number.

16. A yardstick, held vertically on a level surface, casts a shadow 1 foot 8 inches long. Find the tangent of the angle that the rays of the sun make with the horizontal.
17. A roadway rises 55 feet in a horizontal distance of $\frac{1}{2}$ mile. Find the tangent of the angle that it makes with the horizontal.
18. An airplane is descending 225 feet per 1000 feet of horizontal distance covered. What is the cosine of the angle that its path of descent makes with the horizontal?
19. One end of a foot ruler is placed against a vertical wall; the other end of the ruler reaches a point on the floor 9 inches from the base of the wall. Find the sine, cosine, and tangent of the angle that the ruler makes (a) with the wall, (b) with the floor.
20. A box is 3 inches by 4 inches by 1 foot. Find the sine of the angle that a diagonal of the box makes with its longest edge.

3. Functions of complementary angles.

By referring to the definitions of the trigonometric functions (section 2) and to Fig. 1, we see that, for the acute angle B,

$$\begin{aligned} \sin B &= \frac{b}{c}, & \csc B &= \frac{c}{b}, \\ \cos B &= \frac{a}{c}, & \sec B &= \frac{c}{a}, \\ \tan B &= \frac{b}{a}, & \cot B &= \frac{a}{b}. \end{aligned} \tag{1}$$

Comparing these formulas with formulas (1)–(6) of section 2, and making use of the fact that A and B are complementary angles (i.e., $A + B = 90°$), we have

$$\begin{aligned} \sin B &= \sin(90° - A) = \cos A, \\ \cos B &= \cos(90° - A) = \sin A, \\ \tan B &= \tan(90° - A) = \cot A, \\ \csc B &= \csc(90° - A) = \sec A, \\ \sec B &= \sec(90° - A) = \csc A, \\ \cot B &= \cot(90° - A) = \tan A. \end{aligned} \tag{2}$$

It is convenient to arrange the functions in pairs as follows: sine and cosine, tangent and cotangent, secant and cosecant. In any pair, either function may be called the **cofunction** of the other. Relations (2) may then be expressed by the single statement: *Any function of the complement of an angle is equal to the cofunction of the angle.*

EXERCISES I. B

Find the functions of angle B in exercises I. A, 1–14.

4. Finding the other functions of an acute angle when one function is given.

The following examples will illustrate how the remaining functions of an acute angle can be found if the value of one function is given.

Example 1.

Given $\sin A = \frac{5}{13}$, A acute; find the other functions of A.

SOLUTION. Since $\sin A = \frac{a}{c}$, we have $\frac{a}{c} = \frac{5}{13}$. Construct a right triangle with $a = 5$ and $c = 13$ (Fig. 3). (Note that it is not necessary to take $a = 5$ and $c = 13$; we could take $a = 10$ and $c = 26$, for example, or any other numbers in the ratio of 5 to 13.)

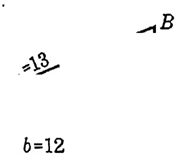


FIG. 3

Making use of the theorem of Pythagoras, that the square of the hypotenuse is equal to the sum of the squares of the sides, we have

$$b^2 = c^2 - a^2 = 169 - 25 = 144, \qquad b = 12.$$

The remaining functions of A can be read from the figure. Thus,

$$\cos A = \frac{12}{13}, \tan A = \frac{5}{12}, \csc A = \frac{13}{5}, \sec A = \frac{13}{12}, \cot A = \frac{12}{5}.$$

Example 2.

If $\tan A = 3$, what are the other functions of A, it being understood that A is acute?

SOLUTION. $$\tan A = 3 = \frac{a}{b}.$$

Take $a = 3$, $b = 1$, and construct a right triangle (Fig. 4). Then,

$$c^2 = a^2 + b^2 = 9 + 1 = 10, \qquad c = \sqrt{10}.$$

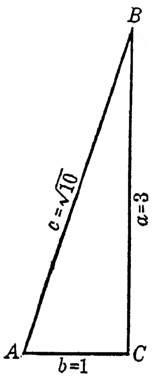


FIG. 4

$$\sin A = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10} = 0.9487,$$

$$\cos A = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10} = 0.3162,$$

$$\csc A = \frac{\sqrt{10}}{3} = 1.054,$$

$$\sec A = \frac{\sqrt{10}}{1} = \sqrt{10} = 3.162$$

$$\cot A = \frac{1}{3} = 0.3333.$$

EXERCISES I. C

Find the other five functions of the acute angle A, given that

1. $\cos A = \frac{4}{5}$.
2. $\tan A = \frac{2}{3}$.
3. $\cot A = \frac{1}{5}$.
4. $\sin A = \frac{2}{5}$.
5. $\sec A = \sqrt{2}$.
6. $\csc A = \frac{41}{9}$.
7. $\sin A = \frac{1}{2}$.
8. $\cos A = \frac{2}{3}$.
9. $\tan A = \frac{2}{5}$.
10. $\csc A = \frac{4}{3}$.
11. $\cot A = \frac{5}{2}$.
12. $\sec A = \frac{5}{4}$.
13. $\sec A = 2$.
14. $\cos A = \frac{1}{4}$.
15. $\tan A = 0.5$.
16. $\sin A = 0.8$.
17. $\sin A = \dfrac{\sqrt{3}}{2}$.
18. $\cos A = \dfrac{\sqrt{2}}{2}$.
19. $\tan A = \dfrac{\sqrt{3}}{3}$.
20. $\csc A = \sqrt{2}$.
21. $\sin A = \frac{2}{7}$.
22. $\tan A = \dfrac{u}{v}$.
23. $\sin A = \dfrac{2mn}{m^2 + n^2}$.

24. Show that if A is an acute angle,

$$\sin^2 A + \cos^2 A = 1.$$

(The notation $\sin^2 A$ means the square of the sine of A. For example, if $\sin A = \frac{2}{3}$, then $\sin^2 A = (\frac{2}{3})^2 = \frac{4}{9}$.)

SOLUTION. $\sin^2 A + \cos^2 A = \left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2$

$$= \frac{a^2}{c^2} + \frac{b^2}{c^2} = \frac{a^2 + b^2}{c^2} = \frac{c^2}{c^2} = 1,$$

since (see Fig. 5), by the Pythagorean theorem, $a^2 + b^2 = c^2$.

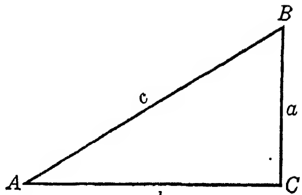


FIG. 5

Show that if A is an acute angle, then

25. $\sec^2 A = 1 + \tan^2 A$.

26. $\csc^2 A = 1 + \cot^2 A$.

27. $\cos A \tan A = \sin A$.

28. $\cot A \cos A = \csc A - \sin A$.

29. $\dfrac{1 + \sin A}{\cos A} = \dfrac{\cos A}{1 - \sin A}$.

30. $\dfrac{\cos^2 A}{1 - \sin A} = 1 + \sin A$.

31. $\dfrac{\sin A + \tan A}{\cot A + \csc A} = \sin A \tan A$.

32. $\dfrac{1 - 2\cos^2 A}{\sin A \cos A} = \tan A - \cot A$.

5. Functions of 45°, 60°, and 30°.

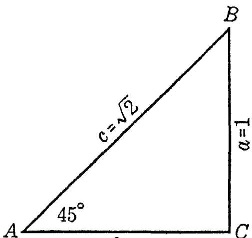


FIG. 6

To find the functions of 45° we construct an isosceles right triangle (Fig. 6). It is convenient to make each leg equal to 1, that is, $a = 1$, $b = 1$. Then,

$$c^2 = a^2 + b^2 = 1 + 1 = 2, \qquad c = \sqrt{2}.$$

From the figure we read

$$\sin 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} = 0.7071, \qquad \csc 45° = \sqrt{2} = 1.414,$$

$$\cos 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} = 0.7071, \qquad \sec 45° = \sqrt{2} = 1.414,$$
$$\tan 45° = 1, \qquad \cot 45° = 1.$$

The decimal values are, of course, merely approximate.

In order to find the functions of 60° we take an equilateral triangle and draw the bisector of one of the angles. (See Fig. 7.) This bisector divides the equilateral triangle into two congruent right triangles whose angles are 60° and 30°. Let us consider one of these, namely ABC. If each side of the original equilateral triangle is 2 units in length, it follows that in ABC, $c = 2$ and $b = 1$, since AC is half the base of the equilateral triangle. Then

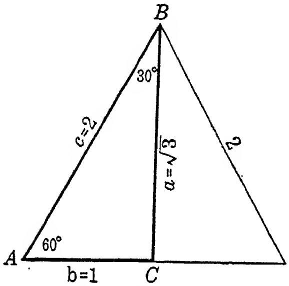


Fig. 7

$$a^2 = c^2 - b^2 = 4 - 1 = 3, \qquad a = \sqrt{3}.$$

From Fig. 7 we read

$$\sin 60° = \frac{\sqrt{3}}{2} = 0.8660, \qquad \csc 60° = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3} = 1.155,$$

$$\cos 60° = \frac{1}{2} = 0.5, \qquad \sec 60° = 2,$$

$$\tan 60° = \sqrt{3} = 1.732, \qquad \cot 60° = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} = 0.5774.$$

From the same figure, or from the relations between the functions of complementary angles, we find

$$\sin 30° = \frac{1}{2} = 0.5,$$

$$\cos 30° = \frac{\sqrt{3}}{2} = 0.8660,$$

$$\tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} = 0.5774,$$

$$\csc 30^\circ = 2,$$

$$\sec 30^\circ = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3} = 1.155,$$

$$\cot 30^\circ = \sqrt{3} = 1.732.$$

6. Tables of functions.

There are very few angles whose functions can be found by the foregoing methods of elementary geometry. It is possible, however, by other means to calculate the functions of any angle. Values of the functions have been calculated and tabulated, as for example in the table on pages 12–14, which gives the values of the sine, cosine, tangent, and cotangent of all angles from 0° to 90° for intervals of ten minutes.

To find a function of an angle less than 45° we locate the angle at the left-hand side of the table and the name of the function at the top of the column. Angles greater than 45° are located at the right-hand side of the table, and the names of their functions are located at the bottom. Opposite the angle, in the appropriate column, is found the value of the function.

For example, we find the sine of 32° 40′ to be 0.5398. Note that this is also the cosine of 57° 20′, the complement of 32° 40′. Because of the relations between the functions of an angle and the functions of its complement, the table does double duty.

EXERCISES I. D

Find, in the table on pages 12–14, the values of the following:

1. cos 28° 20′. **2.** sin 67° 30′. **3.** tan 15° 40′.
4. cot 79° 10′. **5.** sin 45° 20′. **6.** sin 0° 10′.
7. tan 0° 10′. **8.** sin 89°. **9.** tan 89° 50′.

angle	sin	tan	cot	cos	
0° 00′	.0000	.0000	—	1.0000	**90°** 00′
10	.0029	.0029	343.77	1.0000	50
20	.0058	.0058	171.89	1.0000	40
30	.0087	.0087	114.59	1.0000	30
40	.0116	.0116	85.940	.9999	20
50	.0145	.0145	68.750	.9999	10
1° 00′	.0175	.0175	57.290	.9998	**89°** 00′
10	.0204	.0204	49.104	.9998	50
20	.0233	.0233	42.964	.9997	40
30	.0262	.0262	38.188	.9997	30
40	.0291	.0291	34.368	.9996	20
50	.0320	.0320	31.242	.9995	10
2° 00′	.0349	.0349	28.636	.9994	**88°** 00′
10	.0378	.0378	26.432	.9993	50
20	.0407	.0407	24.542	.9992	40
30	.0436	.0437	22.904	.9990	30
40	.0465	.0466	21.470	.9989	20
50	.0494	.0495	20.206	.9988	10
3° 00′	.0523	.0524	19.081	.9986	**87°** 00′
10	.0552	.0553	18.075	.9985	50
20	.0581	.0582	17.169	.9983	40
30	.0610	.0612	16.350	.9981	30
40	.0640	.0641	15.605	.9980	20
50	.0669	.0670	14.924	.9978	10
4° 00′	.0698	.0699	14.301	.9976	**86°** 00′
10	.0727	.0729	13.727	.9974	50
20	.0756	.0758	13.197	.9971	40
30	.0785	.0787	12.706	.9969	30
40	.0814	.0816	12.251	.9967	20
50	.0843	.0846	11.826	.9964	10
5° 00′	.0872	.0875	11.430	.9962	**85°** 00′
10	.0901	.0904	11.059	.9959	50
20	.0929	.0934	10.712	.9957	40
30	.0958	.0963	10.385	.9954	30
40	.0987	.0992	10.078	.9951	20
50	.1016	.1022	9.7882	.9948	10
6° 00′	.1045	.1051	9.5144	.9945	**84°** 00′
10	.1074	.1080	9.2553	.9942	50
20	.1103	.1110	9.0098	.9939	40
30	.1132	.1139	8.7769	.9936	30
40	.1161	.1169	8.5555	.9932	20
50	.1190	.1198	8.3450	.9929	10
7° 00′	.1219	.1228	8.1443	.9925	**83°** 00′
10	.1248	.1257	7.9530	.9922	50
20	.1276	.1287	7.7704	.9918	40
30	.1305	.1317	7.5958	.9914	30
40	.1334	.1346	7.4287	.9911	20
50	.1363	.1376	7.2687	.9907	10
8° 00′	.1392	.1405	7.1154	.9903	**82°** 00′
10	.1421	.1435	6.9682	.9899	50
20	.1449	.1465	6.8269	.9894	40
30	.1478	.1495	6.6912	.9890	30
40	.1507	.1524	6.5606	.9886	20
50	.1536	.1554	6.4348	.9881	10
9° 00′	.1564	.1584	6.3138	.9877	**81°** 00′
	cos	cot	tan	sin	angle

angle	sin	tan	cot	cos	
9° 00′	.1564	.1584	6.3138	.9877	**81°** 00′
10	.1593	.1614	6.1970	.9872	50
20	.1622	.1644	6.0844	.9868	40
30	.1650	.1673	5.9758	.9863	30
40	.1679	.1703	5.8708	.9858	20
50	.1708	.1733	5.7694	.9853	10
10° 00′	.1736	.1763	5.6713	.9848	**80°** 00′
10	.1765	.1793	5.5764	.9843	50
20	.1794	.1823	5.4845	.9838	40
30	.1822	.1853	5.3955	.9833	30
40	.1851	.1883	5.3093	.9827	20
50	.1880	.1914	5.2257	.9822	10
11° 00′	.1908	.1944	5.1446	.9816	**79°** 00′
10	.1937	.1974	5.0658	.9811	50
20	.1965	.2004	4.9894	.9805	40
30	.1994	.2035	4.9152	.9799	30
40	.2022	.2065	4.8430	.9793	20
50	.2051	.2095	4.7729	.9787	10
12° 00′	.2079	.2126	4.7046	.9781	**78°** 00′
10	.2108	.2156	4.6382	.9775	50
20	.2136	.2186	4.5736	.9769	40
30	.2164	.2217	4.5107	.9763	30
40	.2193	.2247	4.4494	.9757	20
50	.2221	.2278	4.3897	.9750	10
13° 00′	.2250	.2309	4.3315	.9744	**77°** 00′
10	.2278	.2339	4.2747	.9737	50
20	.2306	.2370	4.2193	.9730	40
30	.2334	.2401	4.1653	.9724	30
40	.2363	.2432	4.1126	.9717	20
50	.2391	.2462	4.0611	.9710	10
14° 00′	.2419	.2493	4.0108	.9703	**76°** 00′
10	.2447	.2524	3.9617	.9696	50
20	.2476	.2555	3.9136	.9689	40
30	.2504	.2586	3.8667	.9681	30
40	.2532	.2617	3.8208	.9674	20
50	.2560	.2648	3.7760	.9667	10
15° 00′	.2588	.2679	3.7321	.9659	**75°** 00′
10	.2616	.2711	3.6891	.9652	50
20	.2644	.2742	3.6470	.9644	40
30	.2672	.2773	3.6059	.9636	30
40	.2700	.2805	3.5656	.9628	20
50	.2728	.2836	3.5261	.9621	10
16° 00′	.2756	.2867	3.4874	.9613	**74°** 00′
10	.2784	.2899	3.4495	.9605	50
20	.2812	.2931	3.4124	.9596	40
30	.2840	.2962	3.3759	.9588	30
40	.2868	.2994	3.3402	.9580	20
50	.2896	.3026	3.3052	.9572	10
17° 00′	.2924	.3057	3.2709	.9563	**73°** 00′
10	.2952	.3089	3.2371	.9555	50
20	.2979	.3121	3.2041	.9546	40
30	.3007	.3153	3.1716	.9537	30
40	.3035	.3185	3.1397	.9528	20
50	.3062	.3217	3.1084	.9520	10
18° 00′	.3090	.3249	3.0777	.9511	**72°** 00′
	cos	cot	tan	sin	angle

angle	sin	tan	cot	cos	
18° 00′	.3090	.3249	3.0777	.9511	**72°** 00′
10	.3118	.3281	3.0475	.9502	50
20	.3145	.3314	3.0178	.9492	40
30	.3173	.3346	2.9887	.9483	30
40	.3201	.3378	2.9600	.9474	20
50	.3228	.3411	2.9319	.9465	10
19° 00′	.3256	.3443	2.9042	.9455	**71°** 00′
10	.3283	.3476	2.8770	.9446	50
20	.3311	.3508	2.8502	.9436	40
30	.3338	.3541	2.8239	.9426	30
40	.3365	.3574	2.7980	.9417	20
50	.3393	.3607	2.7725	.9407	10
20° 00′	.3420	.3640	2.7475	.9397	**70°** 00′
10	.3448	.3673	2.7228	.9387	50
20	.3475	.3706	2.6985	.9377	40
30	.3502	.3739	2.6746	.9367	30
40	.3529	.3772	2.6511	.9356	20
50	.3557	.3805	2.6279	.9346	10
21° 00′	.3584	.3839	2.6051	.9336	**69°** 00′
10	.3611	.3872	2.5826	.9325	50
20	.3638	.3906	2.5605	.9315	40
30	.3665	.3939	2.5386	.9304	30
40	.3692	.3973	2.5172	.9293	20
50	.3719	.4006	2.4960	.9283	10
22° 00′	.3746	.4040	2.4751	.9272	**68°** 00′
10	.3773	.4074	2.4545	.9261	50
20	.3800	.4108	2.4342	.9250	40
30	.3827	.4142	2.4142	.9239	30
40	.3854	.4176	2.3945	.9228	20
50	.3881	.4210	2.3750	.9216	10
23° 00′	.3907	.4245	2.3559	.9205	**67°** 00′
10	.3934	.4279	2.3369	.9194	50
20	.3961	.4314	2.3183	.9182	40
30	.3987	.4348	2.2998	.9171	30
40	.4014	.4383	2.2817	.9159	20
50	.4041	.4417	2.2637	.9147	10
24° 00′	.4067	.4452	2.2460	.9135	**66°** 00′
10	.4094	.4487	2.2286	.9124	50
20	.4120	.4522	2.2113	.9112	40
30	.4147	.4557	2.1943	.9100	30
40	.4173	.4592	2.1775	.9088	20
50	.4200	.4628	2.1609	.9075	10
25° 00′	.4226	.4663	2.1445	.9063	**65°** 00′
10	.4253	.4699	2.1283	.9051	50
20	.4279	.4734	2.1123	.9038	40
30	.4305	.4770	2.0965	.9026	30
40	.4331	.4806	2.0809	.9013	20
50	.4358	.4841	2.0655	.9001	10
26° 00′	.4384	.4877	2.0503	.8988	**64°** 00′
10	.4410	.4913	2.0353	.8975	50
20	.4436	.4950	2.0204	.8962	40
30	.4462	.4986	2.0057	.8949	30
40	.4488	.5022	1.9912	.8936	20
50	.4514	.5059	1.9768	.8923	10
27° 00′	.4540	.5095	1.9626	.8910	**63°** 00′
	cos	cot	tan	sin	angle

angle	sin	tan	cot	cos	
27° 00′	.4540	.5095	1.9626	.8910	**63°** 00′
10	.4566	.5132	1.9486	.8897	50
20	.4592	.5169	1.9347	.8884	40
30	.4617	.5206	1.9210	.8870	30
40	.4643	.5243	1.9074	.8857	20
50	.4669	.5280	1.8940	.8843	10
28° 00′	.4695	.5317	1.8807	.8829	**62°** 00′
10	.4720	.5354	1.8676	.8816	50
20	.4746	.5392	1.8546	.8802	40
30	.4772	.5430	1.8418	.8788	30
40	.4797	.5467	1.8291	.8774	20
50	.4823	.5505	1.8165	.8760	10
29° 00′	.4848	.5543	1.8040	.8746	**61°** 00′
10	.4874	.5581	1.7917	.8732	50
20	.4899	.5619	1.7796	.8718	40
30	.4924	.5658	1.7675	.8704	30
40	.4950	.5696	1.7556	.8689	20
50	.4975	.5735	1.7437	.8675	10
30° 00′	.5000	.5774	1.7321	.8660	**60°** 00′
10	.5025	.5812	1.7205	.8646	50
20	.5050	.5851	1.7090	.8631	40
30	.5075	.5890	1.6977	.8616	30
40	.5100	.5930	1.6864	.8601	20
50	.5125	.5969	1.6753	.8587	10
31° 00′	.5150	.6009	1.6643	.8572	**59°** 00′
10	.5175	.6048	1.6534	.8557	50
20	.5200	.6088	1.6426	.8542	40
30	.5225	.6128	1.6319	.8526	30
40	.5250	.6168	1.6212	.8511	20
50	.5275	.6208	1.6107	.8496	10
32° 00′	.5299	.6249	1.6003	.8480	**58°** 00′
10	.5324	.6289	1.5900	.8465	50
20	.5348	.6330	1.5798	.8450	40
30	.5373	.6371	1.5697	.8434	30
40	.5398	.6412	1.5597	.8418	20
50	.5422	.6453	1.5497	.8403	10
33° 00′	.5446	.6494	1.5399	.8387	**57°** 00′
10	.5471	.6536	1.5301	.8371	50
20	.5495	.6577	1.5204	.8355	40
30	.5519	.6619	1.5108	.8339	30
40	.5544	.6661	1.5013	.8323	20
50	.5568	.6703	1.4919	.8307	10
34° 00′	.5592	.6745	1.4826	.8290	**56°** 00′
10	.5616	.6787	1.4733	.8274	50
20	.5640	.6830	1.4641	.8258	40
30	.5664	.6873	1.4550	.8241	30
40	.5688	.6916	1.4460	.8225	20
50	.5712	.6959	1.4370	.8208	10
35° 00′	.5736	.7002	1.4281	.8192	**55°** 00′
10	.5760	.7046	1.4193	.8175	50
20	.5783	.7089	1.4106	.8158	40
30	.5807	.7133	1.4019	.8141	30
40	.5831	.7177	1.3934	.8124	20
50	.5854	.7221	1.3848	.8107	10
36° 00′	.5878	.7265	1.3764	.8090	**54°** 00′
	cos	cot	tan	sin	angle

angle	sin	tan	cot	cos	
36° 00	.5878	.7265	1.3764	.8090	**54°** 00
10	.5901	.7310	1.3680	.8073	
20	.5925	.7355	1.3597	.8056	
30	.5948	.7400	1.3514	.8039	
40	5972	.7445	1.3432	.8021	
50	.5995	.7490	1.3351	.8004	
37° 00	.6018	.7536	1.3270	7986	**53°** 00
10	6041	.7581	1.3190	.7969	
20	.6065	.7627	1.3111	7951	
30	6088	.7673	1.3032	7934	
40	.6111	.7720	1.2954	7916	
50	6134	.7766	1.2876	.7898	
° 00	.6157	.7813	1.2799	.7880	**52°** 00′
10	6180	.7860	1.2723	.7862	
20	.6202	.7907	1.2647	.7844	
30	6225	.7954	1.2572	.7826	
40	.6248	.8002	1.2497	.7808	
50	6271	.8050	1.2423	7790	
° 00′	.6293	.8098	1.2349	7771	**51°**
10	6316	.8146	1.2276	[illegible]	
20	.6338	.8195	1.2203	.7735	
30	6361	.8243	1.2131	7716	
40	6383	.8292	1.2059	.7698	
50	.6406	.8342	1.1988	7679	
40° 00	6428	.8391	1.1918	7660	**50°** 00
10	.6450	.8441	1.1847	764[illegible]	
20	.6472	.8491	1.1778	7623	
30	.6494	.8541	.1708	7604	
40	.6517	.8591	.1640	758[illegible]	
50	.6539	.8642	.1571	7566	
41° 00′	.6561	.8693	.1504	7547	**49°** 00
10	.6583	.8744	1.1436	7528	
20	.6604	.8796	1.1369	.7509	
30	.6626	.8847	1.1303	.7490	
40	[illegible]	[illegible]	.1237	.7470	
50	.6670	.8952	.1171	.7451	
42° 00′	.6691	.9004	.1106	.7431	**48°**
10	.6713	.9057	1 1041	.7412	
20	.6734	.9110	0977	7392	
30	.6756	.9163	.0913	7373	
40	.6777	.9217	1.0850	.7353	
50	.6799	.9271	1.0786	.7333	
43° 00	.6820	.9325	1.0724	.7314	**47°** 00
10	.6841	.9380	1.0661	.7294	
20	.6862	.9435	1.0599	.7274	
30	.6884	.9490	1.0538	.7254	
40	.6905	.9545	1.0477	.7234	
50	6926	.9601	1.0416	.7214	
44° 00′	6947	.9657	1.0355	.7193	**46**
10	.6967	.9713	1.0295	.7173	
20	[illegible]	.9770	1.0235	.7153	
30	7009	.9827	1.0176	.7133	
40	7030	.9884	1.0117	.7112	
50	.7050	.9942	1.0058	.7092	
45° 00′	7071	1.0000	1.0000	.7071	**45°** 00
	cos	cot	tan	sin	angle

Find the value of the acute angle A, given that

10. $\sin A = 0.0727$. **11.** $\cos A = 0.8021$. **12.** $\tan A = 2.3183$.
13. $\cot A = 3.2709$. **14.** $\sin A = 0.6202$. **15.** $\cos A = 0.3665$.
16. $\tan A = 0.9601$. **17.** $\cot A = 6.8269$. **18.** $\sin 2A = 0.1994$.
19. $2 \sin A = 1.9500$. **20.** $\sin(A + 30°) = 0.6180$.
21. $\tan(2A - 30°) = 0.3249$. **22.** $2 \cos(\frac{1}{2}A + 10°) = 0.6786$.

23. Find the value of $\sin 20° + \sin 30°$. Is this equal to $\sin 50°$?

CHAPTER II

Solution of Triangles

7. Solution of right triangles.

The use of tables of the trigonometric functions will be illustrated by some examples.

Example 1.

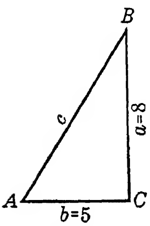


FIG. 8

A vertical pole 8 feet tall casts a shadow 5 feet long on level ground. Find the angle which the rays of the sun make with the horizontal.

SOLUTION. In Fig. 8, a represents the height of the pole, b represents the length of the shadow, A is the angle to be found. We have

$$\tan A = \frac{a}{b} = \frac{8}{5} = 1.6.$$

From the table on pages 12–14 we find $A = 58°$ (to the nearest $10'$).

Example 2.

A surveyor wishes to measure the distance across a stream. He sets up his transit at a point C on the bank of the stream, and sights on a point B on the other bank directly opposite him. Then he turns the transit through a right angle, and measures off a distance of 100 feet to a point A. He moves the transit to A, and measures the angle CAB, which he finds to be $50°$. How wide is the stream?

SOLUTION. The conditions of the problem are illustrated in Fig. 9. To find a, the distance across the stream, we proceed as follows:

$$\frac{a}{b} = \tan A, \qquad a = b \tan A = 100 \tan 50°.$$

From the table on pages 12–14 we find $\tan 50° = 1.1918$. Thus,

$$a = 100 \times 1.1918 = 119.2 \text{ ft.}$$

A triangle is composed of six **parts**, the three sides and the three angles. To **solve** a triangle is to find the unknown parts from the parts that are given. In the case of a right triangle this can always be done if we have given (besides the right angle) two parts, at least one of which is a side.

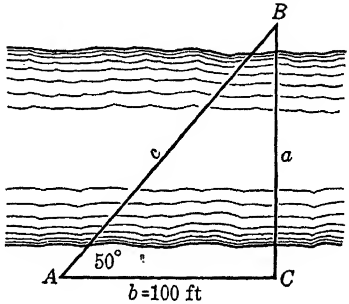


FIG. 9

In problems involving a right triangle ABC, it will ordinarily be understood that the right angle is at C.

In solving right triangles we make use of four of the definitions, namely,

$$\sin A = \frac{a}{c}, \qquad \cos A = \frac{b}{c}, \qquad \tan A = \frac{a}{b}, \qquad \cot A = \frac{b}{a},$$

and of the Pythagorean relation,

$$a^2 + b^2 = c^2.$$

(We seldom use the secant or cosecant, since tables of these functions are not so generally available.) Of course we sometimes find it convenient to use the relation

$$A + B = 90°,$$

and the fact that the functions of B are equal respectively to the corresponding cofunctions of A.

From the foregoing relations we select one which contains the two given, or known, parts and the part which we wish to find.

Example 3.

Solve the right triangle ABC in which $c = 25$, $A = 32° 10'$.

Solution. To find a we use the definition $a/c = \sin A$, which contains the known parts c and A. We get

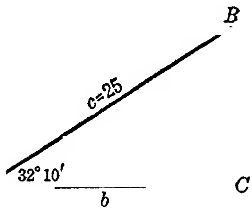


Fig. 10

$$a = c \sin A = 25 \sin 32° 10'$$
$$= 25 \times 0.5324 = 13.3.$$

To find b we use $b/c = \cos A$, from which we get

$$b = c \cos A = 25 \cos 32° 10'$$
$$= 25 \times 0.8465 = 21.2.$$

$$\begin{aligned} 90° &= 89° 60' \\ A &= 32° 10' \\ \hline B &= 57° 50' \end{aligned}$$

Example 4.

Given $a = 27.2$, $b = 10.6$; find A, B, c.

Solution.

$$\tan A = \frac{a}{b} = \frac{27.2}{10.6} = 2.5660, \qquad A = 68° 40'.$$

The value 2.5660 is not to be found in the table on pages 12–14. The value closest to this is 2.5605, which is the tangent of 68° 40'. Consequently, as an approximation, we take

$$A = 68° 40'.$$

In a later section we shall learn how to find a more accurate value for an angle when the given function is between two consecutive values in the table.

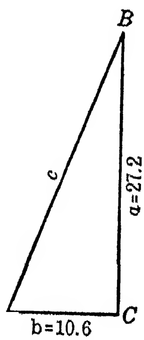


Fig. 11

$$B = 90° - A = 21° 20'.$$

$$\frac{a}{c} = \sin A, \qquad c \sin A = a,$$

$$c = \frac{\quad}{\sin A} = \frac{27.2}{\sin 68° 40'} = \frac{27.2}{0.9315} = 29.2.$$

We could also find c by using the relation $c^2 = a^2 + b^2$, obtaining values from a table of squares, such as is to be found in Table VI of the Macmillan Logarithmic and Trigonometric Tables. Thus,

$$c^2 = (27.2)^2 + (10.6)^2 = 739.84 + 112.36 = 852.20.$$

From Table VI, just referred to, we find

$$c = 29.2.$$

It is recommended that all answers be checked by obtaining the solutions in two different ways.

It is also recommended that a drawing be made to scale. From such a drawing it is possible to make at least a rough check of the results.

EXERCISES II. A

In solving the following exercises, use the nearest values that are to be found in the tables.

Solve the following triangles, in which $C = 90°$.

1. $A = 35°, c = 5.$

2. $a = 6, c = 14.$

3. $A = 37°, b = 53.$

4. $B = 56°, c = 84.$

5. $a = 23, b = 17.$

6. $a = 18.5, c = 37.2.$

7. $B = 17° 30', b = 92.4.$

8. $A = 57° 20', c = 0.0286.$

9. $a = 0.257, b = 0.856.$

10. $b = 189, A = 13° 50'.$

11. A wire is stretched from the top of a vertical pole standing on level ground. The wire reaches to a point on the ground 10 feet from the foot of the pole, and makes an angle of 75° with the horizontal. Find the height of the pole and the length of the wire.

12. A flagpole broken over by the wind forms a right triangle with the ground. If the angle which the broken part makes with the ground is 50°, and the distance from the tip of the pole to the foot is 55 feet, how tall was the pole?

13. A ladder 36 feet long rests against a wall, its foot being at a horizontal distance of 25 feet from the base of the wall. What angle does the ladder make with the ground?

14. If a ladder 40 feet long is placed so as to reach a window

30 feet high, what angle does it make with the level ground, and how far is its foot from the base of the building?

15. A ladder 42 feet long is placed so that it will reach a window 30 feet high on one side of a street; if it is turned over, its foot being held in position, it will reach a window 25 feet high on the other side of the street. How wide is the street from building to building?

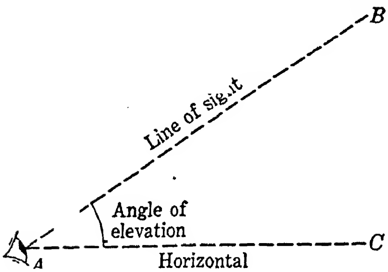


FIG. 12

16. A person on a ship sailing due south at the rate of 15 miles an hour observes a lighthouse due west at 3 p.m. At 5 p.m. the lighthouse is 52° west of north. How far from the lighthouse was the ship at (a) 3 p.m.? (b) 5 p.m.? (c) 4 p.m.?

The **angle of elevation** of an object which is above the eye of an observer is the angle which the line of sight to the object makes with the horizontal. If the object is below the eye of the observer, the angle which the line of sight makes with the horizontal is called the **angle of depression** of the object.

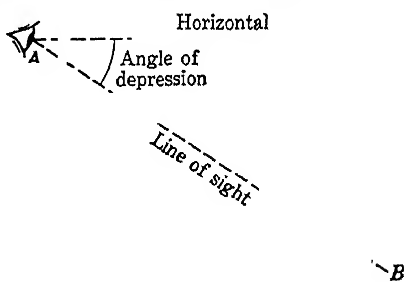


FIG. 13

17. From the top of a cliff 250 feet high the angle of depression of a boat is 10°. How far out is the boat from the foot of the cliff?

18. From a window 30 feet above the level ground, a building 100 feet high, and at a distance of 200 feet, is observed. Find the angle of elevation of the top of the building and the angle of depression of its base.

19. At a point 160 feet from a building, and in a horizontal line with its base, the angle of elevation of the top of the building is 37°. How high is the building?

8. Interpolation.

When an angle such as 18° 47′ cannot be found in the margin of the table on pages 12–14, we can approximate more closely the values of its functions by a process known as **interpolation by proportional parts.** This will be illustrated by means of examples.

Example 1.

Find sin 18° 47′.

Solution. The angle 18° 47′ is between 18° 40′ and 18° 50′. Its sine is between the sines of these two angles. We write the problem in the following form, in which the differences in the angles are shown at the left, and the differences in the values of the function are shown at the right.

$$10' \left\{ \begin{array}{l} \\ 7' \left\{ \begin{array}{l} \\ \\ \end{array} \right. \end{array} \right. \begin{array}{l} \sin 18°\ 50' = .3228 \\ \sin 18°\ 47' = \ \ ? \\ \sin 18°\ 40' = .3201 \end{array} \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} x \qquad .0027$$

Although it is only approximately true, we assume that changes in the function are proportional to changes in the angle. With this assumption, we have

$$\frac{x}{0.0027} = \frac{7}{10} = 0.7, \qquad x = 0.7 \times 0.0027 = 0.00189.$$

We cut this down to four places, since we are dealing with a four-place table, and write $x = 0.0019$. Then,

$$\sin 18°\ 47' = 0.3201 + 0.0019 = 0.3220.$$

This value is correct to four places, as may be verified by consulting more extensive tables.

Example 2.

Find cos 18° 47′.

Solution. The same form of arrangement is used as in example 1. However, it will be noted that the smaller angle has the larger cosine, and to facilitate the subtraction of the functions we

write it above. The quantity x is used, as in example 1, to represent the unknown difference between the function of the smaller angle (not the smaller function) and the function to be found.

$$10' \left\{ 7' \left\{ \begin{array}{l} \cos 18^\circ\, 40' = .9474 \\ \cos 18^\circ\, 47' = \quad ? \end{array} \right\} x \right. \qquad .0009 \atop \cos 18^\circ\, 50' = .9465$$

$$\frac{x}{0.0009} = \frac{7}{10} = 0.7, \qquad x = 0.7 \times 0.0009 = 0.00063.$$

Noting that the function decreases as the angle increases, we have

$$\cos 18^\circ\, 47' = 0.9474 - 0.0006 = 0.9468.$$

If more extensive tables are used, it will be found that the value correct to four places is actually 0.9467.

Likewise, when a function cannot be found exactly in the table, we use inverse interpolation to find the corresponding angle more accurately.

Example 3.

Given $\tan A = 1.1948$; find A.

Solution. The function lies between 1.1918 (corresponding to 50° 00′) and 1.1988 (corresponding to 50° 10′).

$$10' \left\{ \begin{array}{l} \tan 50^\circ\, 10' = 1.1988 \\ x \left\{ \begin{array}{l} \tan A \qquad = 1.1948 \\ \tan 50^\circ\, 00' = 1.1918 \end{array} \right\} .0030 \end{array} \right\} .0070$$

$$\frac{x}{10'} = \frac{0.0030}{0.0070} = 0.4, \qquad x = 4'.$$

$$A = 50^\circ\, 4'.$$

Example 4.

Given $\cos A = 0.7034$; find A.

Solution. The function lies between 0.7030 (corresponding to 45° 20′) and 0.7050 (corresponding to 45° 10′). We write the functions with the largest at the top to facilitate the subtraction.

The quantity x is used to represent the difference between the smaller of the two angles taken from the table and the angle to be found; x will then be the amount to be added to the smaller angle.

$$10' \quad \left.\begin{array}{l} \cos 45^\circ\, 10' = .7050 \\ \cos A = .7034 \end{array}\right\} .0016 \quad .0020$$
$$\cos 45^\circ\, 20' = .7030$$

$$\frac{x}{10'} \quad \frac{0.0016}{0.0020} = 0.8, \qquad x = 8'.$$
$$A = 45^\circ\, 18'.$$

The process of interpolation can be used on any table provided the values are sufficiently close together. For example, it can be used on a table of squares or a table of square roots.

EXERCISES II. B

Find, by interpolation in the table on pages 12–14, the following functions:

1. $\sin 31^\circ\, 14'$. **2.** $\tan 18^\circ\, 6'$. **3.** $\cos 27^\circ\, 18'$.
4. $\cos 39^\circ\, 42'$. **5.** $\sin 55^\circ\, 5'$. **6.** $\cot 43^\circ\, 18'$.
7. $\tan 19^\circ\, 26'$. **8.** $\sin 27^\circ\, 24'$. **9.** $\cos 45^\circ\, 34'$.
10. $\sin 0^\circ\, 3'$. **11.** $\cot 89^\circ\, 51'$. **12.** $\sin 88^\circ\, 22'$.
13. $\tan 88^\circ\, 51'$. **14.** $\cos 74^\circ\, 32'$. **15.** $\cot 65^\circ\, 17'$.

Find angle A by interpolation in the table on pages 12–14, given that

16. $\sin A = 0.4827$. **17.** $\tan A = 0.3899$. **18.** $\cos A = 0.8643$.
19. $\cot A = 2.5626$. **20.** $\tan A = 1.3900$. **21.** $\sin A = 0.3290$.
22. $\sin A = 0.8026$. **23.** $\cos A = 0.3785$. **24.** $\cot A = 0.3785$.
25. $\sin A = 0.0130$. **26.** $\tan A = 0.0130$. **27.** $\sin A = 0.1060$.
28. $\tan A = 0.1060$. **29.** $\cos A = 0.9800$. **30.** $\cot A = 2.0000$.

Solve the following triangles, in which $C = 90^\circ$:

31. $a = 6.84$, $c = 20$. **32.** $a = 23$, $b = 17$.
33. $A = 57^\circ\, 12'$, $c = 0.0286$. **34.** $B = 17^\circ\, 26'$, $b = 92.37$.
35. $a = 18.5$, $c = 37.2$. **36.** $A = 32^\circ\, 24'$, $b = 9.46$.
37. $A = 19^\circ\, 44'$, $a = 22.8$. **38.** $b = 15.4$, $c = 20.2$.

39. $A = 45° 2'$, $b = 8.22$.
40. $B = 15° 53'$, $a = 189$.
41. $a = 0.236$, $c = 1.84$.
42. $a = 17.6$, $b = 16.7$.
43. $A = 11° 1'$, $c = 101.6$.
44. $A = 78° 15'$, $b = 32.22$.
45. $a = 12.34$, $c = 100.3$.
46. $a = 12.34$, $b = 100.3$.

47. A rectangle is 87 feet by 136 feet. Find the length of the diagonal and the angles that it makes with the sides.

48. A surveyor wishes to find the width of a stream without crossing it. He measures a line CB along the bank, C being directly opposite a point A on the farther bank (i.e., angle $ACB = 90°$). The line CB is measured to be 98.25 feet, and the angle ABC to be $55° 56'$. How wide is the stream?

49. Find the height of a vertical pole which casts a shadow 67 feet long on the level ground when the altitude of the sun is $50° 22'$ (i.e., the rays of the sun make an angle of $50° 22'$ with the horizontal).

50. Find the inclination, or angle of ascent, of a road having a $2\frac{1}{2}$ per cent grade (i.e., there is a vertical rise of $2\frac{1}{2}$ feet in a horizontal distance of 100 feet).

51. To measure the height of a building, a surveyor sets up his transit at a distance of 112.2 feet from the building. He finds the angle of elevation of the top of the building to be $48° 17'$. If the telescope of the transit is 5 feet above the base of the building, how high is the building?

52. From the top of a tower 63.2 feet high, the angles of depression of two objects situated in the same horizontal line with the base of the tower, and on the same side of the tower, are $31° 16'$ and $46° 28'$ respectively. Find the distance between the two objects.

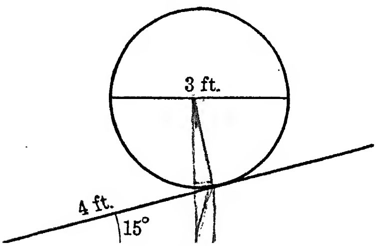


FIG. 14

53. A wheel, 3 feet in diameter, rolls up an incline of 15°. When the point of contact of the wheel with the incline is 4 feet from the base of the incline, what is the height of the center of the wheel above the base of the incline?

54. A roof 20 by 30 feet, the latter being the horizontal dimension,

is inclined at an angle of 30° to the horizontal. Find the angle that a diagonal of the roof makes with the horizontal.

55. A wall extending east and west is 6 feet high. The sun has an altitude of 49° 32′ (see exercise 49) and is 47° 20′ east of south. Find the width of the shadow of the wall on level ground.

56. A 30-foot flagstaff is fixed in the center of a circular tower 40 feet in diameter. From a point in the same horizontal plane as the foot of the tower the angles of elevation of the top of the flagstaff and the top of the tower are found to be 36° and 30° respectively. Find the height of the tower.

57. If, in the preceding exercise, the flagstaff is fixed on the edge of the tower, what is the height of the tower?

58. It is required to measure the height of a tower, CB (Fig. 15), which is inaccessible. From a point A, in the same horizontal plane with the base C, a right angle CAD is turned, and a horizontal line AD, 150 feet in length, is measured. At A the angle of elevation of the top of the tower is 32°, at D the angle of elevation is 28°. Find the height of the tower.

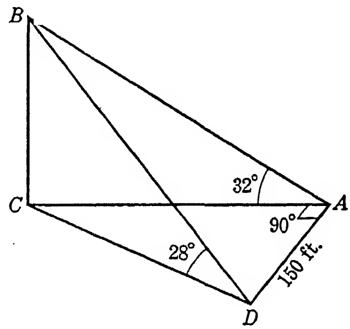


FIG. 15

59. A football player stands at a distance c behind the middle of the goal. He sees the angle of elevation of the nearer crossbar to be u and that of the farther one to be v. Show that the distance between the goals is $c(\tan u \cot v - 1)$.

60. Two points in line with a tower, and in the same horizontal plane with its base, are 160 feet apart. From the point nearer the tower the angle of elevation of the top of the tower is A, from the other point the angle of elevation is B. If $\sin A = 3/5$ and $\cos B = 12/13$, what is the height of the tower?

★9. Components.

The trigonometric functions have direct application in physics and mechanics. A displacement (change of posi-

★ Topics marked with this symbol may be omitted in a short course.

tion), velocity, force, or any other quantity having both magnitude and direction, can be represented by a line having a certain length and a certain direction.

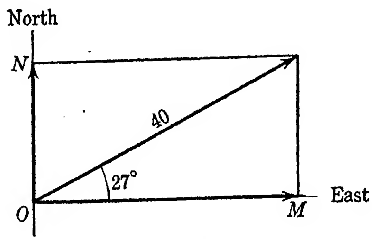


FIG. 16

For example, suppose that an automobile is traveling at the rate of 40 miles an hour along a straight road which makes an angle of 27° to the north of east. Its velocity can be represented by a line OP, 40 units long, extending in the direction shown in Fig. 16. Let M be the projection of P upon an east-west line (that is, the foot of the perpendicular from P to such a line), and let N be its projection on a north-south line. Then,

$$OM = OP \cos 27° = 40 \times 0.8910 = 35.64,$$
$$ON = OP \sin 27° = 40 \times 0.4540 = 18.16.$$

At the end of an hour the automobile will be 35.64 miles east, and 18.16 miles north, of its position at the beginning of the hour. Thus, we may think of its velocity as being composed of an easterly velocity of 35.64 miles an hour and a northerly velocity of 18.16 miles an hour. The projections OM and ON represent the **components** of the velocity represented by OP. We say that OP is **resolved** into its components OM and ON. Conversely, we say that OP is the **resultant** of OM and ON.

Example 1.

A boat, which can travel at the rate of 4 miles an hour in still water, is pointed directly across a stream having a current of 3 miles an hour. What will be the actual speed of the boat, and in what direction will the boat go?

SOLUTION. In still water the boat would go out at right angles to the bank at the rate of 4 miles an hour. But the current carries

it downstream 3 units for every 4 units that it goes across. In Fig. 17, OM represents the velocity of the current, and ON represents the velocity that the boat would have if there were no current. The actual velocity of the boat will be represented by OP. The magnitude of OP is $\sqrt{3^2 + 4^2} = 5$. If A is the angle that OP makes with the bank, then we have $\tan A = \frac{4}{3} = 1.3333$, and $A = 53°$ approximately. That is, the boat will travel at a speed of 5 miles an hour in a direction making an angle of about 53° with the bank.

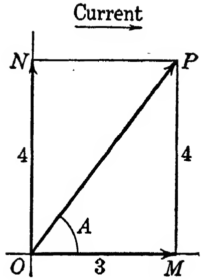


FIG. 17

Example 2.

How must the boat of the preceding example be pointed in order to go straight across the stream?

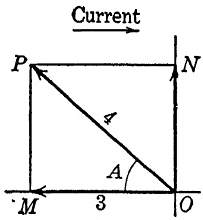


FIG. 18

SOLUTION. The boat must be pointed so that its velocity of 4 miles an hour will have a component parallel to the bank which will exactly offset the effect of the current. That is, it must have an upstream component of 3 miles an hour. From Fig. 18 we see that $\cos A = \frac{3}{4} = 0.75$, and $A = 41.5°$ approximately. Thus, to go straight across the stream, the boat should be pointed at an angle of 41.5° with the upstream direction.

Example 3.

Two forces of 100 pounds and 80 pounds respectively act on a weight as shown in Fig. 19. What will be their horizontal effect, and what will be their vertical or lifting effect?

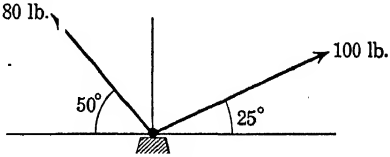


FIG. 19

SOLUTION. The horizontal component of the 100-lb. force is $100 \cos 25° = 90.63$ lb. to the right. The horizontal component of the 80-lb. force is $80 \cos 50° = 51.42$ lb. to the left. Thus, the total horizontal force tending to move the weight to the right is

$$90.63 - 51.42 = 39.21 \text{ lb.}$$

The total lifting force is

$$100 \sin 25° + 80 \sin 50° = 42.26 + 61.28 = 103.54 \text{ lb.}$$

Example 4.

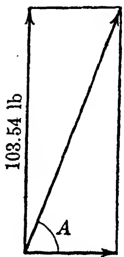


FIG. 20

Find the magnitude and the direction of the resultant force (the single force that is equivalent to the two given forces) in example 3.

SOLUTION. The components of the resultant are 39.21 lb. to the right, and 103.54 lb. upward. The resultant force is

$$\sqrt{(39.21)^2 + (103.54)^2} = 110.7 \text{ lb.}$$

If A is the angle that the resultant makes with the horizontal,

$$\tan A = \frac{103.54}{39.21} = 2.641, \qquad A = 69° 15' \text{ (to nearest } 5').$$

That is, a single force of 110.7 lb., acting at an angle of 69° 15′ with the horizontal and toward the right, will have the same effect as the two given forces.

EXERCISES II. C

1. The westward and southward components of the velocity of a ship are 6.7 knots and 12.5 knots respectively. (See exercise 7.) Find the speed of the ship and the direction in which it is sailing.
2. A force of 150 pounds is acting at an angle of 55° with the horizontal. Find its horizontal and vertical components.
3. A balloon is rising at the rate of 10 feet a second and is being carried horizontally by a wind which has a velocity of 15 miles an hour. Find its actual velocity and the angle that its path makes with the vertical.
4. A boat is being rowed north at the rate of 5 miles an hour, and the tide carries it west at the rate of 3 miles an hour. Find the actual speed of the boat and the direction of its path.
5. A river flows at the rate of 1.5 miles an hour. (a) In what direction must a man swim in order to go straight across, if his

rate of swimming in still water is 2.5 miles an hour? (b) How long will it take him to cross if the river is $\frac{1}{2}$ mile wide?

6. A barge is being towed north at the rate of 15 miles an hour. A man walks across the deck, from west to east, at the rate of 6 feet a second. Find the direction and the magnitude of his actual velocity.

7. A ship is traveling at a speed of 20 knots. (A knot is a nautical mile per hour, a nautical mile being approximately 1.1516 statute miles of 5280 feet each.) When directly opposite a target it fires a gun whose projectile has a velocity of 2000 feet a second. At what angle with the direction of motion of the ship must the gun be pointed in order to hit the target?

8. An airplane which has a speed of 120 miles an hour in calm air is headed southeast. A wind having a velocity of 15 miles an hour is blowing from the southwest. (a) Find the magnitude and the direction of the velocity of the airplane with reference to the ground. (b) How must the airplane be pointed in order to fly southeast, and what will be its actual speed?

9. A weight of 150 pounds is placed on a smooth plane surface which makes an angle of 35° with the horizontal, as shown in Fig. 21. The weight is held in place by a string parallel to the surface and fastened at the top of the plane. Find the pull on the string.

Suggestion. The pull will be equal to the component of the 150-pound weight parallel to the plane.

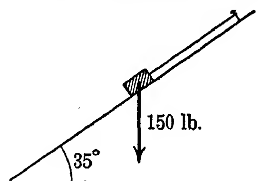


Fig. 21

10. A block is held in position on a smooth inclined plane by means of a string as in Fig. 21. If the pull on the string is 27.3 pounds, and the inclination of the plane is 24° 50′, what is the weight of the block?

★10. Isosceles triangles and regular polygons.

Since the perpendicular from the vertex of an isosceles triangle divides it into two congruent right triangles, we can solve the isosceles triangle by solving one of these right triangles.

To solve a problem involving a regular polygon of n sides, we may first divide it into n congruent isosceles triangles.

Example 1.

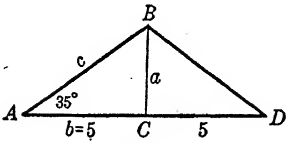


FIG. 22

A garage has a gable roof whose rafters make an angle of 35° with the horizontal. What is the length of a rafter if the width of the garage is 10 feet?

SOLUTION. In Fig. 22, AD represents the width of the garage and AB the length of the rafter.

$$\cos 35° = \frac{b}{c}, \qquad c = \frac{b}{\cos 35°} = \frac{5}{0.8192} = 6.1 \text{ ft.}$$

Example 2.

Find the length of the side of a regular pentagon inscribed in a circle of radius 6 inches.

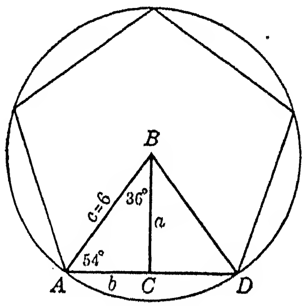


FIG. 23

SOLUTION. Each side of the pentagon subtends a central angle of $\frac{1}{5} \times 360° = 72°$. In Fig. 23, angle $ABC = \frac{1}{2} \times 72° = 36°$, and angle $BAC = 90° - 36° = 54°$. In triangle ABC,

$$\frac{b}{6} = \cos 54°, \qquad b = 6 \cos 54° = 6 \times 0.5878 = 3.527.$$

$$AD = 2b = 7.054 \text{ in.}$$

EXERCISES II. D

1. Each of the equal angles of an isosceles triangle is 40° 15′, the base is 15 inches. Find the remaining parts and the area.
2. Each of the equal sides of an isosceles triangle is 11.52 inches, the vertex angle is 32° 15′. Find the base.
3. The equal sides of a wedge are 4.2 inches, the base is 1.6 inches. Find the angles.

4. Find the radius of a circle in which a 59-foot chord subtends an angle of $12°$ at the center.
5. The radius of a circle is 40 inches, the length of a chord is 70 inches. Find the central angle subtended by the chord.
6. Find the radius of a circle in which a chord of 7.1 inches subtends an angle of $142° 36'$ at the center.
7. Find the chord of a $35°$ arc in a circle of radius 14 inches.
8. Find the length of a belt passing around two pulleys whose radii are 14 inches and 5 inches respectively, and whose distance apart, between centers, is 10 feet.
9. A barn has a gable roof whose rafters are 20 feet long. The width of the barn is 30 feet. Find the angle that the rafters make with the horizontal. Find the area of one of the gable ends (i.e., the triangle in Fig. 24).

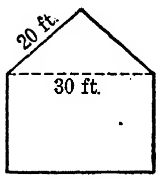


FIG. 24

10. A barn is 30 feet wide by 60 feet long; the rafters make an angle of $40°$ with the horizontal. Find the area of each of the two gable ends and the area of the roof.
11. Find the radius, the apothem (perpendicular distance from the center to a side), and the area of the following regular polygons: (a) a decagon whose side is 10 inches; (b) a 9-sided polygon whose side is 15 inches; (c) a 20-sided polygon whose side is 6.758 inches.
12. The radius of a circle is 100 feet. Find the perimeter and the area of (a) a regular inscribed pentagon; (b) a regular inscribed decagon; (c) a regular circumscribed pentagon; (d) a regular circumscribed decagon.
13. The area of a regular pentagon is 560 square feet. Find the radii of the circumscribed and inscribed circles.
14. A metal nut $\frac{3}{4}$ inch thick is in the shape of a regular hexagon, the distance between the parallel sides being $1\frac{3}{4}$ inches. The circular hole through the center is $\frac{3}{4}$ inch in diameter. Find the amount of metal in the nut.
15. Show that the area of a regular polygon of n sides circumscribed about a circle of radius r is

$$nr^2 \tan \frac{180°}{n}$$

16. Show that the perimeter of a regular polygon of n sides inscribed in a circle of radius r is

$$2nr \sin \frac{180°}{n}$$

*11. Solution of oblique triangles by means of right triangles.

Oblique triangles can always be solved by breaking them up into right triangles. The following examples illustrate the methods used in the four typical cases which arise. Usually, however, it will be found more convenient to employ other methods and formulas for solving oblique triangles. These will be developed in a later chapter.

Case I. Two angles and a side given.

Example 1.

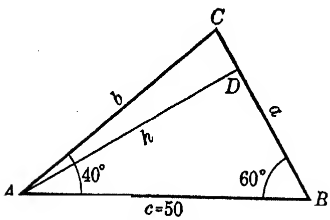


FIG. 25

In the triangle ABC, $A = 40°$, $B = 60°$, $c = 50$. Find the remaining parts.

SOLUTION. $C = 180° - (A + B) = 180° - (40° + 60°) = 80°$. Draw the altitude from one end of the known side. Suppose that this altitude is $AD = h$ (Fig. 25). Then, in the right triangle ABD, $h = 50 \sin 60° = 43.30$.

Now, in the right triangle ADC,

$$b = \frac{h}{\sin C} = \frac{43.30}{\sin 80°} = 44.0.$$

Side a may be found in a similar manner by drawing the altitude from B, or by computing the segments BD and DC and adding them.

Case II. Two sides and the angle opposite one of them given. (See discussion, section 53, pages 84–86.)

Example 2.

Given $A = 75°$, $a = 20$, $b = 10$; find B, C, c.

Solution. Draw the altitude $CD = h$ (Fig. 26). (The altitude must not be drawn from the vertex of the known angle.) In the right triangle ADC,

$$h = b \sin A = 10 \sin 75° = 9.659.$$

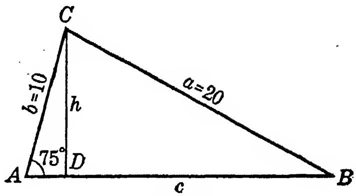


Fig. 26

In the right triangle BDC,

$$\sin B = \frac{h}{a} = \frac{9.659}{20} = 0.48295, \qquad B = 28° 53'.$$

$$C = 180° - (A + B) = 180° - 103° 53' = 76° 7'.$$

Side c may be similarly found by drawing the altitude from B, or by computing the segments AD and DB and adding.

Case III. Two sides and the included angle given.

Example 3.

Given $a = 25$, $b = 30$, $C = 50°$; find the other parts.

Solution. Draw an altitude to one of the known sides, preferably the larger. Suppose that this altitude is $BD = h$, and that it divides the side BC into the segments $CD = m$ and $DA = n$ (Fig. 27). Then,

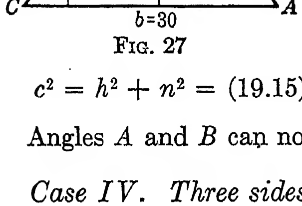


Fig. 27

$$h = a \sin C = 25 \sin 50° = 19.15,$$
$$m = a \cos C = 25 \cos 50° = 16.07,$$
$$n = b - m = 30 - 16.07 = 13.93,$$

$$c^2 = h^2 + n^2 = (19.15)^2 + (13.93)^2 = 560.8, \qquad c = 23.7.$$

Angles A and B can now be found quite easily.

Case IV. Three sides given.

Example 4.

The three sides of a triangle are $a = 5$, $b = 6$, $c = 9$. Find the angles.

SOLUTION. Draw an altitude to one of the sides, preferably the largest. Suppose that this altitude h divides the side AB into segments $AD = m$ and $DB = n$ (Fig. 28). Then,

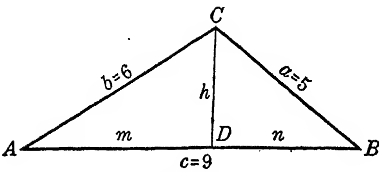


FIG. 28

$$h^2 = 36 - m^2 = 25 - n^2,$$
$$m^2 - n^2 = 36 - 25 = 11,$$
$$(m + n)(m - n) = 11.$$

But,

$$m + n = 9,$$

and consequently, $m - n = \dfrac{11}{9}.$

Solving these simultaneous equations, we get

$$m = \frac{46}{9}, \qquad n = \frac{35}{9}.$$

$$\cos A = \frac{m}{b} = \frac{23}{27} = 0.8519, \qquad A = 31.6°;$$

$$\cos B = \frac{n}{a} \qquad = 0.7778, \qquad B = 39.0°;$$

$$C = 180° - (A + B) = 180° - 70.6° = 109.4°.$$

EXERCISES II. E

Solve the following triangles:

1. $A = 30°, B = 80°, a = 15.$
2. $A = 35°, b = 17, c = 32.$
3. $A = 70°, a = 8, c = 5.$
4. $B = 100°, C = 30°, b = 75.$
5. $a = 2.3, b = 1.5, c = 1.6.$
6. $a = 26, c = 40, B = 62°.$
7. $C = 100°, a = 82, c = 105.$
8. $a = 95, b = 102, c = 150.$
9. From the top of a hill, the angles of depression of two successive milestones on a level road, which leads straight away from the hill, are 5° and 15° respectively. Find the height of the hill.

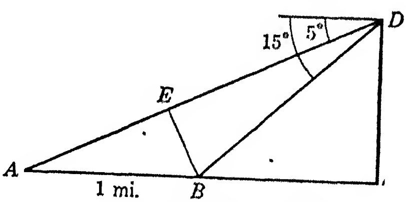


FIG. 29 (not drawn to scale).

SUGGESTION. In Fig. 29 BE is drawn perpendicular to AD. Find BE, then BD, finally CD.

10. At a certain horizontal distance from the foot of a vertical cliff, the angle of elevation of the top of a flagpole 50 feet tall standing on the edge of the cliff is 40°. From the same position, the angle of elevation of the foot of the pole is 35°. How high is the cliff?

11. At a certain point, the angle of elevation of the top of a flagpole, which stands on level ground, is 35°. Seventy-five feet nearer the pole, the angle of elevation is 50°. How high is the pole?

12. Solve the preceding exercise if the angles of elevation are 30° and 45° respectively.

13. From a window 30 feet above the street, the angle of depression of the curb on the near side of the street is 50°, that of the curb on the far side is 13°. How wide is the street from curb to curb?

14. At a point in the same horizontal plane with the foot of a vertical cliff 150 feet high, the angles of elevation of the top and the bottom of a flagpole standing on top of the cliff are 20° and 16° respectively. Find the height of the pole.

15. Points A and B are on opposite sides of a lake. At a point C, which is 456 feet from A and 580 feet from B, the angle subtended by the line AB is $44° 35'$. Find the distance from A to B.

16. The sides of a triangle are 20, 25, and 30. Find the length of the altitude to the longest side.

CHAPTER

Approximate Numbers and Computation

12. Approximate numbers.

An **approximate number** is one which differs slightly from the exact number for which it stands. In trigonometry we deal almost entirely with approximate numbers. With certain exceptions (e.g., $\sin 30° = \frac{1}{2} = 0.5$), all of the tabulated values of the trigonometric functions are approximations. Thus, when we write

$$\sin 45° = \frac{\sqrt{2}}{2} = 0.7071,$$

we do not mean that sin 45° is exactly equal to 0.7071, but that 0.7071 is the four-place number which is closest to the value of sin 45°.

All measurements are approximate numbers. When we measure a line to the nearest tenth of an inch and say that its length is 18.3 inches, we mean that the length is between 18.25 inches and 18.35 inches.

13. Rounding off numbers.

It is often desirable to reduce an approximate number to one of less accuracy. This process is called **rounding off** the number. In rounding off a number we choose the nearest number having the desired number of places. Thus, if we round off 4.2537 to thousandths, we get 4.254. If we round it off to hundredths, we get 4.25.* To tenths, the number is 4.3.

* Here it would be best to write 4.25+. Similarly, in rounding off the

In rounding off a number ending in 5, to a number having one less digit, it is customary to *make the resulting number end in an even digit.* Thus, 17.25 becomes 17.2, while 17.75 becomes 17.8.

*14. Error.

The difference between an approximate value of a quantity and its exact or true value is the **absolute error** of the approximate value. In the approximate number 18.3, the maximum absolute error is 0.05, since 18.3 cannot be less than 18.25 or greater than 18.35. The **relative error** is the quotient of the absolute error divided by the true value. (Ordinarily the true value is not ascertainable, and we are forced to use the approximate value for the divisor. This does not make an appreciable difference in the quotient.) The maximum relative error in the example just given is $0.05/18.3 = 0.003$, or 0.3 per cent.

Relative error is independent of the position of the decimal point. Thus, a measurement of 1.83 inches, although accurate to hundredths, is relatively no more accurate than a measurement of 18.3 inches. For the maximum relative error of the approximate number 1.83 is $0.005/1.83 = 0.003$, and this is exactly the same as the maximum relative error of 18.3.

15. Significant figures.

The illustration of the preceding section indicates that relative accuracy does not depend upon the number of decimal places or upon the position of the decimal point, but upon the number of significant figures that the number contains. A **significant figure** is any one of the digits from 1 to 9 inclusive, and 0 except when it is used to fix the decimal point or to fill the places of unknown or discarded digits.

number 6.347, it would be best to write 6.35−. This is helpful if the number is to be rounded off still further.

The 0's in 0.75 and 0.0024 are not significant figures.

The 0 in 6.80 is a significant figure. In this connection, note that 6.80 means a number between 6.795 and 6.805, whereas 6.8 means a number between 6.75 and 6.85. The number 6.80 has three significant figures, and is more accurate than 6.8, which has only two.

The significance of 0's at the right of a whole number is doubtful. For example, if it is stated that a man's income for a certain calendar year is $5000, it is impossible to say, without further information, which, if any, of the 0's are significant figures. If his income tax return were available and showed his income to be $5043.75, the first 0 in the $5000 would be significant but the other two would not. If the return showed his income to be $5122.80, none of the 0's in the $5000 would be significant.

16. Scientific notation.

The **leading digit** of a number is the first non-zero digit from the left (i.e., the first significant figure). A number is said to be expressed in **scientific notation** when it is written as the product of a number having the decimal point just after the leading digit, and a power of 10. (When the decimal point is just after the leading digit it may be said to be in **standard position.**)

The method of changing from the usual to the scientific notation is illustrated by the following examples:

$$237.65 = 2.3765 \times 100 = 2.3765 \times 10^2,$$
$$0.0054 = 5.4 \div 1000 = 5.4 \times 10^{-3}.$$

It is possible to indicate, by writing a number in scientific notation, whether the 0's at the right of a number are significant. Thus, if in the number 1,300,000 the first two 0's are significant but the last three are not, we could write the number in the form 1.300×10^6.

EXERCISES III. A

1. Round off the following numbers to one less decimal place: 12.34, 29.87, 4.06, 1.396, 0.251, 0.215, 68.2, 63.25, 1.9999, 1.9995, 2.355, 2.345, 2.354, 2.350.
2. Round off the following numbers (a) to three decimal places, (b) to three significant figures: 1.2464, 0.5864, 12.9065, 12.9055, 2.3505, 16.0031, 0.003664.
3. Find the maximum relative error in each of the following approximate numbers: 24.2, 105.16, 38.985, 0.002, 0.00025.
4. How many significant figures are there in each of the following numbers? 39.46, 1.004, 1.400, 0.0014, 100.03, 0.00005, 123892, 200.0.
5. Underline the significant 0's in the following numbers, and put a question mark under each doubtful 0: 10.02, 10.20, 0.20, 0.02, 0.020, 25000, 2506, 0.00300, 0.20500, 20500.
6. Express the following numbers in scientific notation: 256835, 0.000232, 0.000,000,006, 3876.5, 984.876, 1,462,817.
7. Write each of the following numbers in ordinary notation: 1.8×10^7, 2.35×10^{-5}, 8.482×10^8, 3.7×10^{-9}.

*17. Addition and subtraction of approximate numbers.

When two or more approximate numbers are added, the sum cannot be more accurate than the least accurate of the numbers. (This is in the sense of absolute accuracy, not relative accuracy.) For example, consider the sum of the numbers 2.3683, 81.02, 0.0457. The sum cannot be accurate beyond hundredths, so some of the numbers can be rounded off. We carry them, whenever possible, to one more place than the least accurate number, on the theory that the errors in these numbers tend to compensate for each other (that is, that positive and negative errors occur in nearly equal proportions). Thus, we write

$$\begin{array}{r} 2.368 \\ 81.02 \\ 0.046 \\ \hline 83.434 \end{array}$$

The sum should be rounded off to hundredths, giving 83.43.

The above remarks apply also to subtraction.

★18. Multiplication of approximate numbers.

Suppose that the sides of a rectangle are measured as 5.73 and 6.42 inches respectively. The area would be found by multiplying these numbers together; thus,

$$\text{area} = 5.73 \times 6.42 = 36.7866.$$

However, this result is not accurate to as many significant figures as are given. For the approximate number 5.73 means some value between 5.725 and 5.735; similarly, 6.42 means a value between 6.415 and 6.425. Therefore we can merely say that the area is between

$$5.725 \times 6.415 = 36.725875,$$
$$\text{and} \quad 5.735 \times 6.425 = 36.847375.$$

Therefore, in the product 36.7866 we retain only three significant figures, namely 36.8; even then the last digit is not absolutely certain.

In general, we are not justified in retaining more significant figures in a product calculated from approximate numbers than the least accurate of the factors which go to make up the product. Thus, we round off all the factors to the number of such figures in the least accurate factor. The multiplication can then be performed in contracted form, in which the partial products are carried just one place beyond the last place which is to be retained. The following illustration of the multiplication of 6.42 by 5.73 exhibits the method:

$$\begin{array}{r} 6.42 \\ \underline{5.73} \\ 32.10 \\ 4.49 \\ \underline{.19} \\ 36.78 \end{array}$$

The first partial product is obtained by multiplying the multiplicand, 6.42, by the leading digit, 5, of the multiplier; thus, $5 \times 6.42 = 32.10$.

Multiplying by the next digit of the multiplier, we have $7 \times 2 = 14$, and we should write the 4 one place to the right of the 0 in 32.10, and on the next line below, carrying the 1. However, we do not write down the 4, as it does not contribute to the accuracy of our final product, but merely carry the 1. In this way, we find 4.49 as our second partial product.

Before finding our third partial product, we strike out the 2 in the multiplicand. Then we find that $3 \times 4 = 12$, and carry the 1 to add to 3×6. Thus, the third partial product is .19.

The sum of the partial products is rounded off to three significant figures, giving 36.8 as the final product.

★19. Division of approximate numbers.

As in multiplication, so in division, we can show that in general it is useless to retain more figures in the quotient than the number of significant figures in the less accurate of the two numbers, dividend and divisor. Consequently, we note which of these contains the fewer significant figures, and round the other off to the same number of such figures. If, after this has been done, the dividend, *without regard to the decimal point*, is less than the divisor, we restore one digit to the dividend. (See example below.) The quotient is carried to the same number of significant figures as are contained in the divisor. A contracted form of the division process as applied to the example $36.78 \div 6.42$ is shown on page 42.

Here, if the dividend were rounded off to 368 (decimal point omitted), it would be less than the divisor, 642. Hence, we retain four, rather than three, figures in the dividend.

$$\begin{array}{r}
5.73 \\
6.42\overline{)36.78} \\
32\ 10 \\
\hline
4\ 68 \\
4\ 49 \\
\hline
19 \\
19
\end{array}$$

After the first partial product ($5 \times 642 = 3210$) has been subtracted, we do not bring down a 0 from the dividend, but strike out the final digit, 2, in the divisor.

The next digit in the quotient will obviously be 7. We note that $7 \times 2 = 14$, but do not write down the 4; we merely carry the 1. The partial product is 449.

The process is continued as far as possible, cutting down the divisor by one digit at each stage. The final quotient is 5.73.

★20. Square root.

It will be assumed that the student is familiar with the method of extracting square root learned in arithmetic. How a table of squares, such as is to be found in Table VI of the Macmillan Logarithmic and Trigonometric Tables, can be used to expedite the process will be illustrated by extracting the square root of 1350 (considered as an exact, not an approximate, number).

$$\begin{array}{rl}
 & 1\acute{3}5\acute{0}.0\acute{0}\ (36.7 \\
(367)^2 = & 1346\ 89 \\
2 \times 367 = 734 & \overline{|\ 3\ 11}
\end{array}$$

After separating the number into groups of two digits each, starting at the decimal point and going both to left and to right, we note that the largest square contained in the group at the left, namely 13, is the square of 3. Turning to the 300's of Table VI, we find that the largest square just below 135000 is 134689, which is the square of 367.

Subtracting the square of 367, we have a remainder of 311. This is the process previously learned, except that we have subtracted the square of a three-digit number instead of that of a one-digit number.

The process may now be continued as usual. It may be noted, however, that if we have obtained k significant figures in the square root, then $k - 1$ more may be obtained by division. Thus, in the present example, we may divide 311 by 734 and obtain two more significant figures in the square root.

*21. Use of calculating machines.

If a calculating machine is available, the contracted forms of multiplication and division are of course not used. All that has been said about significant digits, however, holds. For example, it would be absurd to carry the quotient of $36.78 \div 6.42$ out to eight or ten figures, even though the division could easily be performed on a machine.

While it is possible to extract square root on a calculating machine, an effective method is to use a table of squares, such as Table VI,* in conjunction with a machine, employing the machine to perform the final division.

EXERCISES III. B

Perform the following operations, retaining the proper number of significant figures:

1. 35.8×41.6.
2. 5.25×48.4.
3. 14.26×3.860.
4. 529.6×29.64.
5. 5028×46.09.
6. 0.1283×127400.
7. $43.8 \times 13.1 \times 32.8$.
8. $0.532 \times 0.00567 \times 12.3$.
9. 13845×89.763.
10. $7.283 \times 283.4 \times 5.437$.
11. $63.1 \div 21.5$.
12. $0.5929 \div 3.801$.
13. $52.96 \div 1.895$.
14. $2.451 \div 1903$.
15. $2500 \div 16.98$.
16. $32.17 \div 712.3$.
17. $(436.5)^2$.
18. $(71.48)^2$.

* Or Barlow's Tables.

19. $\dfrac{35.8 \times 9.86}{136}$. **20.** $\dfrac{12.34 \times 1.}{286.4}$

Extract the square roots of the following quantities, carrying the results to four significant figures:

21. 1683. **22.** 25648. **23.** 17.986.
24. 0.01534. **25.** 0.6843. **26.** 1.0076.

CHAPTER IV

Logarithms

22. Logarithms.

*The **logarithm** of a number to a given base is the exponent of the power to which the base must be raised to yield the number.* It is assumed that the base is positive and different from 1, and that the number is positive.

Thus, since $2^3 = 8$, 3 is the logarithm of 8 to the base 2. This may be written in the form $\log_2 8 = 3$. More generally, we write

$$\log_b N = x, \tag{1}$$

where $$b^x = N \quad (b > 0, \neq 1; N > 0). \tag{2}$$

Forms (1) and (2) are equivalent.

The base in most common use is 10. Since, for example, $10^2 = 100$, we have $\log_{10} 100 = 2$. As we shall deal almost exclusively with logarithms to the base 10 (that is, **common logarithms**), we shall omit the subscript indicating the base, and write simply $\log 100 = 2$. Thus,

$$\begin{array}{rclcrcr}
10^3 &=& 1000, & \text{or} & \log 1000 &=& 3; \\
10^2 &=& 100, & \text{or} & \log 100 &=& 2; \\
10^1 &=& 10, & \text{or} & \log 10 &=& 1; \\
10^0 &=& 1, & \text{or} & \log 1 &=& 0; \\
10^{-1} &=& 0.1, & \text{or} & \log 0.1 &=& -1; \\
10^{-2} &=& 0.01, & \text{or} & \log 0.01 &=& -2; \\
10^{-3} &=& 0.001, & \text{or} & \log 0.001 &=& -3.
\end{array}$$

The logarithms of integral powers of 10, such as the foregoing, can, because of the very meaning of logarithm, be

expressed exactly. Although the logarithm of a number such as 3, for example, cannot be expressed exactly in the decimal notation, we assume that a number x exists for which $10^x = 3$, and that an approximation to this number can be found. Actually, such an approximation, to five decimal places, is 0.47712, and we write $\log 3 = 0.47712$. Similarly, $\log 3.262 = 0.51348$. (How these values are obtained from tables will be explained later.)

23. Mantissa.

Assuming that

$$\log 3.262 = 0.51348,$$

let us write

$$10^{0.51348} = 3.262. \qquad (1)$$

Multiplying both sides by 10, we get

$$10^{1.51348} = 32.62,$$

which, in logarithmic notation, is

$$\log 32.62 = 1.51348.$$

By dividing both sides of (1) by 10, we get

$$10^{0.51348-1} = 0.3262,$$

or

$$\log 0.3262 = 0.51348 \quad 1.$$

This could also be written $\log 0.3262 = -0.48652$,* but it is usually more convenient to keep the decimal part of a logarithm positive. This positive decimal part of a logarithm is called the **mantissa** of the logarithm.

The two examples given above illustrate the fundamental principle: *For numbers having the same sequence of digits,* such as 3.262, 32620, 0.003262, *the mantissa of the logarithm is the same.*†

* Found by subtracting 0.51348 from 1 and prefixing a negative sign.
† Provided that the base is 10.

24. Characteristic.

The integral, or whole-number, part of a logarithm is called the **characteristic.** Thus, since log 32.62 = 1.51348, the characteristic of the logarithm of 32.62 is 1.

Since log 1 = 0, and log 10 = 1, the logarithm of a number between 1 and 10, for example 3.262, is between 0 and 1 in value, and consequently has the characteristic 0.* We shall say that such a number has the decimal point in **standard position,** namely after the first non-zero digit. (See section 16.)

Each time we multiply a number by 10 we move the decimal point one place to the right, and each time we divide by 10 we move the point one place to the left. But each time we multiply a number by 10 we increase the logarithm of the number by 1, and each time we divide a number by 10 we decrease its logarithm by 1, as was seen in the illustration above. Thus, we may state the following rule for finding the characteristic:

If a number has its decimal point in standard position (i.e., after the first non-zero digit), *the characteristic of the logarithm of the number is zero; if the decimal point is not in standard position, the characteristic of the logarithm of the number is equal to the number of places the point has been moved from standard position, and is positive if the point has been moved to the right, negative if it has been moved to the left.*†

For example, in the number 78460, the decimal point has been moved from standard position (after the 7) 4 places to the right (after the 0), and the characteristic of the logarithm of the number is therefore 4.

In the number 0.03262, the point has been moved from standard position 2 places to the left. The characteristic of the logarithm of the number is therefore −2. In fact,

* A characteristic should always be written, even though it is 0.

† Note that *the characteristic is also equal to the exponent of 10 when the number is written in scientific notation.* (See section 16.)

since we saw above that $\log 3.262 = 0.51348$, we may write

$$\log 0.03262 = 0.51348 - 2.$$

It is frequently convenient to write this in the form

$$\log 0.03262 = 8.51348 - 10.$$

The rule given for determining the characteristic also tells us how to point off a number corresponding to a given logarithm. (The number corresponding to a logarithm is called the **antilogarithm.** More precisely, if $\log N = x$, then N is the antilogarithm of x.)

Thus, if we have given

$$\log N = 2.51348,$$

we know from the illustration above that the number N is composed of the sequence of digits 3262. Since the characteristic is 2, the decimal point has been moved 2 places to the right from standard position. Therefore,

$$N = 326.2.$$

EXERCISES IV.

Determine the characteristic of the logarithm of:

1. 436. **2.** 25. **3.** 3280.
4. 4. **5.** 0.136. **6.** 0.2.
7. 0.42. **8.** 0.04. **9.** 0.0075.
10. 1.0075. **11.** 0.1075. **12.** 52.684.
13. 21.64. **14.** 384.6. **15.** 2500.
16. 0.384. **17.** 8.124. **18.** 0.2960.
19. 380000. **20.** 0.006934. **21.** 0.02796.
22. 7.952. **23.** 98. **24.** 98.5.
25. 98.52. **26.** 985. **27.** 9852.
28. 0.9852. **29.** 0.985. **30.** 0.98.
31. 0.098. **32.** 0.000,001,2. **33.** 60,000,000.
34. 6. **35.** 0.6. **36.** 0.600.

25. Finding the mantissa.

In a standard five-place table of logarithms, such as Table I of the Macmillan Logarithmic and Trigonometric Tables, the first three digits of a number are found at the left of the page, the fourth digit at the top or bottom, the corresponding mantissa (decimal point omitted) being in the same row as the first three digits of the number and in the same column as the fourth digit. The student should verify that the mantissa of the logarithm of 3262 is .51348.

To find the logarithm of a number composed of five digits we must use interpolation. (See section 8.)

Example.

Find log 32.627.

SOLUTION. Find the mantissas for the numbers next above and next below 32.62:

Number — **Mantissa** (decimal point omitted)

$$.010\left[\begin{array}{r}32.630\\ .007\left[\begin{array}{l}32.627\\32.620\end{array}\right.\end{array}\right. \qquad \left.\begin{array}{l}51362\\ \left.\begin{array}{c}?\\51348\end{array}\right]x\end{array}\right]14$$

Assuming that the change in the mantissa is proportional to the change in the number,* we have

$$\frac{x}{14} = \frac{0.007}{0.010} = 0.7,$$

$$x = 0.7 \times 14 = 9.8.$$

$$\text{Mantissa} = 51348 + 10 = 51358.$$

$$\log 32.627 = 1.51358.$$

Once the principle of proportionality or proportional parts is understood, the work can be arranged more com-

* This is only approximately true.

pactly in some such way as the following, or may be performed mentally.

$$\begin{array}{rr}
32.63 \sim & 51362 \\
32.62 \sim & \underline{51348} \\
\text{difference} = & 14 \\
& \times \underline{0.7} \\
& 9.8 \\
& \underline{51348} \\
\log 32.627 = & 1.51358
\end{array}$$

(The symbol $\sim$ may here be read "corresponds to.")

EXERCISES IV. B

Find the logarithm of each of the following numbers:

1. 68.	**2.** 68.3.	**3.** 359.
4. 381.	**5.** 2.	**6.** 2.87.
7. 5000.	**8.** 751.5.	**9.** 8428.
10. 0.4313.	**11.** 0.02156.	**12.** 56980.
13. 250000.	**14.** 0.00036.	**15.** 7.851.
16. 1.003.	**17.** 15.95.	**18.** 0.003097.
19. 2.9645.	**20.** 23572.	**21.** 6784.8.
22. 67.843.	**23.** 54326.	**24.** 38.794.
25. 6.3129.	**26.** 0.34732.	**27.** 0.000,876,95.
28. 1.0006.	**29.** 9.9982.	**30.** 99.992.
31. 99998.	**32.** 0.10101.	**33.** 0.000,100,01.
34. 2509.9.	**35.** 829.99.	**36.** 91.119.

26. Finding the antilogarithm.

The process of finding the number corresponding to a given logarithm is illustrated by the following examples:

Example 1.

Find the number whose logarithm is $7.91121 - 10$.

SOLUTION. The mantissa is found exactly in the table. At the left we find 815; at the top we find 1. Thus, the number is composed of the sequence of digits 8151. The characteristic is $7 - 10 = -3$. Consequently, the decimal point must be moved from

standard position (after the 8) 3 places to the left. Therefore the number is 0.008151.

Example 2.

Given $\log N = 1.91123$; find N.

Solution. Here we use inverse interpolation.

$$\begin{array}{c} \textbf{Mantissa} \\ 5\left[\begin{array}{r} 91126 \\ 2\left[\begin{array}{r} 91123 \\ 91121 \end{array}\right. \end{array}\right. \end{array} \qquad\qquad \begin{array}{c} \textbf{Number} \\ \left.\begin{array}{r} 8152 \\ \left.\begin{array}{r} ? \\ 8151 \end{array}\right] x \end{array}\right] 1 \end{array}$$

$$\frac{x}{1} = \frac{2}{5} = 0.4.$$

$$N = 81.514.$$

EXERCISES IV. C

Find the number corresponding to each of the following logarithms:

1. 0.69897.	**2.** 1.76042.	**3.** 2.93601.
4. 4.26174.	**5.** 0.81278 − 1.	**6.** 9.96741 − 10
7. 3.76253 − 10.	**8.** 3.63337.	**9.** 8.84442 − 10
10. 0.63994.	**11.** 0.69085 − 2.	**12.** 1.51416.
13. 7.19767 − 10.	**14.** 1.48762.	**15.** 8.82326 − 10
16. 5.18752.	**17.** 6.15465.	**18.** 9.79029 − 10
19. 0.83445.	**20.** 6.36021 − 10.	**21** 1.94548.
22. 9.00000 − 10.	**23.** 1.00009.	**24.** 0.99998.

27. Laws of logarithms.

Since logarithms are exponents, they obey the laws of exponents, it being assumed that these laws hold for irrational as well as rational exponents.*

I. The logarithm of a product is equal to the sum of the logarithms of its factors.

* See the author's *College Algebra.*

Let $\log_b M = x, \qquad \log_b N = y.$

Then, $M = b^x, \qquad N = b^y,$

$$MN = b^x b^y = b^{x+y},$$

$$\log_b MN = x + y,$$

or $$\mathbf{\log_b MN = \log_b M + \log_b N.}$$

The proof can easily be extended to cover the case of any finite number of factors.

II. The logarithm of a quotient is equal to the logarithm of the dividend minus the logarithm of the divisor.

Using the same notation as above, we have

$$\frac{M}{N} = \frac{b^x}{b^y} = b^{x-y},$$

$$\log_b \frac{M}{N} = x - y,$$

or $$\mathbf{\log_b \frac{M}{N} = \log_b M - \log_b N.}$$

III. The logarithm of the mth power of a number is equal to m times the logarithm of the number.

If $\log_b N = x$, then $N = b^x$, and

$$N^m = (b^x)^m = b^{mx},$$

$$\log_b N^m = mx,$$

or $$\mathbf{\log_b N^m = m \log_b N.}$$

IV. The logarithm of the mth real positive root of a number is equal to one mth of the logarithm of the number.

This is really the same as III, since $\sqrt[m]{N} = N^{1/m}$. Thus,

$$\mathbf{\log_b \sqrt[m]{N} = \frac{1}{m} \log_b N.}$$

28. Logarithmic computation of products and quotients.

The advantage of logarithms in performing multiplication and division is that these operations can be replaced

by the simpler operations of addition and subtraction respectively.

It must be realized that results are only approximate.

Example 1.

Find the value of 32.62×8.673.

Solution. Denoting the product by x, we have

$$\log x = \log 32.62 + \log 8.673.$$

We arrange the work as follows:

$$\begin{array}{rl} \log 32.62 & 1.51348 \\ \log 8.673 & 0.93817 \\ \hline \text{sum} = \log x & 2.45165 \\ x & 282.9 \end{array}$$

Example 2.

Find the value of $8.673 \div 32.62$.

Solution. Let the quotient be denoted by x. Then

$$\log x = \log 8.673 - \log 32.62.$$

$$\begin{array}{rl} \log 8.673 & 0.93817 \\ \log 32.62 & 1.51348 \\ \hline \end{array}$$

Here we are subtracting the larger quantity from the smaller. In order to keep the mantissa positive, we add 10 to, and subtract 10 from, the logarithm of 8.673, getting

$$\begin{array}{rl} \log 8.673 & 10.93817 - 10 \\ \log 32.62 & 1.51348 \\ \hline \text{difference} = \log x & 9.42469 - 10 \\ x & 0.2659 \end{array}$$

Example 3.

Find the value of

$$\frac{3262 \times 1.786}{532.1 \times 0.8673}.$$

SOLUTION. We note that

$$\log \text{fraction} = \log \text{numerator} - \log \text{denominator},$$

and arrange the work as follows:

log 3262	3.51348
log 1.786	0.25188
log numerator	3.76536
log denominator	2.66416
log fraction	1.10120
fraction	12.62

log 532.1	2.72599
log 0.8673	9.93817 − 10
log denominator	12.66416 − 10

Note that we do not interpolate to find a fifth figure in the antilogarithm because of the rules for computation with approximate numbers.

29. Cologarithm.

When one number is to be divided by another we may change the problem to one of multiplication by using the reciprocal of the divisor. For example, $3 \div 2 = 3 \times \frac{1}{2}$.

The logarithm of the reciprocal of a number is called the **cologarithm** of the number and is abbreviated **colog.** That is,

$$\text{colog}\, N = \log \frac{1}{N} = \log 1 - \log N = -\log N.$$

Thus, *the cologarithm of a number is the negative of the logarithm of the number.* Consequently, in solving a problem in division by means of logarithms, we may either subtract the logarithm of the divisor or add its cologarithm. There is no advantage, but rather a disadvantage, in using the cologarithm when only two numbers are involved in a division problem. There is, however, some advantage, particularly in the arrangement of the solution, when more than one number occurs in the denominator of an expression.

The cologarithm of a number is obtained by subtracting

the logarithm of the number from log 1, that is, from 0. The 0 is usually written in the form 10 − 10, and the subtraction can be performed mentally, after some practice, by the following method: *Begin at the left, and subtract from 9 each digit of the logarithm except the last non-zero digit, which must be subtracted from 10.*

Examples.

log 32.62 = 1.51348, log 0.01508 = 8.17840 − 10,

colog 32.62 = 8.48652 − 10, colog 0.01508 = 1.82160.

Following is a solution of example 3 above which employs cologarithms:

log 3262	3.51348
log 1.786	0.25188
colog 532.1	7.27401 − 10
colog 0.8673	0.06183
log fraction	11.10120 − 10
fraction	12.62

30. Logarithmic computation of powers and roots.

The operations of raising to powers and of extracting roots are greatly facilitated by the use of logarithms, because it replaces these operations by the simpler ones of multiplication and division.

Example 1.

Evaluate $(3.262)^4$.

SOLUTION. Let $x = (3.262)^4$; then $\log x = 4 \log 3.262$.

log 3.262	0.51348
	× 4
log x	2.0539*
x	113.2

* Only five significant figures are retained here because of the rules for computation with approximate numbers.

Example 2.

Find the cube root of 3.262.

SOLUTION. If x is the desired cube root, then

$$\log x = \frac{1}{3} \log 3.262.$$

$$\begin{array}{rl} \log 3.262 & \underline{0.51348} \ (\div 3 \\ \log x & 0.17116 \\ x & 1.4831 \end{array}$$

Example 3.

Find the cube root of 0.3262.

SOLUTION. If x is the desired cube root, then

$$\log x = \frac{1}{3} \log 0.3262 = \frac{1}{3} (9.51348 - 10).$$

In order to make the negative part of the characteristic exactly divisible by 3, add 20 and subtract 20:

$$\begin{array}{rl} \log 0.3262 & \underline{29.51348 - 30} \ (\div 3 \\ \log x & 9.83783 - 10 \\ x & 0.68838 \end{array}$$

EXERCISES IV. D

Find the value of each of the following expressions by means of logarithms:

1. 41.6×35.8.
2. 4.84×5.25.
3. $41.6 \div 35.8$.
4. $4.84 \div 5.25$.
5. 529.6×29.64.
6. 127400×0.1283.
7. 123.4×9.866.
8. $(3.482)^3$.
9. $5.832 \div 25.96$.
10. $7.283 \times 283.4 \times 5.437$.
11. $\sqrt{26.18}$.
12. $\sqrt[3]{1.546}$.
13. $\sqrt{0.9146}$.
14. $\sqrt[5]{3}$.
15. 24284×3789.5.
16. $0.82371 \times 0.001{,}985{,}7$.
17. $1.3336 \div 2.1248$.
18. $1.7321 \div 0.73205$.
19. $0.41831 \div 0.057864$.
20. $48.252 \times 9.6384 \times 0.96384$.

21. $53201 \times 56784 \times 12619.$

22. $472.48 \times 45.990 \times 0.87723.$

23. $\sqrt{89897}.$

24. $\sqrt[3]{4.6123}.$

25. $\sqrt[3]{0.92468}.$

26. $\sqrt[3]{0.092468}.$

27. $\dfrac{9.812 \times 18.76}{405.1}.$

28. $\dfrac{32.64}{19.23 \times 0.7191}.$

29. $\dfrac{54.321 \times 2.7183}{3.1416}.$

30. $\dfrac{1776.4}{24.683 \times 1.0054}.$

31. $(648.35)^5.$

32. $(648.35)^{1/5}.$

33. $\sqrt{5.2683 \times 0.84216}.$

34. $(1.0025)^{-1/2}.$

35. $\dfrac{538.21 \times 1.7864}{0.40752 \times 863.76}.$

36. 97.304×71.486

37. $\dfrac{\sqrt[3]{25.321}}{\sqrt{1.0048}}.$

38. ([illegible] $)^{3/4}$

39. $\dfrac{0.15630(-3.6251)^3}{-36.714\sqrt[5]{-91850}}.$

Note. Although negative numbers have no real logarithms, we can treat this problem as if all the numbers involved were positive, and then prefix the proper sign to the result. Here we have, symbolically,

$$\frac{(+)(-)^3}{(-)\sqrt[5]{-}} = \frac{(+)(-)}{(-)(-)} = \frac{-}{+} = -$$

Thus, a negative sign should precede the final result.

40. $(-1.2381)^2 \div (-7.9564)^3.$

41. $\sqrt[3]{-9999.4}.$

42. $\dfrac{6.8213 \times (-3.4868)}{12.863}.$

43. $\dfrac{(-25.868)^2\sqrt[3]{-0.88255}}{-32.759}.$

31. Logarithms of the trigonometric functions.

Tables giving the values of the trigonometric functions of angles are called tables of "natural functions" to distinguish them from tables which give the logarithms of these functions. We might in all cases find the natural function, and then the logarithm of that function from a table of logarithms of numbers. However, we have tables

which omit one step in this process by giving the logarithm of the function directly, when the value of the angle is known (e.g., Table III of the Macmillan Logarithmic and Trigonometric Tables).

The process of finding the value of the logarithm of a trigonometric function is quite like that of finding the value of the natural function, even when interpolation is required. Similarly, the process of finding the angle, when the logarithm of the function is given, is in no respect different from that of finding the angle when the natural function is given.

Example 1.

Find log cos 17° 25.8′.

SOLUTION. The interpolation can be carried out as in section 8, or it can be arranged as follows (cf. section 25):

$$\begin{aligned} \log\cos 17^\circ 25' &= 9.97962 - 10 \\ \log\cos 17^\circ 26' &= \underline{9.97658} - 10 \\ \text{difference} &= 4 \\ &\ \ \underline{\times\, 0.8} \\ &3.2 \end{aligned}$$

$$\begin{aligned} \log\cos 17^\circ 25' &= 9.97962 - 10 \\ \text{negative correction} &= \underline{3} \\ \log\cos 17^\circ 25.8' &= 9.97959 - 10 \end{aligned}$$

Example 2.

Given log tan A = 0.10860; find the acute angle A.

SOLUTION.

$$1'\left\{\begin{array}{l} \qquad \log\tan 52^\circ 6' = 0.10875 \\ x\left\{\begin{array}{l} \log\tan A \qquad = 0.10860 \\ \log\tan 52^\circ 5' = 0.10849 \end{array}\right\}11 \end{array}\right\}26$$

$$\frac{x}{1'} = \frac{11}{26}, \qquad x = \frac{11}{26} \times 1' = 0.4'.$$

$$A = 52^\circ 5.4'.$$

EXERCISES IV. E

Find the following by using tables of logarithms of the trigonometric functions:

1. $\log \sin 29°$.
2. $\log \cos 31°$.
3. $\log \sin 78° 10'$.
4. $\log \tan 74° 20'$.
5. $\log \cot 17° 17'$.
6. $\log \cot 80° 22'$.
7. $\log \tan 12° 25'$.
8. $\log \sin 31° 52'$.
9. $\log \cos 49° 12'$.
10. $\log \sin 6° 31'$.
11. $\log \sin 7° 46'$.
12. $\log \cos 53° 21'$.
13. $\log \cot 30° 26'$.
14. $\log \sin 26° 45'$.
15. $\log \tan 35° 15.3'$.
16. $\log \sin 12° 13.2'$.
17. $\log \cos 58° 37.8'$.
18. $\log \cot 81° 25.1'$.
19. $\log \sin 9° 41.4'$.
20. $\log \tan 54° 22.2'$.
21. $\log \sin 57° 17.7'$.
22. $\log \cos 45° 2.3'$.
23. $\log \cot 10° 59.9'$.
24. $\log \tan 88° 59.8'$.

Find the acute angle A, given that

25. $\log \sin A = 9.53888 - 10$.
26. $\log \cos A = 9.99484 - 10$.
27. $\log \tan A = 0.30575$.
28. $\log \cot A = 1.54493$.
29. $\log \tan A = 0.18762$.
30. $\log \sin A = 9.71708 - 10$.
31. $\log \tan A = 9.28875 - 10$.
32. $\log \cos A = 9.53871 - 10$.
33. $\log \cos A = 9.72868 - 10$.
34. $\log \cos A = 9.88150 - 10$.
35. $\log \cos A = 9.89530 - 10$.
36. $\log \sin A = 8.90150 - 10$.
37. $\log \sin A = 9.80070 - 10$.
38. $\log \sin A = 9.99483 - 10$.
39. $\log \cot A = 9.18854 - 10$.
40. $\log \cot A = 0.18750$.
41. $\log \tan A = 0.06735$.
42. $\log \tan A = 0.10235$.
43. $\log \tan A = 1.55553$.
44. $\log \cot A = 8.99983 - 10$.
45. $\log \sin A = 9.99950 - 10$.
46. $\log \tan A = 1.00000$.
47. $\log \cos A = 0.17182$.
48. $\log \sin A = 0.11111$.

Find, by using logarithms, the value of each of the following expressions:

49. $12.38 \sin 13° 20'$.
50. $485.6 \cos 22° 28'$.
51. $204.65 \sin 28° 18.2'$.
52. $98.128 \tan 33° 35.6'$.
53. $0.18622 \cos 14° 8.3'$.
54. $57663 \cot 40° 40.8'$.
55. $152.98 \sin 74' 22.9'$.
56. $3004.2 \tan 66° 33.4'$.
57. $1.2346 \cos 45° 45.4'$.
58. $19.897 \sin 38° 59.6'$.

59. $\dfrac{543.21 \sin 72° \, 14.3'}{\sin 22° \, 18.9'}$

60. $\dfrac{2381.4 \tan 44° \, 18.3'}{4561.8}$

Find the value of the acute angle A, given that

61. $\sin A = \dfrac{548.26 \sin 75° \, 43.3'}{865.27}.$

62. $\sin A = \dfrac{9753.6 \sin 18° \, 36.6'}{8910.4}.$

CHAPTER V

Logarithmic Solution of Right Triangles

32. Logarithmic solution of right triangles.

The general instructions of section 7 apply to the logarithmic solution of right triangles. It should be noted that the theorem of Pythagoras is not adapted to the use of logarithms if it is written in the form $c^2 = a^2 + b^2$. However, if the hypotenuse, c, is one of the known parts, we can write

$$a^2 = c^2 - b^2 = (c + b)(c - b), \quad \text{or} \quad b^2 = (c + a)(c - a),$$

and to these forms logarithms can be applied.

An outline, like that in the model solution shown on page 62, should first be made out. Begin with the known parts and conclude with the check. The outline should be complete before any numerical values are written in.

The following general rules will be of use in determining the degree of accuracy to be expected *when dealing with approximate numbers,* not only in connection with right triangles, but for all trigonometric work:

Lengths expressed to two significant figures call for angles to be expressed to the nearest 30′, and vice versa.

Lengths expressed to three significant figures call for angles to be expressed to the nearest 5′, and vice versa.

Lengths expressed to four significant figures call for angles to be expressed to the nearest minute, and vice versa.

Lengths expressed to five significant figures call for angles to be expressed to the nearest tenth of a minute, and vice versa.

It is thus convenient, in dealing with lengths expressed

to three significant figures and angles expressed to the nearest 5′, to use a four-place table of natural functions, such as the table on pages 12–14, without interpolation, or with very rough interpolation. For lengths expressed to four significant figures and angles to the nearest minute, four-place tables of the natural functions or four-place logarithmic tables may be used; in either case interpolation should be employed. Also, for this degree of accuracy, five-place logarithmic tables may be used without interpolation. For lengths expressed to five significant figures and angles to the nearest tenth of a minute, five-place logarithmic tables should be used with interpolation.

Example.

Solve the right triangle in which a 16.84, $c = 20.36$.

Solution.

$\sin A = \dfrac{a}{c},$	a	16.84
	c	20.36
$\log \sin A = \log a - \log c.$	$\log a$	1.22634
	$\log c$	1.30878
	$\log \sin A$	9.91756 − 10
	A	**55° 48′**
$B = 90° - A.$	B	**34° 12′**
	$c + a$	37.20
$b = \sqrt{(c+a)(c-a)},$	$c - a$	3.52
	$\log(c + a)$	1.57054
$\log b = \frac{1}{2}[\log(c+a) + \log(c-a)].$	$\log(c - a)$	0.54654
	$\log b^2$	2.11708
	$\log b$	1.05854
Check.	b	**11.44**
$b = c \cos A,$	$\log c$	1.30878
$\log b = \log c + \log \cos A.$	$\log \cos A$	9.74980 − 10
	$\log b$	1.05858

The work is checked, since the values of $\log b$, obtained by two different methods, agree except in the last place.

EXERCISES V. A

Find the remaining parts, and also the areas, of the following right triangles ($C = 90°$) by logarithms:

1. $a = 793.6$, $b = 965.5$.
2. $A = 52° 41'$, $a = 55.71$.
3. $a = 0.2042$, $c = 0.2753$.
4. $A = 10° 51'$, $b = 7.123$.
5. $b = 5012$, $c = 8117$.
6. $A = 30° 18'$, $c = 0.02040$.
7. $B = 58° 15'$, $a = 48.04$.
8. $B = 6° 31'$, $b = 0.3691$.
9. $B = 23° 9'$, $b = 754.8$.
10. $A = 43° 49.2'$, $b = 22.568$.
11. $a = 2841.6$, $c = 6394.7$.
12. $A = 45° 11.6'$, $b = 61.496$.
13. $b = 862.35$, $c = 1036.0$.
14. $A = 14° 21.1'$, $c = 9.4726$.
15. $B = 26° 17.2'$, $a = 335.88$.
16. $a = 0.18709$, $b = 0.22115$.
17. $B = 52° 9.8'$, $c = 73.211$.
18. $B = 34° 14.6'$, $b = 1202.2$.
19. (a) Find the base of an isosceles triangle whose vertex angle is $38° 27.2'$, and each of whose legs is 153.42. (b) Find the area of the triangle.
20. Find the side of a regular pentagon inscribed in a circle whose radius is 10.354 inches.
21. Find the radius of a circle in which a chord of 23.546 centimeters subtends an angle of $141° 18.4'$ at the center.
22. Find the area of a regular 5-pointed star inscribed in a circle of radius 12.517 inches.

Additional material for practice in the logarithmic solution of right triangles may be obtained from the exercises of Chapter II.

CHAPTER VI

Trigonometric Functions of Any Angle

33. Generation of an angle.

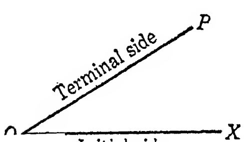


FIG. 30

The angle at O in Fig. 30 may be thought of as generated by the rotation of the line OP, from coincidence with OX to its present position. The line OX is called the **initial side** of the angle, OP is its **terminal side.**

34. Positive and negative angles.

It is evident that there is a choice of directions for rotating the generating line from the position OX to the position OP. One of these is that of the motion of the hands of a clock, and is called **clockwise,** the other is called **counterclockwise.** If the rotation of the generating line is counterclockwise, the angle is **positive** $(+)$; if the rotation is clockwise, the angle is **negative** $(-)$.* A small curved arrow, starting from the initial side and ending with its tip on the terminal side, is often used to indicate the direction of motion. (See Fig. 31.)

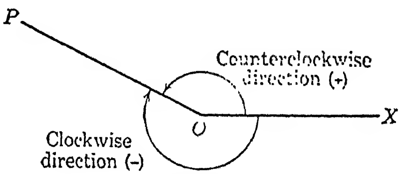


FIG. 31

It is evident that an angle may be of any magnitude

* There is no intrinsic reason why a counterclockwise rotation should give a positive angle and a clockwise rotation a negative angle. This designation, however, is the customary one.

(either positive or negative) whatever, for the generating line may rotate any number of times in either direction.

Any given position of OP represents an unlimited number of positive and negative angles.* On the other hand, to each angle, whether positive, negative, or zero, there corresponds one and only one position of OP.

Angles are equal if they are generated by the same amount of rotation in the same direction.

35. Rectangular coordinates.

Let us take two straight lines, OX and OY, intersecting at right angles at the point O. (See Fig. 32.) On each line we mark off a scale (same scale on each); positive numbers are to the right on the horizontal line OX, above on the vertical line OY; negative numbers are to the left on OX, below on OY. Line OX is called the **x-axis,** line OY the **y-axis,** point O the **origin.**

Now take any point P. The distance of the point from the y-axis is called the **abscissa** of the point and is denoted by x, its distance from the x-axis is called its **ordinate** and is denoted by y. The abscissa and ordinate together are called the **coordinates** (more specifically, **rectangular coordinates**) of the point. The point P in Fig. 32 has the abscissa -3 and the ordinate 4. For such a point it is customary to write $P(-3, 4)$, the abscissa being written first.

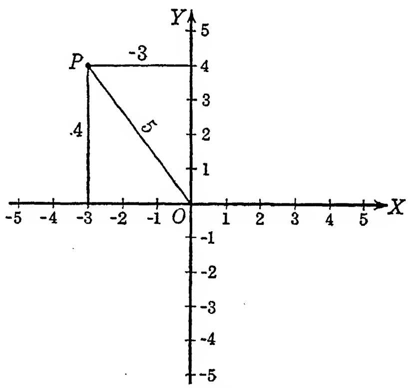


FIG. 32

Locating and marking the position of a point whose coordinates are given is called **plotting** the point.

* These angles may be called **coterminal,** since they have the same terminal side.

Besides the coordinates of the point, we find it convenient to consider its distance from the origin, which may be termed its **radius vector,** or simply its **radius,** and which we shall denote by r. Unless otherwise stated, r will for the present always be regarded as positive. (But see section 72.) Obviously we have $r^2 = x^2 + y^2$, and for the point P in the figure, $r = \sqrt{9 + 16} = 5$. Thus, for this particular point, we have $x = -3$, $y = 4$, $r = 5$.

36. Quadrants.

It will be noted that the coordinate axes divide the plane into four parts, called **quadrants,** numbered as shown in Fig. 33. The order of numbering is in accordance with counterclockwise rotation. That is, a line starting from coincidence with the positive end of the x-axis, and rotating about the origin O so as to generate a positive angle, turns first through quadrant I, then through quadrant II, and so on. Angles between 0° and 90° are in quadrant I, angles between 90° and 180° are in quadrant II, those between 180° and 270° are in quadrant III, those between 270° and 360° are in quadrant IV. Angles between 360° and 450° are in the first quadrant, and so on.

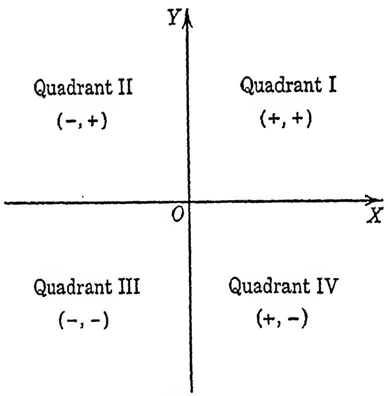


Fig. 33

The signs of x and y in each of the various quadrants are shown in Fig. 33 (the sign of x is written first) and in the following table:

Quadrant	I		III	IV
x (abscissa)	$+$			$+$
y (ordinate)	$+$	$+$		

As already stated, the radius r will for the present be considered as always positive.

37. Trigonometric functions of any angle.

The definitions of the trigonometric functions given in section 2 suffice for acute angles only. In order to deal with the solution of oblique triangles and with other phases of trigonometry, it is necessary to generalize these definitions so that they will apply to any angle.

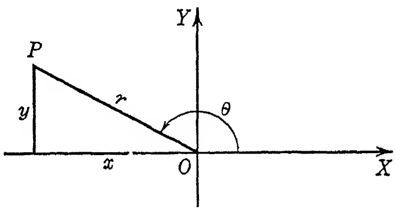


FIG. 34

To this end, let us consider the angle θ (Fig. 34), which has been generated by a line rotating about the origin, starting from coincidence with OX. Take any point P on its terminal side. With this point are associated three values: the abscissa x, the ordinate y, and the radius r. We define

$$\sin\theta = \frac{\text{ordinate}}{\text{radius}} = \frac{y}{r}, \qquad \csc\theta = \frac{\text{radius}}{\text{ordinate}} = \frac{r}{y},$$

$$\cos\theta = \frac{\text{abscissa}}{\text{radius}} = \frac{x}{r}, \qquad \sec\theta = \frac{\text{radius}}{\text{abscissa}} = \frac{r}{x}, \qquad (1)$$

$$\tan\theta = \frac{\text{ordinate}}{\text{abscissa}} = \frac{y}{x}, \qquad \cot\theta = \frac{\text{abscissa}}{\text{ordinate}} = \frac{x}{y}.$$

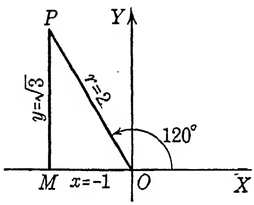


FIG. 35

These new definitions agree with those previously given (section 2) if the angle θ is in the first quadrant. As an illustration of their meanings for other angles, let us find the functions of 120°.

On the terminal side of an angle of 120°, whose initial side is the x-axis, take the point P so that $r = 2$. (See Fig. 35.) Then, angle $MOP = 60°$ and $x = -1$, from which we find, by

using the theorem of Pythagoras, that $y = \sqrt{3}$. The functions may now be read from the figure as follows:

$$\sin 120° = \frac{y}{r} = \frac{\sqrt{3}}{2},$$

$$\cos 120° = \frac{x}{r} = \frac{-1}{2} = -\frac{1}{2},$$

$$\tan 120° = \frac{y}{x} = \frac{\sqrt{3}}{-1} = -\sqrt{3},$$

$$\csc 120° = \frac{r}{y} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3},$$

$$\sec 120° = \frac{r}{x} = \frac{2}{-1} = -2,$$

$$\cot 120° = \frac{x}{y} = \frac{-1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}.$$

EXERCISE

Show that the signs of the functions in the various quadrants are as shown in the following table.

quadrant	sin	cos	tan	csc	sec	cot
I	+	+	+	+	+	+
II	+	−	−	+	−	−
III	−	−	+	−	−	+
IV	−	+	−	−	+	−

38. Functions of 0°, 90°, 180°, 270°.

$(x=1, y=0, r=1)$

Fig. 36

We may consider that we have an angle of 0° if there has been no rotation of the generating line. Take a point P on the terminal side of the angle, which of course coincides with the initial side, with any convenient abscissa, say 1. (See Fig. 36.) Then $x = 1, y = 0, r = 1$, and we have

$$\sin 0° = \frac{y}{r} = \frac{0}{1} = 0,$$

$$\cos 0° = \frac{x}{r} = \frac{1}{1} = 1,$$

$$\tan 0° = \frac{y}{x} = \frac{0}{1} = 0,$$

$$\csc 0° = \frac{r}{y} = \frac{1}{0}, \text{ undefined},$$

$$\sec 0° = \frac{r}{x} = \frac{1}{1} = 1,$$

$$\cot 0° = \frac{x}{y} = \frac{1}{0}, \text{ undefined}.$$

Note that csc 0° and cot 0° do not exist, since the ratios which would represent them have zero for denominator, and division by zero is impossible. However, as the angle θ shrinks to zero, cot θ * becomes numerically larger and larger without bound (e.g., cot $1' = 3437.7$, cot $1'' = 206265$). It is customary to express this fact by writing

$$\cot \theta \to \infty \text{ as } \theta \to 0, \tag{1}$$

where the symbol $\to$ is read " approaches " and the symbol ∞ is called **infinity**. The fact may also be written in the form

$$\lim_{\theta \to 0} \cot \theta = \infty, \tag{2}$$

which is read " the limit, as θ approaches zero, of cot θ is infinity." Either (1) or (2) is merely a shorthand notation for indicating that as the angle gets closer and closer to the value zero, the cotangent increases numerically without bound. It must be insisted that infinity (∞) is not a number.

* We select cot θ merely for purposes of illustration. A similar discussion holds for csc θ.

Similarly, from Fig. 37, in which each of the points P_1, P_2, P_3 is at a numerical distance of 1 from the origin, we can read off the functions of 90°, 180°, 270°. The values of these functions, as well as the functions of 0°, are tabulated below. The student should check them as an exercise. It is clear that the functions of 360° are the same as the functions of 0°. In the table the symbol ∞ is used to indicate that as the angle approaches the specified value, the corresponding function increases in numerical value without bound.

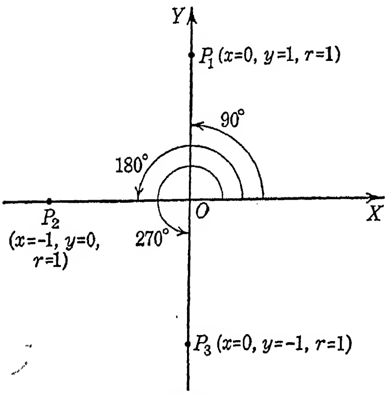


FIG. 37

angle	sin	cos	tan	csc	sec	cot
0°	0	1	0	∞	1	∞
90°	1	0	∞	1	∞	0
180°	0	−1	0	∞	−1	∞
270°	−1	0	∞	−1	∞	0

EXERCISES VI. A

Find the six functions of

1. 135°. **2.** 150°. **3.** 210°. **4.** 240°.
5. 225°. **6.** 300°. **7.** 330°. **8.** 315°.

Find the values of the following expressions:

9. $\sin 150° + \tan 225° + \cos 330°$.
10. $\cos 150° - 3 \tan 300° + 2 \sin 90°$.
11. $3 \tan 240° - \sin^2 135° + 2 \cot 210°$.
12. $3 \sin 135° + 2 \cos 225° - \tan 315°$.

13. $2 \cos 150° - 3 \sin 90° + \tan 210°$.
14. $(\cos 225° + \tan 45°)(\sin 135° + \cos 0°)$.
15. $(\tan 240° - \cos 300°)(2 \sin 300° + \frac{1}{2} \cot 225°)$.
16. $\sin^2 315° + \cos^2 270° + \tan^2 225°$.
17. $(\sin 315° + \cos 270° + \tan 225°)^2$.
18. $2 \cot 300° + 3 \cos 180° + \sin 270° \tan 150°$.
19. $\csc 150° + 2 \sec 330° + 5 \sin 180°$.
20. $3 \sec 135° - 2 \csc 225° + 4 \sin 315°$.
21. $\sec 150° \tan 300° + \tan 225° \csc^2 315°$.
22. $(5 \cos 270° + \sec 180° - \frac{1}{3} \sin 360°)^3$.
23. $(\frac{1}{2} \sec 240° + \csc^2 315° - \cot 135°)^2$.
24. $\sqrt{2} \tan 135° + \sqrt{3} \sin 240° + \sqrt{5} \csc 270°$.
25. $\dfrac{\cos 300° + \cos 360°}{\sin 150° + \sec 300°}$.
26. $\dfrac{3 \tan 135° + 2 \cos 225°}{\sin 240° + \tan 300°}$.
27. $\dfrac{\cot 225° + \sin 270°}{\sec 225° - \tan 300°}$.

39. Functions of $-\theta$.

Let us consider the functions of $-\theta$, where θ is any angle whatever. In Fig. 38 the angle θ is, for definiteness, shown in the first quadrant, but in the following considerations θ is not restricted to the first, or to any other quadrant. It is readily seen that in the congruent right triangles OMP' and OMP, $x' = x$, $y' = -y$ (since MP' and MP extend in opposite directions), and $r' = r$ (since the radius is to be regarded as positive). Consequently,

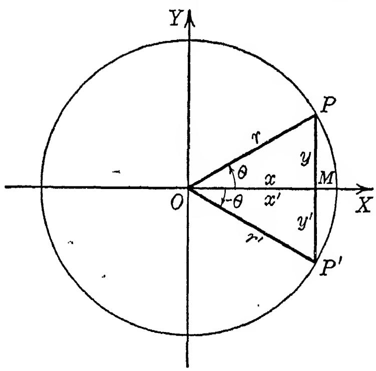


Fig. 38

$$\sin(-\theta) = \frac{y'}{r'} = \frac{-y}{r} = -\frac{y}{r} = -\sin \theta,$$

$$\cos(-\theta) = \frac{x'}{r'} = \frac{x}{r} = \cos\theta,$$

$$\tan(-\theta) = \frac{y'}{x'} = \frac{-y}{x} = -\frac{y}{x} = -\tan\theta,$$

$$\csc(-\theta) = \frac{r'}{y'} = \frac{r}{-y} = -\frac{r}{y} = -\csc\theta,$$

$$\sec(-\theta) = \frac{r'}{x'} = \frac{r}{x} = \sec\theta,$$

$$\cot(-\theta) = \frac{x'}{y'} = \frac{x}{-y} = -\frac{x}{y} = -\cot\theta.$$

EXERCISE

Prove the formulas of section 39 by means of a figure in which θ is an angle in (a) quadrant II, (b) quadrant III, (c) quadrant IV.

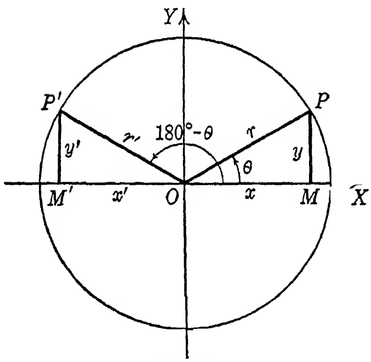


FIG. 39

40. Functions of $180° - \theta$.

Let us now consider the functions of $180° - \theta$, where again θ may be any angle whatever. Reference to Fig. 39, in which $OM'P'$ and OMP are congruent right triangles, shows that

$$\sin(180° - \theta) = \frac{y'}{r'} = \frac{y}{r} = \sin\theta,$$

$$\cos(180° - \theta) = \frac{x'}{r'} = \frac{-x}{r} = -\frac{x}{r} = -\cos\theta,$$

$$\tan(180° - \theta) = \frac{y'}{x'} = \frac{y}{-x} = -\frac{y}{x} = -\tan\theta,$$

$$\csc(180° - \theta) = \frac{r'}{y'} = \frac{r}{y} = \csc\theta,$$

$$\sec(180° - \theta) = \frac{r}{x'} = \frac{r}{-x} = -\frac{r}{x} = -\sec\theta,$$

$$\cot(180° - \theta) = \frac{x'}{y'} = \frac{-x}{y} = -\frac{x}{y} = -\cot\theta.$$

EXERCISE

Prove the formulas of section 40 by means of a figure in which θ is an angle in (a) quadrant II, (b) quadrant III, (c) quadrant IV.

41. Functions of $180° + \theta$.

By the same method of proof, it can be shown from Fig. 40, that

$$
\begin{aligned}
\sin(180° + \theta) &= -\sin\theta,\\
\csc(180° + \theta) &= -\csc\theta,\\
\cos(180° + \theta) &= -\cos\theta,\\
\sec(180° + \theta) &= -\sec\theta,\\
\tan(180° + \theta) &= \tan\theta,\\
\cot(180° + \theta) &= \cot\theta.
\end{aligned}
$$

This is left as an exercise for the student.

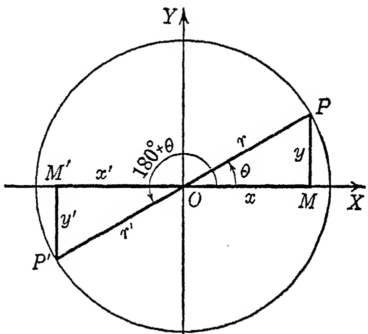


FIG. 40

42. Functions of $360° - \theta$.

From Fig. 38, it is evident that the functions of $360° - \theta$ are the same as the functions of $-\theta$. Thus,

$$
\begin{aligned}
\sin(360° - \theta) &= -\sin\theta, & \csc(360° - \theta) &= -\csc\theta,\\
\cos(360° - \theta) &= \cos\theta, & \sec(360° - \theta) &= \sec\theta,\\
\tan(360° - \theta) &= -\tan\theta, & \cot(360° - \theta) &= -\cot\theta.
\end{aligned}
$$

43. Functions of $360° + \theta$.

It should be quite clear that the functions of $360° + \theta$ are the same as the corresponding functions of θ, since these two angles are coterminal. (See footnote, page 65.)

44. Functions of $90° - \theta$.

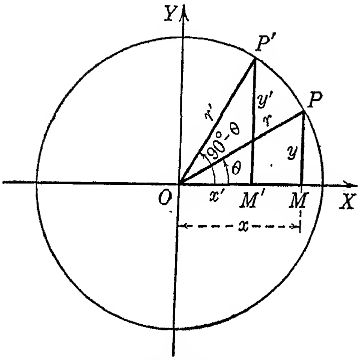


FIG. 41

It was shown in section 3 that, for any acute angle A, $\sin(90° - A) = \cos A$, etc. That is, any function of an acute angle is equal to the cofunction of the complementary angle. That formulas (2) of section 3 are true for any angle may be shown by Fig. 41 as follows:

Right triangles $OM'P'$ and OMP are congruent, and consequently $x' = y$, $y' = x$, $r' = r$. Therefore,

$$\sin(90° - \theta) = \frac{y'}{r'} = \frac{x}{r} = \cos\theta,$$

$$\cos(90° - \theta) = \frac{x'}{r'} = \frac{y}{r} = \sin\theta,$$

$$\tan(90° - \theta) = \frac{y'}{x'} = \frac{x}{y} = \cot\theta,$$

$$\csc(90° - \theta) = \frac{r'}{y'} = \frac{r}{x} = \sec\theta,$$

$$\sec(90° - \theta) = \frac{r'}{x'} = \frac{r}{y} = \csc\theta,$$

$$\cot(90° - \theta) = \frac{x'}{y'} = \frac{y}{x} = \tan\theta.$$

EXERCISE

Prove the formulas of section 44 by means of a figure in which θ is an angle in (a) quadrant II, (b) quadrant III, (c) quadrant IV.

45. Functions of 90° + θ.

It is seen that in Fig. 42, x' and y are numerically equal but have opposite signs; that is, $x' = -y$. Similarly, y' and x are numerically equal and have the same sign; that is, $y' = x$. Also, $r' = r$. It follows that

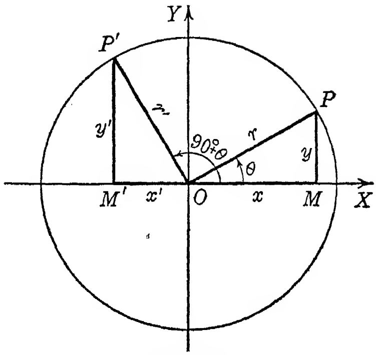


FIG. 42

$$\sin(90° + \theta) = \frac{y'}{r'} = \frac{x}{r} = \cos\theta,$$

$$\cos(90° + \theta) = \frac{x'}{r'} = \frac{-y}{r} = -\frac{y}{r} = -\sin\theta,$$

$$\tan(90° + \theta) = \frac{y'}{x'} = \frac{x}{-y} = -\frac{x}{y} = -\cot\theta,$$

$$\csc(90° + \theta) = \frac{r'}{y'} = \frac{r}{x} = \sec\theta,$$

$$\sec(90° + \theta) = \frac{r'}{x'} = \frac{r}{-y} = \qquad = -\csc$$

$$\cot(90° + \theta) = \frac{x'}{y'} = \frac{-y}{x} = \qquad -\tan\theta.$$

EXERCISE

Prove the formulas of section 45 by means of a figure in which θ is an angle in (a) quadrant II, (b) quadrant III, (c) quadrant IV.

46. Functions of 270° − θ.

In Fig. 43, $x' = -y$, $y' = -x$, $r' = r$, and it can readily be proved that

$\sin(270° - \theta) = -\cos\theta,$ $\qquad$ $\csc(270° - \theta) = -\sec\theta,$

$\cos(270° - \theta) = -\sin\theta,$ $\sec(270° - \theta) = -\csc\theta,$

$\tan(270° - \theta) = \cot\theta,$ $\cot(270° - \theta) = \tan\theta.$

Proofs are left as exercises for the student.

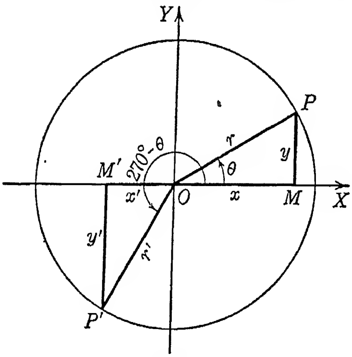


FIG. 43

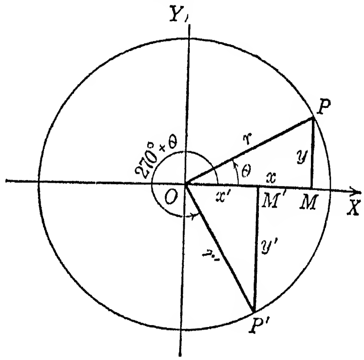


FIG. 44

47. Functions of $270° + \theta$.

In Fig. 44, $x' = y$, $y' = -x$, $r' = r$, and it follows that

$\sin(270° + \theta) = -\cos\theta,$ $\csc(270° + \theta) = -\sec\theta,$

$\cos(270° + \theta) = \sin\theta,$ $\sec(270° + \theta) = \csc\theta,$

$\tan(270° + \theta) = -\cot\theta,$ $\cot(270° + \theta) = -\tan\theta.$

Proofs are left as exercises.

48. Summary.

The formulas of sections 39–47 may be summarized as in the accompanying table. The upper sign preceding a function corresponds to the upper sign in the angle at the left of the same row, and similarly for the lower sign.

angle	sin	cos	tan	csc	sec	cot
$-\theta$	$-\sin\theta$	$\cos\theta$	$-\tan\theta$	$-\csc\theta$	$\sec\theta$	$-\cot\theta$
$90° \pm \theta$	$\cos\theta$	$\mp\sin\theta$	$\mp\cot\theta$	$\sec\theta$	$\mp\csc\theta$	$\mp\tan\theta$
$180° \pm \theta$	$\mp\sin\theta$	$-\cos\theta$	$\pm\tan\theta$	$\mp\csc\theta$	$-\sec\theta$	$\pm\cot\theta$
$270° \pm \theta$	$-\cos\theta$	$\pm\sin\theta$	$\mp\cot\theta$	$-\sec\theta$	$\pm\csc\theta$	$\mp\tan\theta$
$360° \pm \theta$	$\pm\sin\theta$	$\cos\theta$	$\pm\tan\theta$	$\pm\csc\theta$	$\sec\theta$	$\pm\cot\theta$

Note that in any column we have the same function as that at the head of the column, except for the rows having $90° \pm \theta$ and $270° \pm \theta$ at the left; in these rows we find the cofunctions.

The student should make no attempt to memorize this table, but he should be able to work out any of the results listed in it by the methods of the preceding sections; that is, by drawing a figure for each separate problem as needed.

For the important special case in which θ is an acute angle the following statements may prove helpful: If an angle is written in the form $-\theta$, $180° \pm \theta$, or $360° \pm \theta$ we may say that it is **referred to the x-axis**; if it is written in the form $90° \pm \theta$ or $270° \pm \theta$, we may say that it is **referred to the y-axis**; in either case we shall call θ the **reference angle.** *The function of any angle referred to the x-axis is numerically equal to the same function of the reference angle; the function of any angle referred to the y-axis is numerically equal to the cofunction of the reference angle. The sign to be prefixed to the resulting function of θ is that of the original function, as determined by the quadrant in which the original angle is situated.*

49. Reduction of functions of any angle to functions of an acute angle.

We are now in a position to find the functions of any angle whatever.

Example 1.

Find sine, cosine, and tangent of 110°.

SOLUTION. Since $110° = 180° - 70°$, we have

$$\begin{aligned}
\sin 110° &= \sin(180° - 70°) = \sin 70° = 0.9397,\\
\cos 110° &= \cos(180° - 70°) = -\cos 70° = -0.3420,\\
\tan 110° &= \tan(180° - 70°) = -\tan 70° = -2.7475.
\end{aligned}$$

Or, since $110° = 90° + 20°$,

$$\begin{aligned}
\sin 110° &= \sin(90° + 20°) = \cos 20° = 0.9397,\\
\cos 110° &= \cos(90° + 20°) = -\sin 20° = -0.3420,\\
\tan 110° &= \tan(90° + 20°) = -\cot 20° = -2.7475.
\end{aligned}$$

Example 2.

Find sine, cosine, and tangent of 615°.

SOLUTION. Since $615° = 360° + 255°$, the functions of 615° are exactly the same as those of 255°. But $255° = 180° + 75°$. Thus,

$$\begin{aligned}
\sin 615° &= \sin 255° = \sin(180° + 75°) = -\sin 75° = -0.9659,\\
\cos 615° &= \cos 255° = \cos(180° + 75°) = -\cos 75° = -0.2588,\\
\tan 615° &= \tan 255° = \tan(180° + 75°) = \tan 75° = 3.7321.
\end{aligned}$$

Or, we could express 255° as $270° - 15°$.

EXERCISES VI. B

1. Express each of the following as a function of a positive acute angle:

(a) $\sin 160°$, (b) $\cos 145°$, (c) $\tan 100°$,
(d) $\csc 130°$, (e) $\sec 172°$, (f) $\cot 98°$,
(g) $\sin 137°$, (h) $\cos 95° 10'$, (i) $\tan 162° 4'$,
(j) $\cot 125° 18'$, (k) $\sin 114° 21'$, (l) $\cos 92° 12.8'$.

2. Reduce each of the following to a function of a positive angle less than 45°:

(a) $\sin 175°$, (b) $\cos(-167°)$, (c) $\tan 520°$,
(d) $\cot 125° 26'$, (e) $\sec 267° 28'$, (f) $\csc 325° 41.8'$,
(g) $\sin 215° 5'$, (h) $\cos 281° 22'$, (i) $\tan 197° 35'$,
(j) $\cot 312° 54'$, (k) $\sin 356° 56'$, (l) $\cos 95° 6.5'$.

3. Find the numerical value of

(a) $\sin 145°$, (b) $\cos 246°$, (c) $\tan 285°$,
(d) $\cot 572° 38'$, (e) $\cos 321°$, (f) $\sin 642° 50.5'$,
(g) $\cot 121° 13.6'$, (h) $\sin 462° 31.1'$, (i) $\sin(-162° 45')$,
(j) $\cos(-72° 15')$, (k) $\tan(-200°)$, (l) $\cot(-275° 18')$.

Find the value of

4. $\cos 240° \cos 120° - \sin 120° \cos 150°$.
5. $\tan 315° \sec 900° + \cot 495° \csc 450°$.
6. $\sin(90° + \theta) \sin(180° + \theta) + \cos(90° + \theta) \cos(180° - \theta)$.
7. Given that θ is the angle of a triangle, find θ if

   (a) $\sin \theta = 0.3090$, (b) $\cos \theta = 0.4975$, (c) $\tan \theta = 2.8770$,
   (d) $\cot \theta = 1.7090$, (e) $\sin \theta = 0.6713$, (f) $\cos \theta = -0.7716$.

8. Express as functions of θ:

   (a) $\sin(810° - \theta)$, (b) $\tan(990° - \theta)$, (c) $\cot(\theta - 360°)$,
   (d) $\sec(\theta - 90°)$, (e) $\cos(-180° - \theta)$, (f) $\csc(630° + \theta)$.

CHAPTER VII

Solution of Oblique Triangles

50. The four cases.

We shall now take up the solution of oblique triangles by methods that do not require breaking them up into right triangles, as was done in section 11. Problems in the solution of oblique triangles may be classified into the following four cases, already mentioned in that section:

Case I. Two angles and a side given.

Case II. Two sides and the angle opposite one of them given.

Case III. Two sides and the included angle given.

Case IV. Three sides given.

Certain formulas are necessary for handling the various cases, and these will be developed as needed.

51. Law of sines.

Fig. 45(a) represents an acute triangle, Fig. 45(b) an ob-

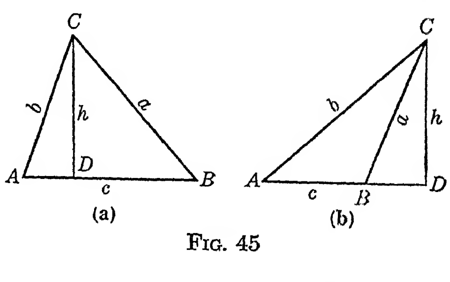


Fig. 45

tuse triangle, B being the obtuse angle. In each figure we draw the altitude CD and designate its length by h. Then, in Fig. 45(a),

$$\sin B = \frac{h}{a}, \qquad \text{or} \qquad h = a \sin B, \tag{1}$$

and the same relation holds in Fig. 45(b), since

$$\sin(180° - B) = \sin B.$$

In either figure,

$$\sin A = \frac{h}{b}, \qquad \text{or} \qquad h = b \sin A. \tag{2}$$

Equating the values of h in (1) and (2), we have

$$a \sin B = b \sin A, \tag{3}$$

and dividing both sides of (3) by $\sin A \sin B$, we get

$$\frac{a}{\sin A} = \frac{b}{\sin B}. \tag{4}$$

Similarly, by drawing the altitude from A, we can prove that

$$\sin B \qquad \sin C \tag{5}$$

Combining (4) and (5), we obtain the **law of sines,**

$$\frac{}{\sin A} = \frac{}{\sin B} = \frac{}{\sin C}, \tag{6}$$

which may be stated in words as follows: *The sides of a triangle are proportional to the sines of the opposite angles.*

EXERCISE

Prove that if $C = 90°$, formula (6) reduces to the definitions of $\sin A$ and $\sin B$.

A formula for the area of a triangle is easily derivable from formula (2) for the altitude. Since the area is equal

to half the product of the base and the altitude, we have

$$\text{area} = \frac{1}{2} bc \sin A. \tag{7}$$

The area is also of course equal to $\frac{1}{2} ac \sin B$ and $\frac{1}{2} ab \sin C$. In words, *the area of a triangle is equal to one-half the product of any two sides and the sine of the included angle.*

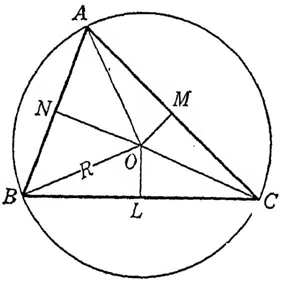


FIG. 46

The following proof of the law of sines gives a geometric meaning to the equal ratios in (6):

Draw the perpendicular bisectors of the sides of the triangle ABC (Fig. 46). They will meet in a point O, which is the center of the circumscribed circle. Draw this circle, and connect its center with the vertices of the triangle. Let R be the radius of the circle, and, as usual, let A, B, C represent the angles of the triangle.

Then, angle $BOC = 2A$. (Why?)
Hence, angle $BOL = A$.
Consequently,

$$\sin A = \sin BOL = \frac{BL}{R} = \frac{\frac{1}{2}a}{R} = \frac{a}{2R}.$$

Similarly,

$$\sin B = \frac{b}{2R}, \qquad \sin C = \frac{c}{2R},$$

and it follows that

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R = D, \tag{8}$$

where D is the diameter of the circumscribed circle.

If one of the angles of the triangle is obtuse, the proof requires a slight modification.

52. Solution of Case I.

This case, in which there are *two angles and a side given*, can be solved by the law of sines.

Example.

Solve the triangle $A = 40°$, $B = 60°$, $c = 50$.

SOLUTION. $C = 180° - (A + B) = 80°$.
From the law of sines,

$$a = \frac{c \sin A}{\sin C} \quad \frac{50 \sin 40°}{\sin 80°} \quad \frac{50 \times 0.6428}{0.9848} = 32.6,$$

$$b = \frac{c \sin B}{\sin C} \quad \frac{50 \sin 60°}{\sin 80°} \quad \frac{50 \times 0.8660}{0.9848} = 44.0.$$

These results may be checked by using the relation $a/\sin A = b/\sin B$, or by means of **Mollweide's equation,**

$$\frac{a + b}{c} = \frac{\cos \frac{1}{2}(A - B)}{\sin \frac{1}{2}C}, \tag{1}$$

which is proved in section 61. (If $B > A$, interchange A and B, a and b, respectively, in the formula.)

They may also be checked by using one of the following relations, proofs of which are left as exercises:

$$a = b \cos C + c \cos B, \qquad b = a \cos C + c \cos A, \qquad c = a \cos B + b \cos A. \tag{2}$$

EXERCISES VII. A

Solve the following triangles:

1. $A = 70°$, $B = 80°$, $a = 12$.
2. $A = 70°$, $B = 80°$, $c = 12$.
3. $A = 58° 10'$, $C = 84° 40'$, $b = 2.5$.
4. $B = 132° 10'$, $C = 18° 20'$, $c = 10.2$.
5. $B = 10° 50'$, $C = 75° 30'$, $b = 61$.
6. $A = 95° 40'$, $C = 45° 20'$, $a = 8.2$.

7. The bases of a trapezoid are 22 and 12 respectively. The angles at the extremities of one base are 65° and 40° respectively. Find the two legs.
8. Two observers, who are 2 miles apart on a horizontal plane, observe a balloon in the same vertical plane with themselves. The angles of elevation are 50° and 65° respectively. Find the height of the balloon, (a) if it is between the observers; (b) if it is on the same side of both of them.
9. One diagonal of a parallelogram is 16.5. It makes angles of 36° 10′ and 14° 30′ respectively with the sides. Find the sides.
10. A line AB, 125 feet long, is measured along the straight bank of a river. A point C is on the opposite bank. Angles ABC and BAC are found to be 65° 40′ and 54° 30′ respectively. How wide is the river?
11. From a certain point the angle of elevation of the top of a building is 38°. From a point 75 feet nearer the building the angle of elevation is 65°. Find the height of the building.
12. From a given position an observer notes that the angle of elevation of a rock is 47°. After walking 1000 feet towards the rock, up a slope of 32°, he finds the angle of elevation to be 75°. Find the vertical distance of the rock above each point of observation.
13. A flagpole 25 feet tall stands on top of a building. From a point in the same horizontal plane with the base of the building the angles of elevation of the top and the bottom of the flagpole are 61° 30′ and 56° 20′ respectively. How high is the building?
14. Find the radius of the circle circumscribed about the triangle for which $A = 50°$, $B = 20°$, $a = 35$.

53. Solution of Case II.

This case, in which we have *two sides and the angle opposite one of them given,* presents difficulties that are not found in the other cases. This is because we sometimes find two solutions for the problem; that is, we find two triangles having the given parts. Sometimes we find only one triangle, and sometimes, indeed, we do not find any; that

is, the problem is impossible. A carefully constructed figure will usually show how many solutions there are, but the following discussion explains how this can be determined accurately:

Let us suppose that the given parts are a, b, A.

We consider first the case in which A is acute. Construct this angle, and mark off the point C on one of its sides so that $AC = b$. Extend the other side indefinitely. (See Fig. 47.)

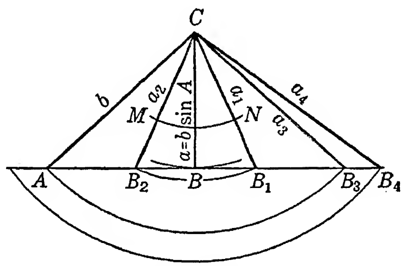


FIG. 47

The perpendicular distance from C to this extended side is $b \sin A$, and it is evident that various cases may occur, depending upon the length of a as compared with b and with $b \sin A$.

Let us take a pair of compasses, and with C as center and a as radius, test these various cases by constructing arcs.

If a is less than $b \sin A$, the arc will be like MN, and there will be no triangle.

If $a = b \sin A$, the arc will be tangent to the base line (that is, the extended side) at the point B, and there will be but one triangle, the right triangle ABC.

If a is greater than $b \sin A$ but less than b, the arc will cut the base line in two points, such as B_1 and B_2. Consequently, we get two triangles, AB_1C and AB_2C. Under these conditions, Case II is said to be **ambiguous**, that is, there is not a unique solution. Since either of the triangles satisfies the requirements of the problem, we must solve both.

If $a = b$, the arc passes through A, and we get but one solution, the isosceles triangle AB_3C.

If a is greater than b, there is but one triangle, such as AB_4C.

There are no other possible conditions when A is acute.

If A is a right angle, as shown in Fig. 48, it is evident that we cannot have a triangle unless a is greater than b, under which condition we have only one construction.

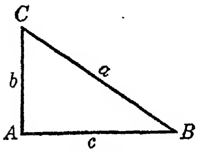


FIG. 48

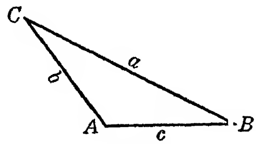


FIG. 49

If A is obtuse, as in Fig. 49, the arc having a as radius cannot cut the base line on the proper side of the point A unless a is greater than b. Thus, we have no triangle unless a is greater than b, and then we have only one.

Our conclusions may be summarized as follows:

$A < 90°$	
$a < b \sin A$	no solution
$a = b \sin A$	one solution (right triangle)
$b \sin A < a < b$	two solutions
$a = b$	one solution (isosceles triangle)
$a > b$	one solution

$A \geqq 90°$	
$a \leqq b$	no solution
$a > b$	one solution

If the given parts are other than a, b, A, the foregoing summary must, of course, be modified accordingly.

Case II is solved by the application of the law of sines.

Example.

Solve the triangle $a = 20$, $b = 10$, $A = 75°$.

SOLUTION. It is apparent here that there is only one solution. From the law of sines, we have

$$\sin B = \frac{b \sin A}{a} = \frac{10 \sin 75°}{20} = \frac{10 \times 0.9659}{20} = 0.4830,$$

$$B = 28° \, 50',$$

$$C = 180° - (A + B) = 180° - 103° \, 50' = 76° \, 10',$$

$$c = \frac{a \sin C}{\sin A} = \frac{20 \sin 76^\circ 10'}{\sin 75^\circ} = \frac{20 \times 0.9710}{0.9659} = 20.1.$$

The results may be checked by computing c from the relation $c = b \sin C/\sin B$, or by using Mollweide's equation (1) of the preceding section.

Note that from the value $\sin B = 0.4830$ we could also have $B = 180^\circ - 28^\circ 50' = 151^\circ 10'$. However, if we should attempt to find C by adding A and B and subtracting their sum from 180°, we should find $A + B = 75^\circ + 151^\circ 10' = 226^\circ 10'$, which is impossible. This method will always show whether there is a second solution.

EXERCISES VII. B

Solve the following triangles:

1. $A = 40^\circ$,	$a = 8$,	$b = 5$.
2. $A = 30^\circ$,	$a = 5$,	$b = 8$.
3. $B = 36^\circ 10'$,	$a = 21.2$,	$b = 31.0$.
4. $C = 108^\circ 20'$,	$b = 12.2$,	$c = 25.1$.
5. $A = 73^\circ 20'$,	$a = 2.5$,	$b = 1.8$.
6. $B = 30^\circ$,	$b = 99$,	$a = 198$.
7. $C = 15^\circ 40'$,	$a = 35$,	$c = 9.5$.
8. $B = 65^\circ 30'$,	$a = 17.6$,	$b = 15.9$.

9. A side and a diagonal of a parallelogram are 12 inches and 19 inches respectively. The angle between the diagonals, opposite the given side, is 124°. Find the length of the other diagonal and the length of the other side.
10. A lighthouse is 10 miles northeast of a dock. A ship leaves the dock at noon, and sails east at a speed of 12 miles an hour. At what time will it be 8 miles from the lighthouse?
11. A vertical pole 35 feet high, standing on sloping ground, is braced by a wire which extends from the top of the pole to a point on the ground 25 feet from the foot of the pole. If the pole subtends an angle of 30° at the point where the wire reaches the ground, how long is the wire?
12. A tower 125 feet high stands on the side of a hill. At a point 240 feet from the foot of the tower, measured straight down the hill, the tower subtends an angle of 25°. What angle does the side of the hill make with the horizontal?

54. Law of cosines.

In Fig. 50(a), angle A is acute; in Fig. 50(b), angle A is obtuse. In each figure let us draw the altitude CD, whose numerical value we set equal to h. Further, let $AD = m$. Then, in Fig. 50(a),

$$a^2 = h^2 + (c - m)^2 = h^2 + c^2 - 2\,cm + m^2, \tag{1}$$

while in Fig. 50(b),

$$a^2 = h^2 + (c + m)^2 = h^2 + c^2 + 2\,cm + m^2. \tag{2}$$

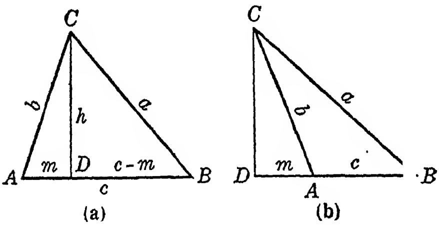


FIG. 50

Since, in either figure, $h^2 + m^2 = b^2$, (1) and (2) reduce respectively to

$$a^2 = b^2 + c^2 - 2cm, \tag{3}$$

and

$$a^2 = b^2 + c^2 + 2cm. \tag{4}$$

But in Fig. 50(a),

$$m = b \cos A,$$

and in Fig. 50(b),

$$m = b\cos(180° - A) = -b \cos A.$$

Substituting these values of m in (3) and (4) respectively, we obtain

$$a^2 = b^2 + c^2 - 2bc \cos A. \tag{5}$$

Similarly, $$b^2 = c^2 + a^2 - 2ca \cos B, \tag{6}$$

and $$c^2 = a^2 + b^2 - 2ab \cos C. \tag{7}$$

These three formulas constitute the **law of cosines**, which states that *the square of any side of a triangle is equal to the sum of the squares of the other two sides minus twice the product of these two sides times the cosine of the angle between them.*

NOTE. The law of cosines combines into one statement the following three theorems of plane geometry:

I. The square of the hypotenuse of a right triangle is equal to the sum of the squares of the two sides.

II. In any triangle, the square of the side opposite an acute angle is equal to the sum of the squares of the other two sides diminished by twice the product of either of those sides by the projection of the other upon it.

III. In any obtuse triangle, the square of the side opposite the obtuse angle is equal to the sum of the squares of the other two sides increased by twice the product of one of those sides by the projection of the other upon it.

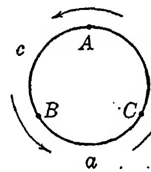


FIG. 51

Formulas (6) and (7) may be obtained from (5) by what is termed a **cyclic** change of letters. This may be effected in the following way:

Arrange the letters around the circumference of a circle, as in Fig. 51. Then replace each letter in the given formula by the next in order. Thus, a new formula is obtained if

a is replaced by **b**,

b is replaced by **c**,

c is replaced by **a**,

and similarly for the capital letters.

In this manner (5) is changed into (6), which in turn may be changed into (7).

Note that if C is a right angle, (7) becomes the Pythagorean relation, $c^2 = a^2 + b^2$, since $\cos 90° = 0$.

EXERCISE

Show that if $C = 90°$, (5) and (6) reduce to the definitions of $\cos A$ and $\cos B$ respectively.

55. Solution of Case III.

The law of cosines is useful in solving Case III, in which we have *two sides and the included angle given.*

Example.

Solve the triangle $a = 25$, $b = 30$, $C = 50°$

SOLUTION. $c^2 = a^2 + b^2 - 2ab \cos C$

$$= (25)^2 + (30)^2 - 2 \times 25 \times 30 \times \cos 50°$$
$$= 625 + 900 - 1500 \times 0.6428 = 560.8,$$
$$c = 23.7.$$

Angles A and B may be found by the law of sines.

The smaller of these angles should be found first, for if the larger is obtuse some confusion may arise.

A check is afforded by Mollweide's equation (1) of section 52.

EXERCISES VII. C

Solve the following triangles:

1. $a = 5$, $c = 6$, $B = 60°$.
2. $a = 2$, $b = 3$, $C = 130°$.
3. $b = 1.7$, $c = 2.2$, $A = 17° 20'$.
4. $a = 0.35$, $b = 0.24$, $C = 75° 40'$.
5. $a = 230$, $b = 150$, $C = 95°$.
6. $b = 80.1$, $c = 106$, $A = 165° 50'$.
7. Two ships leave a dock at the same time. One sails northeast at the rate of 8.5 miles an hour, the other sails north at the rate of 10 miles an hour. How far apart are they at the end of 2 hours?
8. If the slower ship in the preceding exercise leaves at noon, and the other at 1 p.m., how far apart are they at 2 p.m.?
9. The diagonals of a parallelogram are 7 inches and 9 inches respectively; they intersect at an angle of 52°. Find the sides of the parallelogram.
10. A military observer notes two enemy batteries which subtend, at his observation post, an angle of 40°. The interval between the flash and the report of a gun is 5 seconds for one battery, and 4 seconds for the other. If the velocity of sound is 1140 feet a second, how far apart are the batteries?
11. Points A and B are separated by an obstacle. In order to find the distance between them, a third point C is selected which is 120 yards from A and 150 yards from B. The angle

ACB is measured to be 80° 10′. Find the distance from A to B.

12. Two circles, whose radii are 12 inches and 16 inches respectively, intersect. The angle between the tangents at either of the points of intersection is 29° 30′. Find the distance between the centers of the circles.

56. Solution of Case IV.

Case IV, *three sides given*, can also be solved by the law of cosines.

Example.

Solve the triangle $a = 5$, $b = 6$, $c = 9$.

SOLUTION. Solving the law of cosines $a^2 = b^2 + c^2 - 2bc \cos A$ for $\cos A$, we get

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{36 + 81 - 25}{2 \times 6 \times 9} = \frac{92}{108} = 0.8519,$$

$$A = 31° \, 35'.$$

Similarly,

$$\cos B = \frac{c^2 + a^2 - b^2}{2ca} = \frac{81 + 25 - 36}{2 \times 9 \times 5} = \frac{70}{90} = 0.7778,$$

$$B = 38° \, 57';$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{25 + 36 - 81}{2 \times 5 \times 6} = -\frac{20}{60} = -0.3333,$$

$$C = 180° - 70° \, 32' = 109° \, 28'.$$

CHECK. $A + B + C = 180°$.

EXERCISES VII. D

Find the angles of the following triangles:

1. $a = 2$, $b = 3$, $c = 4$.
2. $a = 0.013$, $b = 0.014$, $c = 0.015$.
3. $a = 8.4$, $b = 7.2$, $c = 6.5$.
4. $a = 45$, $b = 32$, $c = 71$.
5. $a = 1.4$, $b = 4.8$, $c = 5.0$.
6. $a = 24$, $b = 7$, $c = 25$.
7. $a = 13.2$, $b = 11.8$, $c = 20.1$.
8. $a = 20.1$, $b = 21.0$, $c = 15.5$.

9. Three towns, A, B, and C, are situated so that $AB = 300$ miles, $AC = 194$ miles, and $BC = 160$ miles, B being due north of C. Find the direction from B to A.
10. A ladder 20 feet long is set with one end at a horizontal distance of 7 feet from a sloping wall. The other end of the ladder reaches 15 feet up the face of the wall. What angle does the wall make with the horizontal?
11. The sides of a parallelogram are 11.7 inches and 15.0 inches respectively; one diagonal is 13.1 inches. Find the angles. Also find the other diagonal.
12. If the sides of a triangle are 16, 20, and 27 respectively, what is the length of the bisector of the largest angle?
13. Find the length of the median to the longest side in the preceding exercise.
14. Three circles of radii 3, 4, and 5 inches respectively are tangent to each other externally. Find the angles of the triangle formed by joining the centers.

*57. Application of law of cosines to Case II.

It may be noted that Case II can be handled by the law of cosines.

Example.

Solve the triangle $a = 20$, $b = 10$, $A = 75°$.

Solution. Substitute the given values in the equation

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

This gives
$$\begin{aligned} 400 &= 100 + c^2 - 2 \times 10 \times c \times \cos 75° \\ &= 100 + c^2 - 20c \times 0.2588, \end{aligned}$$

which reduces to the quadratic equation

$$c^2 - 5.176c - 300 = 0.$$

$$\frac{\qquad}{2} = 20.1.$$

There is also a negative root of the equation, but it is discarded. If there are two positive roots, it means that there are two solutions.

The method is particularly useful if it is not required to find the remaining two angles. However, if they are required, they may be found either by the law of sines or by the law of cosines.

EXERCISE

Solve, by using the law of cosines, exercise VII. B, 10; also such other exercises of VII. B as the instructor may assign.

58. Logarithmic solution of Case I.

The solution of this case by logarithms follows the same steps as the solution in section 52. The only difference is that logarithms are employed in performing the computations.

Example.

Solve the triangle $A = 79° 59.3'$, $B = 46° 36.4'$, $a = 804.32$.

SOLUTION.

$C = 180° - (A + B)$.

$$b = \frac{a \sin B}{\sin A},$$

$\log b = \log a + \log \sin B + \operatorname{colog} \sin A.$

$$c = \frac{a \sin C}{\sin A},$$

$\log c = \log a + \log \sin C + \operatorname{colog} \sin A.$

A	$79° 59.3'$	
B	$46° 36.4'$	
$A + B$	$126° 35.7'$	
C	**$53° 24.3'$**	
a	804.32	
$\log \sin B$	9.86133	10
$\log a$	2.90543	
$\operatorname{colog} \sin A$	0.00666	
$\log \sin C$	9.90465	$- 10$
$\log b$	2.77342	
$\log c$	2.81674	
b	**593.50**	
c	**655.75**	

CHECK.

$$\frac{a + b}{c} = \frac{\cos \frac{1}{2}(A - B)}{\sin \frac{1}{2}C} = x,$$

$\log x = \log(a + b) - \log c,$

$\log x = \log \cos \frac{1}{2}(A - B) - \log \sin \frac{1}{2}C.$

$a + b$	1397.82	
$A - B$	$33° 22.9'$	
$\frac{1}{2}(A - B)$	$16° 41.45'$	
$\frac{1}{2}C$	$26° 42.15'$	
$\log(a + b)$	3.14545	
$\log c$	2.81674	
$\log x$	0.32871	
$\log \cos \frac{1}{2}(A - B)$	9.98131	$- 10$
$\log \sin \frac{1}{2}C$	9.65259	$- 10$
$\log x$	0.32872	

It should be noted that, in checking, we do not need to find the quantities $(a + b)/c$ and $\cos \frac{1}{2}(A - B)/\sin \frac{1}{2}C$; it is sufficient if the logarithms of these quantities agree. Slight discrepancies in the last place are to be expected.

EXERCISES VII. E

Find the remaining parts, and also the areas, of the following triangles:

1. $B = 65° 25.5'$,	$C = 81° 24.6'$,	$b = 724.32$.
2. $B = 38° 37.4'$,	$C = 75° 32.8'$,	$c = 129.63$.
3. $A = 48° 29.2'$,	$C = 115° 33.8'$,	$a = 14.829$.
4. $A = 68° 41.5'$,	$C = 110° 16.5'$,	$c = 9.4326$.
5. $A = 11° 11.3'$,	$C = 57° 37.4'$,	$c = 444.79$.
6. $B = 20° 20.2'$,	$C = 12° 28.5'$,	$a = 673.75$.
7. $A = 28° 14.7'$,	$C = 109° 32.5'$,	$b = 730.80$.
8. $B = 102° 38.3'$,	$C = 20° 3.2'$,	$b = 479.36$.
9. $B = 30° 36.8'$,	$C = 107° 15.5'$,	$b = 0.14379$.
10. $A = 36° 14.2'$,	$B = 14° 26.7'$,	$c = 16.583$.

11. One diagonal of a parallelogram is 21.871 inches. It makes angles of 43° 20.5′ and 56° 14.2′ respectively with the sides. Find the sides of the parallelogram.

12. At a certain point in the same horizontal plane as the base of a radio tower, the angle of elevation of the top of the tower is 13° 25.4′. At a point which is 156.25 feet nearer the tower the angle of elevation is 18° 10.5′. Find the height of the tower.

59. Logarithmic solution of Case II.

Case II can also be solved logarithmically by using the law of sines. The solution may be checked by formula (1) of section 52 (page 83) or by the law of tangents. (See section 60.)

Example.

Solve the triangle $A = 38° 14.2'$, $a = 8.7161$, $b = 9.7869$.

SOLUTION.

$$\sin B = \frac{b \sin A}{a},$$

$\log \sin B$

$= \log b + \log \sin A$

$+ \operatorname{colog} a.$

$C = 180° - (A + B).$

$$c = \frac{a \sin C}{\sin A},$$

$\log c$

$= \log a + \log \sin C$

$+ \operatorname{colog} \sin A.$

a	8.7161	
b	9.7869	
A	38° 14.2′	
log b	0.99065	
log sin A	9.79163 − 10	
colog a	9.05968 − 10	
log sin B	9.84196 − 10	
B	**44° 1.5′**, B' =	**135° 58.5′**
A	38° 14.2′	38° 14.2′
$A + B$	82° 15.7′	174° 12.7′
C	**97° 44.3′**, C' =	**5° 47.3′**
log sin C	9.99602 − 10	
log a	0.94032	
colog sin A	0.20837	
log sin C'	9.00369 − 10	
log c	1.14471	
log c'	0.15238	
c	**13.954**	
c'	**1.4203**	

CHECK. 1st solution.

$$\frac{b + a}{c} = \frac{\cos \frac{1}{2}(B - A)}{\sin \frac{1}{2}C} = x,$$

$\log x = \log(b + a) - \log c,$

$\log x$

$= \log \cos \frac{1}{2}(B - A)$

$- \log \sin \frac{1}{2}C.$

$b + a$	18.5030
$B - A$	5° 47.3′
$\frac{1}{2}(B - A)$	2° 53.65′
$\frac{1}{2}C$	48° 52.15′
log$(b + a)$	1.26724
log c	1.14471
log x	0.12253
log cos $\frac{1}{2}(B - A)$	9.99944 − 10
log sin $\frac{1}{2}C$	9.87692 − 10
log x	0.12252

EXERCISES VII. F

Solve all possible triangles in the following set, and find their areas:

1. $a = 62.518$, $b = 72.932$, $B = 98° 23.5'$.
2. $a = 429.15$, $c = 328.12$, $A = 130° 33.7'$.
3. $b = 3912.7$, $c = 3526.5$, $C = 35° 25.8'$.
4. $b = 12968$, $c = 1529.6$, $B = 38° 28.6'$.
5. $a = 86.425$, $c = 73.463$, $C = 49° 18.9'$.
6. $b = 223.46$, $c = 327.92$, $C = 116° 19.6'$.

7. $b = 0.32492$, $c = 0.52392$, $B = 27° 49.3'$.
8. $a = 5660.1$, $c = 8442.0$, $A = 42° 6.2'$.
9. $b = 45.872$, $c = 56.321$, $B = 20° 14.5'$.
10. $a = 57.147$, $b = 46.703$, $B = 19° 17.8'$.
11. $a = 515.55$, $c = 524.31$, $A = 80° 52.2'$.

12. Two lighthouses are 3.276 miles apart, and a certain rock is 4.835 miles from one of them. The angle subtended by the two lighthouses at the rock is 15° 22′. How far is the rock from the other lighthouse? (Two solutions.)

13. The diagonals of a parallelogram intersect at an angle of 52° 10.2′. One diagonal is 3325 feet and one side is 2995 feet. Find the other diagonal. (Two solutions.)

60. Law of tangents.

Case III was solved by the law of cosines, but the method is not adapted to the use of logarithms. In the present sec-

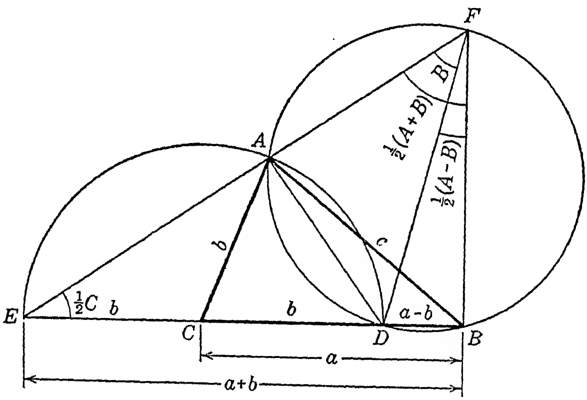


FIG. 52

tion we shall develop a formula which enables us to use logarithms in solving this case.

In triangle ABC, suppose that a is greater than b (Fig. 52). With C as center and b as radius, draw a circle cutting BC in D, and BC extended in E. Then,

$$BD = a - b, \qquad BE = a + b. \tag{1}$$

At B draw a perpendicular to BE. Draw EA and extend to meet this perpendicular in F. On DF as diameter construct a circle. This circle will pass through A; for FAD is a right angle, since it is supplementary to EAD, which is inscribed in a semicircle. The circle will also pass through B, since DBF is a right angle by construction.

It follows that $BEA = \frac{1}{2}C$, and that $BFE = \frac{1}{2}(A + B)$, since BFE is the complement of $\frac{1}{2}C$. Also, DFA and B are equal, since they are inscribed angles intercepting the same arc, AD. By subtraction we find $BFD = \frac{1}{2}(A - B)$.

Now in right triangles BDF and BEF we have respectively,

$$\frac{a - b}{BF} = \tan \tfrac{1}{2}(A - B), \qquad \frac{a + b}{BF} = \tan \tfrac{1}{2}(A + B). \tag{2}$$

Dividing the first of the foregoing equations by the second, we obtain

$$\frac{a - b}{a + b} = \frac{\tan \frac{1}{2}(A - B)}{\tan \frac{1}{2}(A + B)}. \tag{3}$$

This formula is one form of the **law of tangents.** Other forms may be obtained by a cyclic change of letters. If b were greater than a, we could interchange a and b, A and B, in (3). If a and b were equal the formula would still hold, but would be trivial, since both sides of the equation would be zero.

*61. Mollweide's equations.

From Fig. 52 we can obtain two formulas which are very serviceable in checking solutions of triangles.

Applying the law of sines to triangle ABD, we get

$$\frac{\sin DAB}{\sin BDA} \tag{1}$$

But $DAB = \frac{1}{2}(A - B)$, since DAB and DFB are inscribed angles intercepting the same arc. BD; and BDA

$= 90° + \frac{1}{2}C$, since BDA is an exterior angle of the triangle ADE. Since $\sin(90° + \frac{1}{2}C) = \cos \frac{1}{2}C$, (1) reduces to

$$\frac{a-b}{c} = \frac{\sin \frac{1}{2}(A-B)}{\cos \frac{1}{2}C}. \tag{2}$$

Applying the law of sines to triangle ABE, we get

$$\frac{a+b}{c} = \frac{\sin BAE}{\sin \frac{1}{2}C}. \tag{3}$$

But $BAE = A + \frac{1}{2}C = \frac{1}{2}(A + B + C) + \frac{1}{2}(A - B) = 90° + \frac{1}{2}(A - B)$. Thus, $\sin BAE = \cos \frac{1}{2}(A - B)$, and (3) becomes

$$\frac{a+b}{c} = \frac{\cos \frac{1}{2}(A-B)}{\sin \frac{1}{2}C}. \tag{4}$$

Formulas (2) and (4) are sometimes called **Mollweide's equations.*** Their advantage as checking formulas is that each contains all six parts of a triangle, and hence an error will be detected by a lack of agreement between the two members of one of these equations.

62. Logarithmic solution of Case III.

We are now ready to solve Case III by means of logarithms. The two angles are found by the law of tangents; the third side is then found by the law of sines. A check may be made by the law of sines or by one of Mollweide's equations.

Example.

Solve the triangle $a = 55.138$, $b = 33.094$, $C = 30° \, 24.6'$.

SOLUTION.

$$A + B = 180° - C.$$

$$\tan \frac{1}{2}(A - B) = \frac{a-b}{a+b} \tan \frac{1}{2}(A + B),$$

* The law of tangents can be obtained from Mollweide's equations by division.

$$\log\tan\tfrac{1}{2}(A-B)=\log(a-b)+\operatorname{colog}(a+b)$$
$$+\log\tan\tfrac{1}{2}(A+B).$$

a	55.138
b	33.094
C	30° 24.6′
$a-b$	22.044
$a+b$	88.232
$A+B$	149° 35.4′
$\frac{1}{2}(A+B)$	74° 47.7′
$\log(a-b)$	1.34329
$\operatorname{colog}(a+b)$	8.05437 − 10
$\log\tan\frac{1}{2}(A+B)$	0.56577
$\log\tan\frac{1}{2}(A-B)$	9.96343 − 10
$\frac{1}{2}(A-B)$	42° 35.4′
$\frac{1}{2}(A+B)$	74° 47.7′
A	**117° 23.1′**
B	**32° 12.3′**

$$c=\frac{b\sin C}{\sin B},$$

$$\log c=\log b+\log\sin C+\operatorname{colog}\sin B.$$

$\log b$	1.51975
$\log\sin C$	9.70431 − 10
$\operatorname{colog}\sin B$	0.27331
$\log c$	1.49737
c	**31.432**

CHECK.

$$c=\frac{a\sin C}{\sin A},$$

$$\log c=\log a+\log\sin C+\operatorname{colog}\sin A.$$

$\log a$	1.74145
$\log\sin C$	9.70431 − 10
$\operatorname{colog}\sin A$	0.05162
$\log c$	1.49738

EXERCISES VII. G

Solve the following triangles, and find their areas:

1. $a = 284.3$, $b = 286.5$, $C = 63°\,38'$.
2. $a = 49.366$, $b = 36.427$, $C = 47°\,16.6'$

3. $a = 36.508$, $b = 8.9156$, $C = 132° 18.3'$.
4. $b = 247.81$, $c = 513.58$, $A = 147° 8.8'$.
5. $a = 67.375$, $c = 36.858$, $B = 12° 28.5'$.
6. $b = 284.12$, $c = 362.12$, $A = 126° 32.2'$.
7. $a = 482.33$, $c = 395.71$, $B = 137° 31.2'$.
8. $a = 0.06350$, $c = 0.10391$, $B = 83° 29.4'$.
9. $b = 17976$, $c = 24824$, $A = 43° 36.2'$.
10. $a = 4216.4$, $b = 3125.2$, $C = 88° 10.1'$.
11. Two points, A and B, are at opposite ends of a lake. To find the distance between them, a point C is selected so that it is possible to measure a straight line from A to C and also from B to C. The distances AC and BC are measured and found to be 3472 feet and 2956 feet respectively. The angle ACB is measured by means of a transit, and is found to be 46° 25'. What is the distance from A to B?
12. Two sides of a triangular plot of ground are 256.8 feet and 198.2 feet respectively, the included angle being 65° 22'. Find (a) the length of fence required to enclose the plot, (b) the area of the plot.

★63. Heron's formula.

In this section and the following we shall derive formulas for the logarithmic solution of Case IV.

From formula (7) of section 51 we have

$$(\text{area})^2 = \tfrac{1}{4}b^2c^2 \sin^2 A, \tag{1}$$

and, since by exercise I. C, 24,*

$$\sin^2 A = 1 - \cos^2 A = (1 + \cos A)(1 - \cos A),$$

we have

$$(\text{area})^2 = \tfrac{1}{4}b^2c^2(1 + \cos A)(1 - \cos A). \tag{2}$$

By the law of cosines,

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}, \tag{3}$$

* This exercise covers only the case in which A is acute. The case in which A is obtuse is covered by (4) of section 68.

and consequently,

$$1 + \cos A = \frac{2bc + b^2 + c^2 - a^2}{2bc} = \frac{(b + c)^2 - a^2}{2bc}$$

$$= \frac{(b + c + a)(b + c - a)}{2bc}, \tag{4}$$

$$1 - \cos A = \frac{2bc - b^2 - c^2 + a^2}{2bc} = \frac{a^2 - (b - c)^2}{2bc}$$

$$= \frac{(a + b - c)(a - b + c)}{2bc}. \tag{5}$$

If we let

$$s = \tfrac{1}{2}(a + b + c), \tag{6}$$

then it can easily be shown that

$$b + c - a = 2(s - a), \qquad a + c - b = 2(s - b), \qquad a + b - c = 2(s - c). \tag{7}$$

Making use of (6) and (7) in (4) and (5), we find that

$$1 + \cos A = \frac{2s(s - a)}{bc},$$
$$1 - \cos A = \frac{2(s - b)(s - c)}{bc}. \tag{8}$$

Substituting these values in (2) and extracting the square root, we obtain **Heron's formula** for the area of a triangle:

$$\text{area} = \sqrt{s(s - a)(s - b)(s - c)}, \tag{9}$$

in which s is defined by (6), that is, it is the semiperimeter of the triangle.

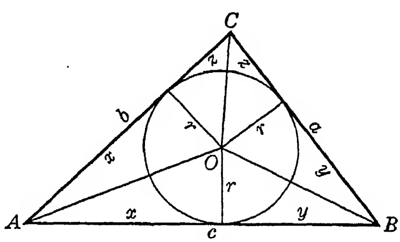


FIG. 53

64. Half-angle formulas.

In Fig. 53 the radius of the circle inscribed in triangle ABC is r. Then r is the altitude of each of the triangles AOB, BOC, COA, which have as a common vertex the center, O, of the circle. It

is readily seen that the area of the triangle ABC is given by the formula

$$\text{area} = \tfrac{1}{2}r(a + b + c) = rs, \tag{1}$$

where, as before $s = \frac{1}{2}(a + b + c)$.

But, by Heron's formula,

$$\text{area} = \sqrt{s(s - a)(s - b)(s - c)}. \tag{2}$$

Equating the two expressions for the area, we find that

$$r = \sqrt{\frac{(s - a)(s - b)(s - c)}{s}}. \tag{3}$$

Now let the equal tangents from A be denoted by x, those from B by y, and those from C by z. Adding all of these tangents, we get the perimeter of the triangle, or

$$2x + 2y + 2z = a + b + c = 2s. \tag{4}$$

From this it follows that $x + y + z = s$, and

$$x = s - y - z = s - a, \qquad y = s - b, \qquad z = s - c.$$

Consequently,

$$\tan \tfrac{1}{2}A = \frac{r}{s - a}, \quad \tan \tfrac{1}{2}B = \frac{r}{s - b}, \quad \tan \tfrac{1}{2}C = \frac{r}{s - c}, \tag{5}$$

in which r is given by (3), and

$$s = \tfrac{1}{2}(a + b + c). \tag{6}$$

Formulas (5) may be termed the **half-angle formulas.**

65. Logarithmic solution of Case IV.

The half-angle formulas enable us to use logarithms in solving Case IV.

Example.

Solve the triangle $a = 51.286$, $b = 65.353$, $c = 20.001$.

SOLUTION.

$s = \frac{1}{2}(a + b + c).$

$$r = \sqrt{\frac{(s-a)(s-b)(s-c)}{s}},$$

$\log r = \frac{1}{2}[\log(s-a) + \log(s-b) + \log(s-c) + \operatorname{colog} s].$

$$\tan \tfrac{1}{2}A = \frac{r}{s-a}, \text{ etc.,}$$

$\log \tan \frac{1}{2}A = \log r - \log(s-a)$, etc.

a	51.286
b	65.353
c	20.001
$2s$	136.640
s	68.320
$s-a$	17.034
$s-b$	2.967
$s-c$	48.319
CHECK. s	68.320
$\log(s-a)$	1.23131
$\log(s-b)$	0.47232
$\log(s-c)$	1.68412
$\operatorname{colog} s$	8.16545 − 10
$\log r^2$	1.55320
$\log r$	0.77660
$\log \tan \frac{1}{2}A$	9.54529 − 10
$\log \tan \frac{1}{2}B$	0.30428
$\log \tan \frac{1}{2}C$	9.09248 − 10
$\frac{1}{2}A$	19° 20.4′
$\frac{1}{2}B$	63° 36.4′
$\frac{1}{2}C$	7° 3.2′
A	**38° 40.8′**
B	**127° 12.8′**
C	**14° 6.4′**
$A+B+C$	180° 0.0′

CHECK. $A + B + C = 180°.$

It is an easy and valuable check to add the values of $s - a$, $s - b$, and $s - c$, as soon as these have been found. Since this gives $3s - a - b - c = 3s - 2s = s$, the sum should be equal to s. This simple check often prevents working the entire problem with an incorrect value for one of the expressions $s - a$, $s - b$, $s - c$.

For convenience in computing $\log \tan \frac{1}{2}A$, etc., $\log r$ may be written at the bottom of a slip of paper, and placed in turn above $\log(s - a)$, $\log(s - b)$, $\log(s - c)$.

EXERCISES VII. H

Solve the following triangles, and find their areas:

1. $a = 125.36, \qquad b = 176.43, \qquad c = 101.23.$

2. $a = 23.586$, $b = 25.743$, $c = 10.047$.
3. $a = 10.057$, $b = 19.436$, $c = 15.067$.
4. $a = 2249.8$, $b = 2467.2$, $c = 3152.6$.
5. $a = 50014$, $b = 70023$, $c = 90054$.
6. $a = 121.62$, $b = 9.8210$, $c = 113.94$.
7. $a = 42.391$, $b = 23.168$, $c = 51.833$.
8. $a = 0.98452$, $b = 0.67514$, $c = 0.81106$.
9. $a = 1.8943$, $b = 2.2465$, $c = 3.5488$.
10. $a = 0.11056$, $b = 0.05264$, $c = 0.17842$.
11. The sides of a triangular lot are 156.8 feet, 132.4 feet, and 148.3 feet respectively. Find the radius of the largest upright cylindrical tank that can be constructed on the lot.
12. In a triangle ABC, $a = 25.864$, $b = 26.232$, and the median from A is 20.866. Find the angles of the triangle, also side c.

66. Summary of methods.

The methods of solving oblique triangles are recapitulated below.

Case I. Two angles and a side given.	Use **law of sines**. Check by Mollweide's equation.
Case II. Two sides and the angle opposite one of them given.	Use **law of sines**. (Law of cosines may be used.) Note number of solutions. Check by Mollweide's equation.
Case III. Two sides and the included angle given.	If the sides are given to a small number of significant figures, or if only the third side is desired, **law of cosines** may be used. Find angles by **law of sines.** For logarithmic solution, use **law of tangents** to find angles. Find third side by **law of sines.** Check by Mollweide's equation.
	If the sides are given to a small number of significant figures, or if only one angle is desired, **law of cosines** may be used. For logarithmic solution, use **half-angle formulas.** Check by $A + B + C = 180°$

Note that an alternative check to Mollweide's equations is provided by the law of tangents.

To find the area of a triangle we can always resort to the fundamental formula of half the product of the base and the altitude. However, the formula

$$\text{area} = \tfrac{1}{2}bc \sin A$$

(and the others obtained from it by a cyclic change of letters) and Heron's formula are sometimes useful. (See also exercise VII. I, 47.)

MISCELLANEOUS EXERCISES VII. I

Solve the following triangles, and find their areas:

1.	$A = 55° 23.2'$,	$B = 72° 20.9'$,	$a = 537.14$.
2.	$A = 87° 58.4'$,	$a = 119.51$,	$b = 72.486$.
3.	$B = 19° 58.4'$,	$C = 94° 39.8'$,	$a = 4.3612$.
4.	$A = 34° 39.6'$,	$b = 61.519$,	$c = 47.612$.
5.	$a = 0.74261$,	$b = 0.10398$,	$c = 0.67517$.
6.	$C = 11° 14.3'$,	$b = 14.433$,	$c = 9.4670$.
7.	$C = 26° 36.6'$,	$a = 273.18$,	$b = 479.63$.
8.	$a = 1960.4$,	$b = 1093.3$,	$c = 2601.3$.
9.	$B = 127° 9.3'$,	$a = 67517$,	$c = 10398$.
10.	$B = 32° 18.0'$,	$a = 480.01$,	$b = 312.39$.
11.	$A = 53° 7.8'$,	$C = 45° 40.0'$,	$b = 374.85$.
12.	$B = 73° 44.4'$,	$C = 87° 20.1'$,	$c = 712.25$.
13.	$B = 104° 15.0'$,	$a = 7.3515$,	$c = 4.9764$.
14.	$B = 75° 45.0'$,	$a = 735.15$,	$b = 983.97$.
15.	$a = 31.628$,	$b = 68.235$,	$c = 52.063$.
16.	$a = 592.45$,	$b = 285.77$,	$c = 585.48$.
17.	$A = 43° 36.2'$,	$B = 102° 40.8'$,	$c = 392.37$.
18.	$C = 43° 35.6'$,	$b = 74.591$,	$c = 34.191$.
19.	$C = 51° 59.9'$,	$a = 228.15$,	$b = 109.84$.
20.	$a = 0.45562$,	$b = 0.32897$,	$c = 0.43129$.

21. Two sides of a parallelogram are 694.50 feet and 418.32 feet respectively; one diagonal is 602.94 feet. Find the length of the other diagonal.

22. The bases of a trapezoid are 397.62 and 254.15 respectively;

the angles that the sides make with the longer base are $68° 39.2'$ and $72° 6.0'$. Find the sides and the diagonals.

23. The sides of a triangular field are $AB = 193.8$ feet, $BC = 139.8$ feet, and $CA = 218.3$ feet. If the bearing of AB is N 20° E,* find the bearings of BC and CA, it being given that C is west of AB.

24. Let A, B, C represent three consecutive mileposts on a straight road. From each of these a distant spire is observed. At A it is northeast, at B it is east, and at C it is E 30° S. Find the distance of the spire from B, and the shortest distance from the road to the spire.

25. Along one bank of a river with parallel banks, a surveyor lays off a base line, AB, 600.0 feet long. From each end of the line an object C on the opposite bank is sighted. The angles which the lines of sight make with the base line are $62° 5.3'$ and $81° 34.7'$ respectively. Find the width of the river.

26. Points A and B are on opposite sides of a body of water, and soundings are to be taken in the line AB at points one-quarter, one-half, and three-quarters of the distance from A to B. On the shore, a base line AC is laid off, and it is found that angle $BAC = 63° 19'$, angle $ACB = 78° 43'$. What angles must be turned from CA at C in order to line up the boat from which the soundings are made at the proper points on the line AB?

27. In order to measure the distance between two inaccessible points, A and B, a base line, CD, 1168.2 feet in length was laid off. The following angles were then measured: $ACD = 132° 29'$, $ACB = 82° 20'$, $ADC = 45° 59'$, $BDC = 124° 48'$. Find the distance AB.

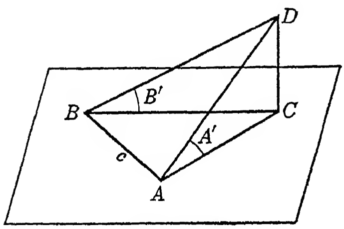


FIG. 54

28. It is required to find the horizontal distance and the vertical distance from a point A to an inaccessible point D, when it is not convenient to measure a base line in the same vertical plane with D. (See Fig. 54.) Draw AB, of length c, in any

* This means that the line drawn from A to B makes an angle of 20° with north, measured toward east.

convenient direction, in a horizontal plane. Let C be the foot of the perpendicular from D to this plane. Let A' and B' be the angles of elevation of D from A and B respectively. Show that

$$AC = \frac{c \sin B}{\sin C}, \qquad BC = \frac{c \sin A}{\sin C},$$

$$CD = \frac{c \sin A \tan B'}{\sin C} \qquad \frac{c \sin B \tan A'}{\sin C}.$$

where A, B, C are the angles of the triangle ABC. The height CD can be found from both formulas in order to check.

29. In the preceding exercise let $AB = 1255$ feet, $ABC = 46° 27'$, $BAC = 54° 40'$, $A' = 38° 42'$. Find AC, CD, B'.
30. Two boundary lines of a piece of property intersect at an angle of 85°. It is desired to cut off a triangular portion of the property which will be one acre (43560 square feet) in area by means of a straight fence. If the fence begins at a point on one boundary 250 feet from the corner of the property, and runs in a straight line to the other boundary, what angles does it make with the boundary lines, and how long is it?
31. To measure across a pond from A to B, a point C is selected so that $AC = 489$ feet, $BC = 674$ feet, and angle $ACB = 78° 45'$. Find the distance AB.
32. The diagonals of a parallelogram are 56.5 yards and 78.4 yards respectively. They intersect at an angle of 51° 35'. Find the area of the parallelogram.
33. A chimney projects 6 feet above a roof. At a point 10 feet 8 inches down the roof from the base of the chimney, the chimney subtends an angle of 17° 40'. Find the angle at which the roof is inclined to the horizontal.
34. The sides of a triangle are 14.832, 16.987, 18.645 respectively. Find the length of the perpendicular from the vertex of the largest angle to the side opposite.
35. The sides of a triangular grass plot are 47.5, 64.5, and 85 feet respectively. Find the minimum radius of action of an automatic lawn sprinkler which will water all parts of the plot simultaneously.

36. Find the radius of the largest circular flower bed which can be constructed on the plot of the preceding exercise.

37. The sum of the sides of a triangle is 100 inches. The angles are in the continued proportion 1 : 2 : 4. Find the sides.

38. Find the number of square yards of canvas in a conical tent, if the angle between the axis of the cone and an element is 30°, and the center pole is 14 feet high.

39. The sides of a triangular field which contains 15 acres are in the continued proportion 3 : 5 : 7. Find the sides. (1 acre = 160 sq. rd.)

40. Prove that the area of a quadrilateral is equal to half the product of its diagonals multiplied by the sine of their included angle.

41. A point A is in the same horizontal plane as the base of a radio tower. From this point a horizontal line AB, of length d, is drawn directly toward the tower. If the angle of elevation of the top of the tower from the point A is denoted by A, and the angle of elevation from the point B is denoted by B, show that the height of the tower is

$$\frac{d \sin A \sin B}{\sin(B - A)}.$$

42. A flagpole of height k stands on top of a building. From a certain point of observation in the same horizontal plane as the base of the building, the angle of elevation of the top of the pole is A, the angle of elevation of the bottom of the pole is B. Show that the distance d to the building from the point of observation, and the height h of the building are

$$d = \frac{k \cos A \cos B}{\sin(A - B)}, \qquad h = \frac{k \cos A \sin B}{\sin(A - B)}$$

43. In a triangle ABC, D is the intersection of the median from A and the bisector of angle C. Prove that

$$a \times \text{area } ABC = (a + 2b) \times \text{area } BCD.$$

44. On the sides of a triangle ABC are constructed isosceles triangles with their vertices on the circumference of the circumscribed circle of the given triangle. Show that their areas are in the ratio

$$b^2$$

$$s - a \qquad - b \qquad - c$$

where $s = \frac{1}{2}(a + b + c)$.

45. Prove the formulas:

$$\sin \tfrac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{bc}} \qquad \cos \tfrac{1}{2}A = \sqrt{\frac{s(s-a)}{bc}}$$

46. Prove that the area of a triangle is given by the formula

$$\frac{c^2 \sin A \sin B}{2 \sin(A + B)}.$$

47. Prove that the area of a triangle is given by the formula $abc/4R$, where R is the radius of the circumscribed circle.

48. Find the angle between the diagonal of a cube and the diagonal of a face of the cube, both diagonals drawn from the same vertex.

49. From one corner of a cube lines are drawn in two of its faces, making angles of 30° and 40° respectively with the common edge of these faces. Find the angle between the two lines.

50. A rectangular solid is 5 inches long, 4 inches wide, and 3 inches high. From one vertex a diagonal is drawn in each of the three faces having this vertex in common. Find the angles between these diagonals.

*67. Vectors.

If an object is at the point A in Fig. 55, and is displaced (i.e. moved) to the point B, the displacement may be represented by the directed line segment AB. (The arrow indicates the direction.) It will be noted that this line segment represents both the amount and the direction of the displacement. Now let BC represent another displacement. If an object originally at A is given both of these displacements it will arrive at the point C. The order in which these displacements occur is immaterial; that is, the object may be moved from A to B and then from B to C, or it may be

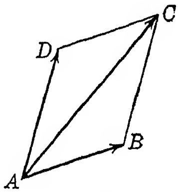


FIG. 55

moved from A to D (the displacement AD is equal and parallel to BC) and then from D to C. The displacement AC is called the **resultant** of the displacements AB and AD. (Cf. section 9.) Obviously, the resultant is a diagonal of the parallelogram of which AB and AD are sides. The displacements AB and AD are called **components** of AC.

It can be proved experimentally that two forces acting at the same point also combine into a resultant according to this so-called parallelogram law. Thus, if in Fig. 55, AB and AD represent, in magnitude and direction, two forces acting on an object at A, then the diagonal AC will represent, in magnitude and direction, the resultant of the two given forces. That is, the single force represented by AC will have the same effect on the object as the two forces represented by AB and AD.

Velocities and many other directed quantities (those which have direction as well as magnitude) also combine according to the parallelogram law. Such a quantity is called a **vector quantity.** The directed line segment representing the vector quantity is called a **vector.**

The resultant of any two vectors may of course be found graphically or geometrically by completing the parallelogram of which they form the adjacent sides, and drawing the diagonal. This is called the " addition " of the vectors. They may also be " added " by placing the initial point of one on the terminal point of the other, preserving the proper direction of each, and then drawing a third vector from the initial point of the first to the terminal point of the second. This can be seen by reference to Fig. 55.

A knowledge of trigonometry is essential in dealing with vectors. Its application may be illustrated by the following examples.

Example 1.

Three forces of 20, 30, and 40 pounds, respectively, are in equilibrium. Find the angles that they make with each other.

SOLUTION. Since the forces are in equilibrium, any one of them must be equal in magnitude and opposite in direction to the resultant of the other two. That is, we have a parallelogram in which the diagonal is, for example, 40, and in which the two sides are 20 and 30. (See Fig. 56.) Our problem is thus reduced to that of finding the angles of a triangle whose sides are 20, 30, and 40. This may be done by employing the law of cosines or the law of tangents. Since the numbers are simple, we shall use the former. Referring to the figure, we see that

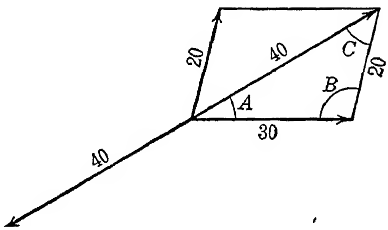


FIG. 56

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{(40)^2 + (30)^2 - (20)^2}{2 \cdot 40 \cdot 30} = 0.8750,$$

$$\cos B = \frac{c^2 + a^2 - b^2}{2ca} = \frac{(30)^2 + (20)^2 - (40)^2}{2 \cdot 30 \cdot 20} = -0.2500,$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{(20)^2 + (40)^2 - (30)^2}{2 \cdot 20 \cdot 40} = 0.6875;$$

$$A = 28° 57', \qquad B = 104° 29', \qquad C = 46° 34'.$$

CHECK. $A + B + C = 180° 00'.$

Therefore,

angle between 40-lb. and 30-lb. forces $= 180° - A = 151° \ 3'$,
angle between 30-lb. and 20-lb. forces $= 180° - B = 75° \ 31'$,
angle between 20-lb. and 40-lb. forces $= 180° - C = 133° \ 26'$.
CHECK. $360° \ 00'$.

It may be noted that since the forces are represented by the sides of the triangle ABC, the forces are proportional to the sines of the opposite angles.

Example 2.

An airplane having a speed of 120 miles an hour in calm air is pointed in a direction 30° east of north. A wind having a velocity

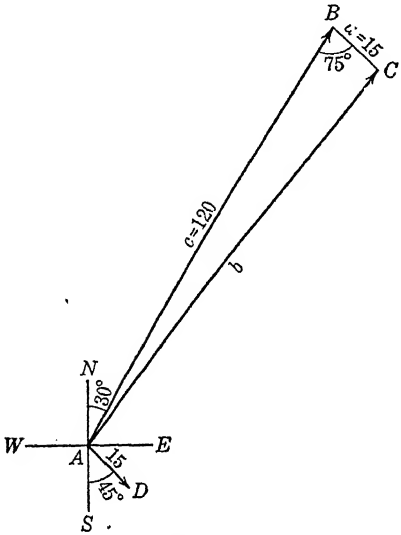


Fig. 57

of 15 miles an hour is blowing from the northwest. Find the speed and direction of the airplane relative to the ground.

Solution. Referring to Fig. 57, we see that the vector AB represents the velocity of the airplane due to its own power, and that the vector AD represents the velocity of the wind. We draw BC parallel and equal to AD, and connect A and C. Then AC represents the velocity of the airplane relative to the ground and is the vector required.

It is readily seen, if we draw a north-south line through B, that angle $B = 30° + 45° = 75$. Thus, in the triangle ABC, we have $a = 15$, $c = 120$, $B = 75°$. The numbers are simple, and we use the law of cosines, finding

$$\begin{aligned} b^2 &= a^2 + c^2 - 2ac\cos B \\ &= (15)^2 + (120)^2 - 2 \cdot 15 \cdot 120 \cdot \cos 75° \\ &= 13693.25, \\ b &= 117.0. \end{aligned}$$

Further,

$$\sin A = \frac{a \sin B}{b} = \frac{15 \sin 75}{117.0} = 0.1238,$$

$$A\ (= BAC) = 7°\,7', \qquad NAC = 30° + 7°\,7' = 37°\,7'.$$

Thus, the airplane actually travels in a direction 37° 7′ east of north at a speed of 117 miles per hour relative to the ground.

EXERCISES VII. J

1. Two forces of 8 and 11 pounds respectively act at an angle of 75° with each other. Find the magnitude of their resultant, and the angle that it makes with the 8-pound force.
2. Three forces of 7, 9, and 13 pounds respectively are in equilibrium. Find the angles that they make with each other.

3. A train is traveling at the rate of 30 miles an hour, and rain is falling with a velocity of 22 feet a second, at an angle of 30° with the vertical and in the same direction as the motion of the train. Find the direction of the splashes made on the windows of the coaches by the raindrops.
4. A motorboat which has a speed of 15 miles an hour in still water sets out to cross a stream which has a current of 5 miles an hour. The boat points upstream at an angle of 30° with the bank. Find its actual speed and the actual direction that it takes.
5. If a force of 100 pounds is resolved into components of 60 pounds and 50 pounds respectively, what angle do these components make with each other?
6. An airplane has a speed of 150 miles an hour in still air. The pilot wishes to fly in a direction 65° east of north. A 15-mile wind is blowing from the southeast. In what direction must the airplane be pointed?
7. The actual velocity of a motorboat is 25 miles an hour due north. The wind is blowing from the direction N 50° W at the rate of 15 miles an hour. What is the apparent velocity of the wind, and from what direction does it seem to strike the boat?
8. Two forces of 475 and 530 pounds respectively, making an angle of 36° 35′ with each other, act at the same point. Find the magnitude of their resultant, and the angle that it makes with the smaller force.
9. Three forces of 255, 320, and 195 pounds respectively are in equilibrium. What angles do they make with each other?
10. An airplane has a speed of 120 miles an hour in still air. A 20-mile wind is blowing from the northwest. A pilot wishes to fly 200 miles west and return to his original position. In what direction must he point the airplane (a) on the outward trip? (b) on the return trip?

CHAPTER VIII

Trigonometric Formulas and Identities

68. Fundamental relations among the functions.

It is readily seen, from the generalized definitions of section 37, that the functions of any angle satisfy the same **reciprocal relations** as the functions of an acute angle, namely,

$$
\begin{aligned}
\csc\theta &= \frac{1}{\sin\theta}, & \sin\theta &= \frac{1}{\csc\theta}, \\
\sec\theta &= \frac{1}{\cos\theta}, & \cos\theta &= \frac{1}{\sec\theta}, \\
\cot\theta &= \frac{1}{\tan\theta}, & \tan\theta &= \frac{1}{\cot\theta}
\end{aligned}
\tag{1}
$$

The following relations can also be readily proved:

$$\tan\theta = \frac{\sin\theta}{\cos\theta}, \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}. \tag{2}$$

The first can be proved by making use of the definitions of the functions. For,

$$\frac{\sin\theta}{\cos\theta} = \frac{\frac{y}{r}}{\frac{x}{r}} = \frac{y}{x} = \tan\theta.$$

The second follows from the fact that $\cot\theta = 1/\tan\theta$, or it can be proved independently.

Starting from the equation

$$x^2 + y^2 = r^2, \tag{3}$$

which may be obtained from Fig. 34 (page 67) by applying the theorem of Pythagoras, we can derive three more fundamental relations.

Dividing (3) by r^2, we get

$$\frac{x^2}{r^2} + \frac{y^2}{r^2} = 1,$$

which, since $x/r = \cos\theta$ and $y/r = \sin\theta$, can be written

$$\cos^2\theta + \sin^2\theta = 1. \tag{4}$$

Dividing (3) by x^2, we get

$$1 + \frac{y^2}{x^2} = \frac{r^2}{x^2},$$

which becomes

$$1 + \tan^2\theta = \sec^2\theta. \tag{5}$$

Finally, dividing (3) by y^2, we get

$$\frac{x^2}{y^2} + 1 = \frac{r^2}{y^2},$$

or

$$\cot^2\theta + 1 = \csc^2\theta. \tag{6}$$

Relations (4), (5), (6) may be termed the **Pythagorean relations.** They may be written in different forms if desirable; for example, (4) may be transformed as follows:

$$\cos^2\theta = 1 - \sin^2\theta, \qquad \text{or} \qquad \cos\theta = \pm\sqrt{1 - \sin^2\theta}.$$

69. Finding the other functions of an angle when one function is given.

The foregoing formulas may be used to find the values of the functions of an angle when the value of one function is given. However, the method used in section 4 for functions of acute angles is preferable.

Example 1.

Given $\sin\theta = \frac{3}{5}$; find the other functions of θ.

SOLUTION. Since $\sin\theta = y/r$, we may take $r = 5$, from which it follows that $y = 3$. Draw a circle with its center at the origin and having a radius of 5 units (Fig. 58). Take a point on the y-axis at a distance of 3 units above the x-axis. A line through this point parallel to the x-axis will cut the circle in two points, and consequently there will be two positions for the angle θ: θ_1 in quadrant I, and θ_2 in quadrant II, as shown in the figure.

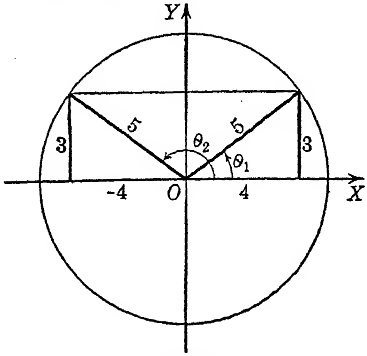


FIG. 58

Now,

$$x^2 = 5^2 - 3^2 = 16, \qquad x = \pm 4.$$

Thus, corresponding to the angle in quadrant I we have an abscissa 4, and corresponding to the angle in quadrant II we have an abscissa -4. We can now read all of the functions of both angles directly from the figure.

Quadrant I	Quadrant II
$\sin\theta_1 = \frac{3}{5}$,	$\sin\theta_2 = \frac{3}{5}$,
$\cos\theta_1 = \frac{4}{5}$,	$\cos\theta_2 = -\frac{4}{5}$,
$\tan\theta_1 = \frac{3}{4}$,	$\tan\theta_2 = -\frac{3}{4}$,
$\csc\theta_1 = \frac{5}{3}$,	$\csc\theta_2 = \frac{5}{3}$,
$\sec\theta_1 = \frac{5}{4}$,	$\sec\theta_2 = -\frac{5}{4}$,
$\cot\theta_1 = \frac{4}{3}$.	$\cot\theta_2 = -\frac{4}{3}$.

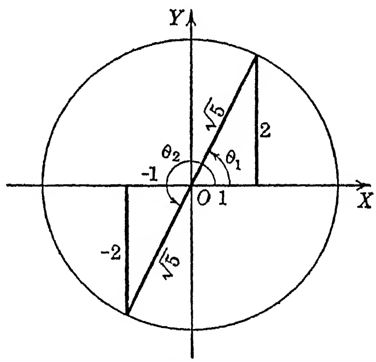


FIG. 59

Example 2.

Given $\tan\theta = 2$; find the other functions.

SOLUTION. Since $\tan\theta = y/x$, we may take $y = 2$ and $x = 1$, or $y = -2$ and $x = -1$ (Fig. 59). There are two angles, one in quadrant I, the other in quadrant III. In either case,

$$r^2 = 1^2 + 2^2 = 5, \quad r = \sqrt{5}.$$

(We take only the positive square root as the value of r, according to the agreement of section 35.) From the figure we read

Quadrant I	Quadrant III
$\sin\theta_1 = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$	$\sin\theta_2 = \frac{-2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$
$\cos \quad \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5},$	$\cos\theta_2 = \frac{-1}{\sqrt{5}} = \quad \frac{\sqrt{5}}{5}$
$\tan\theta_1 = 2,$	$\tan\theta_2 = \frac{-2}{-1} = 2,$
$\csc\theta_1 = \frac{\sqrt{5}}{2},$	$\csc\theta_2 = \frac{\sqrt{5}}{-2} = -\frac{\sqrt{5}}{2},$
$\sec\theta_1 = \sqrt{5},$	$\sec\theta_2 = \frac{\sqrt{5}}{-1} = -\sqrt{5},$
$\cot\theta_1 = \frac{1}{2}.$	$\cot\theta_2 = \frac{-1}{-2} = \frac{1}{2}.$

EXERCISES VIII. A

Find the other functions of θ, given that

1. $\sin\theta = \frac{12}{13}$, θ in quadrant I.
2. $\cos\theta = -\frac{4}{5}$, θ in quadrant III.
3. $\tan\theta = -\frac{2}{3}$, θ in quadrant IV.
4. $\cot\theta = \frac{1}{5}$, θ in quadrant III.
5. $\cos\theta = -\frac{2}{5}$, θ in quadrant II.
6. $\csc\theta = -\frac{41}{9}$, θ in quadrant IV.
7. $\sec\theta = \sqrt{2}$, θ in quadrant IV.
8. $\sin\theta = \frac{5}{6}$, θ in quadrant II.
9. $\tan\theta = \frac{7}{24}$, θ in quadrant III.
10. $\csc\theta = \frac{17}{15}$, θ in quadrant II.

Find the other functions of θ if

11. $\sin\theta = \frac{1}{2}$.	**12.** $\cos\theta = \frac{2}{3}$.
13. $\tan\theta = -\frac{2}{5}$.	**14.** $\csc\theta = \frac{4}{3}$.
15. $\cot\theta = \frac{5}{2}$.	**16.** $\sec\theta = \frac{5}{4}$.
17. $\sec\theta = -2$.	**18.** $\cos\theta = -\frac{1}{4}$.
19. $\tan\theta = 0.5$.	**20.** $\sin\theta = -0.8$
21. $\csc\theta = 3$.	**22.** $\cos\theta = 0.2$.

23. $\tan\theta = -\sqrt{3}$.
24. $\csc\theta = -\frac{5}{3}$.
25. $\cos\theta = -\frac{1}{3}$.
26. $\tan\theta = -5$.
27. $\cot\theta = 0.1$.
28. $\sin\theta = -\frac{5}{6}$.
29. $\tan\theta = \sqrt{2}$.
30. $\cot\theta = 1$.

31. If $\sin\theta = \frac{7}{25}$ and $\cos\phi = \frac{15}{17}$, find all possible values of

(a) $\tan\theta + \tan\phi$,
(b) $\cos\theta + \sin\phi$,
(c) $5\sin\theta - 2\sin\phi$,
(d) $\sec\theta\tan\phi$,
(e) $\dfrac{1+\cot\theta}{\sin\phi}$,
(f) $\dfrac{1-\cos\theta}{1+\tan\phi}$,
(g) $(2+\cos\theta)(3-2\sin\phi)$,
(h) $(m+n\tan\theta)(m+n\cot\phi)$.

32. If $\tan\theta = \frac{21}{20}$ and $\cot\phi = -\frac{9}{40}$, find all possible values of

(a) $\sin\theta + \sin\phi$,
(b) $\cos\theta + \tan\phi$,
(c) $\frac{1}{3}\sin\theta + \frac{1}{5}\sin\phi$,
(d) $\sec\theta(2-3\cos\phi)$,
(e) $\csc\theta\sec\phi$,
(f) $\sin\theta\cos\phi + \cos\theta\sin\phi$,
(g) $\dfrac{\sec\phi}{1+\frac{1}{2}\cos\theta}$,
(h) $\dfrac{\tan\theta - \tan\phi}{1+\tan\theta\tan\phi}$

70. Identities.

Formulas (1), (2), (4), (5), (6) of section 68 are **identities**, in the sense that they are satisfied by all possible values of θ for which their left-hand and right-hand members are defined. By means of them it is possible to prove other identities, and consequently to change an expression involving trigonometric functions into a different but equivalent form which is more suitable for the purpose at hand.

Example 1.

Prove: $\tan\theta + \cot\theta = \sec\theta\csc\theta.$

SOLUTION. To reduce the expression on the left to that on the right we first make use of (2) of section 68:

$$\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = \frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}.$$

But by (6) of section 68, the last numerator is equal to 1, and the above expression reduces to

$$\frac{1}{\cos\theta\sin\theta},$$

which, because of the reciprocal relations, is equal to sec θ csc θ. Thus, we have reduced the left-hand side to the right-hand side and have consequently proved the identity.

Example 2.

Prove: $$\frac{1 + \tan^2 \theta}{\csc \theta} = \sec \theta \tan \theta.$$

Solution. Applying the Pythagorean relation (5) of section 68 to the numerator on the left, we reduce the fraction to

$$\frac{\sec^2}{\csc \theta} \qquad \frac{\sec \theta \dfrac{1}{\cos \theta}}{\dfrac{1}{\sin \theta}} = \sec \theta \frac{\sin \theta}{\cos \theta}.$$

This, by the first of equations (2) of section 68, reduces to sec θ tan θ, and the identity is established.

Ordinarily, in proving an identity, one must transform one side into the other. No general method of proof can be given. However, a thorough familiarity with the fundamental identities is essential. These should be kept constantly in mind, and careful consideration should be given to the question of which one of them is appropriate to the situation. There should also be kept in mind the expression toward which one is working. It is usually better to work with the more complicated side of the identity, endeavoring to reduce it to the form of the simpler side.

Frequently, if all functions are expressed in terms of sines and cosines, a clue will be obtained as to the next step to take.

If one side of the identity involves but one function, it may be best to express everything on the other side in terms of that function.

It is usually best to avoid radical expressions when possible.

EXERCISES VIII. B

Prove the following identities:

1. $\cos\theta\tan\theta = \sin\theta.$
2. $\cot\theta\cos\theta = \csc\theta - \sin\theta.$
3. $\dfrac{1+\sin\theta}{\cos\theta} = \dfrac{\cos\theta}{1-\sin\theta}.$
4. $(\tan\theta - \sin\theta)^2 + (1-\cos\theta)^2 = (1-\sec\theta)^2.$
5. $\dfrac{\cos^2\theta}{1-\sin\theta} = 1+\sin\theta.$
6. $\cot\theta + \tan\theta = \dfrac{\csc^2\theta+\sec^2\theta}{\csc\theta\sec\theta}.$
7. $\dfrac{\sin\theta+\tan\theta}{\cot\theta+\csc\theta} = \sin\theta\tan\theta.$
8. $\dfrac{1-2\cos^2\theta}{\sin\theta\cos\theta} = \tan\theta \quad \cot\theta.$
9. $(\sin\theta+\cos\theta)^2 + (\sin\theta-\cos\theta)^2 = 2.$
10. $\sin^4\theta - \cos^4\theta = \sin^2\theta - \cos^2\theta.$
11. $\tan^2\theta - \sin^2\theta = \tan^2\theta\sin^2\theta.$
12. $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta.$
13. $\dfrac{\csc\theta}{\csc\theta-1} + \dfrac{\csc\theta}{\csc\theta+1} = 2\sec^2\theta.$
14. $\dfrac{1-\tan\theta}{1+\tan\theta} = \dfrac{\cot\theta-1}{\cot\theta+1}$
15. $\dfrac{\tan^2\theta}{\sec^2\theta} + \dfrac{\cot^2\theta}{\csc^2\theta} = 1.$
16. $\dfrac{\sin\theta+\cos\phi}{\sin\theta-\cos\phi} = \dfrac{\sec\phi+\csc\theta}{\sec\phi-\csc\theta}.$
17. $(\tan\theta+\cot\phi)(\cot\theta-\tan\phi) = \cot\theta\cot\phi - \tan\theta\tan\phi.$
18. $(\tan\theta-\sec\phi)(\cot\theta+\cos\phi) = \tan\theta\cos\phi - \cot\theta\sec\phi.$
19. $\sin^2\theta(1+\cot^2\theta) = 1.$
20. $\cos\theta(1+\tan^2\theta) = \sec\theta.$
21. $\sin\theta(1+\cot^2\theta) = \csc\theta.$
22. $\dfrac{1+\sec\theta}{1-\sec\theta} = \dfrac{\cos\theta+1}{\cos\theta-1}.$
23. $\sec\theta - \sin\theta\tan\theta = \cos\theta.$

24. $\dfrac{1 - \tan^2 \theta}{1 - \cot^2 \theta} = 1 - \sec^2$

25. $\tan \theta + \tan(90° - \theta) = \sec \theta \csc \theta.$

26. $\dfrac{\tan \theta + \sin \theta}{\tan \quad - \sin \theta} - \dfrac{\sec \theta + 1}{\sec \theta - 1}.$

27. $\dfrac{\text{si} \quad \hat{}}{1 + \cos \theta} = \csc \theta - \cot \theta.$

28. $\sec^4 \theta - \tan^4 \theta = 1 + 2 \tan^2 \theta.$

29. $\dfrac{1 - \tan^2 \theta}{1 + \tan^2 \theta} = \cos^2 \theta - \sin^2 \theta.$

30. $\dfrac{\tan \theta - \tan \phi}{\cot \theta - \cot \phi} = -\tan \theta \tan \phi.$

31. $\dfrac{\cos \theta}{\cos \theta - \sin \theta} = \dfrac{1}{1 - \tan \theta}.$

32. $\dfrac{\tan \theta}{\sin^2 \theta} = \pm\sqrt{\dfrac{1 + \tan^2 \theta}{1 - \cos^2}}$

33. $\dfrac{\tan \theta + \tan \phi}{\cot \theta + \cot \phi} = \tan \theta \tan \phi.$

34. $(1 - \cos^2 \theta)(1 + \cot^2 \theta) = 1.$

35. $\dfrac{1}{\sec \theta + \tan \theta} = \sec \theta - \tan \theta.$

36. $\dfrac{\sin \theta + \tan \theta}{1 + \sec \theta} = \sin \theta.$

37. $\dfrac{\cos \theta}{\sec \theta} \quad \dfrac{\sin \theta}{\cot \theta} \quad \dfrac{\cos \theta \cot \theta - \tan}{\csc \theta}$

38. $\dfrac{}{1 - \sin \theta} = \dfrac{\cot \theta}{\cot \theta - \cos \theta}$

39. $\dfrac{\cos \theta}{1 - \tan \theta} + \dfrac{\sin \theta}{1 - \cot} = \sin \theta + \cos \theta.$

40. Express $\sin \theta$ in terms of $\tan \theta$.

SOLUTION.

$$\frac{\sin \theta}{\cos \theta} = \tan \theta,$$

$\tan \theta,$

$$\frac{\sin^2\theta}{1-\sin^2\theta} = \tan^2\theta,$$

$$\sin^2\theta = \tan^2\theta - \tan^2\theta\sin^2\theta,$$

$$(1+\tan^2\theta)\sin^2\theta = \tan^2\theta,$$

$$\sin^2\theta = \frac{\tan^2\theta}{1+\tan^2\theta},$$

$$\sin\theta = \pm\frac{\tan\theta}{\sqrt{1+\tan^2\theta}}$$

The exercise can also be solved as follows: Draw a right triangle having an acute angle θ. Mark the opposite side $\tan\theta$, the adjacent side 1. Then the hypotenuse will be $\sqrt{1+\tan^2\theta}$. The value of $\sin\theta$ can now be read from the figure. (Cf. section 69.) The double sign should be used with the radical.

41. Construct a table giving each of the functions in terms of the other functions.

71. Directed line segments.

In defining rectangular coordinates, we introduced the idea of a positive and a negative direction on a line. Thus, the positive direction on the x-axis is to the right, the positive direction on the y-axis is upward. Any line, such as one of these axes, on which the positive direction has been specified, is a **directed line.** A portion of a directed line, such as AB in Fig. 60, is called a **directed line segment.** The point A may be called the **initial point** and the point B the **terminal point** of the line segment AB.

FIG. 60

Two line segments may be added by placing the initial point of the second on the terminal point of the first; the sum is the segment from the initial point of the first segment to the terminal point of the second. (It is immaterial which segment is considered the first and which the second.) The proper direction must, of course, be preserved for each segment.

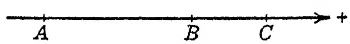


FIG. 61

Thus, if A, B, C are points arranged in any order on a directed line, we may write

$$AB + BC = AC,$$

which merely states that if we go from A to B and then from B to C, we reach the same position that we reach by going directly from A to C.

Subtraction of two directed line segments is accomplished by changing the direction of the segment to be subtracted, and then proceeding as in addition.

Several segments can be added by carrying out successively the process described for two segments.

72. Functions of the sum and the difference of two angles.

To derive a formula for $\cos(\theta + \phi)$, place the angles θ and ϕ with reference to the coordinate axes as shown in Fig. 62. Take a point P on the terminal side of the angle $\theta + \phi$, and drop a perpendicular PQ to the terminal side of θ. Draw PM and QN perpendicular to the x-axis.

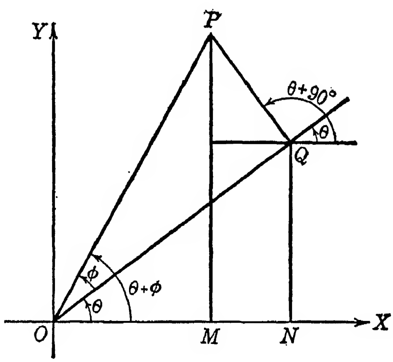


FIG. 62

Now, if we take into consideration the signs of the line segments involved, we have

$$OM = ON + NM. \tag{1}$$

But
$$\begin{aligned} OM &= OP\cos(\theta + \phi), \qquad ON = OQ\cos\theta, \\ NM &= QP\cos(90^\circ + \theta) = -QP\sin\theta. \end{aligned} \tag{2}$$

Substituting these values in (1), we get

$$OP\cos(\theta + \phi) = OQ\cos\theta - QP\sin\theta.$$

Division by OP gives

$$\cos(\theta + \phi) = \frac{OQ}{OP}\cos\theta - \frac{QP}{OP}\sin\theta.$$

But
$$\frac{OQ}{OP} = \cos\phi, \qquad \frac{QP}{OP} = \sin\phi,$$

and consequently,

$$\boldsymbol{\cos(\theta + \phi) = \cos\theta\cos\phi - \sin\theta\sin\phi.} \tag{3}$$

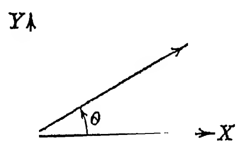


FIG. 63

The foregoing proof will hold for all values of θ and ϕ if we are careful to take into consideration the proper sign of each function and of each line segment involved. It will be necessary, however, to consider as negative a segment measured backward along the terminal side of an angle, such as segment OP in Fig. 63. In this figure r would be considered negative.

If in (3) we replace ϕ by $-\phi$, we get

$$\cos(\theta - \phi) = \cos\theta\cos(-\phi) - \sin\theta\sin(-\phi),$$

or
$$\boldsymbol{\cos(\theta - \phi) = \cos\theta\cos\phi + \sin\theta\sin\phi.} \tag{4}$$

To develop a formula for $\sin(\theta + \phi)$, we use (3), replacing θ by $90° - \theta$, and ϕ by $-\phi$. We get

$$\begin{aligned}\cos(90° - \theta - \phi) &= \cos[(90° - \theta) + (-\phi)] \\ &= \cos(90° - \theta)\cos(-\phi) - \sin(90° - \theta)\sin(-\phi),\end{aligned}$$

which becomes

$$\boldsymbol{\sin(\theta + \phi) = \sin\theta\cos\phi + \cos\theta\sin\phi.} \tag{5}$$

The foregoing formula can also be derived by dropping

perpendiculars from the points P and Q in Fig. 62 to the y-axis, and proceeding somewhat as in the proof of (3).

If in (5) we replace ϕ by $-\phi$, we get

$$\sin(\theta - \phi) = \sin\theta\cos(-\phi) + \cos\theta\sin(-\phi),$$

or
$$\boldsymbol{\sin(\theta - \phi) = \sin\theta\cos\phi - \cos\theta\sin\phi.} \qquad (6)$$

Formulas (3) and (5) are sometimes called the **addition formulas** for the cosine and sine respectively. Similarly, (4) and (6) may be called their **subtraction formulas.**

To find the tangent of $\theta + \phi$ and of $\theta - \phi$, we proceed as follows:

$$\tan(\theta + \phi) = \frac{\sin(\theta + \phi)}{\cos(\theta + \phi)} = \frac{\sin\theta\cos\phi + \cos\theta\sin\phi}{\cos\theta\cos\phi - \sin\theta\sin\phi}.$$

If it is desired to express $\tan(\theta + \phi)$ in terms of $\tan\theta$ and $\tan\phi$, we divide numerator and denominator of the last fraction by $\cos\theta\cos\phi$, obtaining

$$\tan(\theta + \phi) = \frac{\dfrac{\sin\theta\cos\phi}{\cos\theta\cos\phi} + \dfrac{\cos\theta\sin\phi}{\cos\theta\cos\phi}}{\dfrac{\cos\theta\cos\phi}{\cos\theta\cos\phi} - \dfrac{\sin\theta\sin\phi}{\cos\theta\cos\phi}}$$

which reduces to

$$\boldsymbol{\tan(\theta + \phi) = \frac{\tan\theta + \tan\phi}{1 - \tan\theta\tan\phi}.} \qquad (7)$$

In like manner, or by replacing ϕ by $-\phi$ in (7), we find that

$$\boldsymbol{\tan(\theta - \phi) = \frac{\tan\theta - \tan\phi}{1 + \tan\theta\tan\phi}.} \qquad (8)$$

For the cotangent we obtain the following formulas:

$$\cot(\theta + \phi) = \frac{\cot\theta\cot\phi - 1}{\cot\phi + \cot\theta} \qquad (9)$$

$$\cot(\theta - \phi) = \frac{\cot\theta\cot\phi + 1}{\cot\phi - \cot\theta} \qquad (10)$$

Proofs of (9) and (10) are left as exercises.

EXERCISES VIII. C

1. Find $\sin 75°$ by setting $\theta = 45°$, $\phi = 30°$ in (5) of section 72.

SOLUTION. $\sin 75° = \sin(45° + 30°)$

$$= \sin 45° \cos 30° + \cos 45° \sin 30°$$

$$= \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\frac{1}{2} = \tfrac{1}{4}(\sqrt{6} + \sqrt{2}).$$

2. Find $\cos 75°$, $\tan 75°$, $\cot 75°$.
3. Find $\sin 15°$, $\cos 15°$, $\tan 15°$, $\cot 15°$.
4. Verify the values of $\sin 90°$, $\cos 90°$, $\cot 90°$ by setting $\theta = 60°$, $\phi = 30°$ in (5), (3), (7), respectively, of section 72.
5. Verify the values of $\sin 30°$, $\cos 30°$, $\tan 30°$, $\cot 30°$ by setting $\theta = 60°$, $\phi = 30°$ in (6), (4), (8), (10), respectively, of section 72.
6. Find $\sin 105°$, $\cos 105°$, $\tan 105°$, $\cot 105°$.
7. Prove the formulas for $\sin(90° + \theta)$, $\cos(90° + \theta)$, $\tan(90° + \theta)$, $\cot(90° + \theta)$ by means of the addition formulas.
8. Prove the formulas for $\sin(180° - \theta)$, $\cos(180° - \theta)$, $\tan(180° - \theta)$, $\cot(180° - \theta)$ by means of the subtraction formulas.

Simplify the following expressions:

9. $\sin(\theta + 30°) + \cos(\theta + 60°)$.
10. $\sin(\theta + 60°) - \cos(\theta + 30°)$.
11. $\tan(\theta + 45°) + \cot(\theta - 45°)$.
12. $\cos(30° - \theta) - \cos(30° + \theta)$.

Prove the following identities:

13. $\sin(\theta + \phi)\sin(\theta - \phi) = \sin^2\theta - \sin^2\phi$.
14. $\cos(\theta + \phi)\cos(\theta - \phi) = \cos^2\theta - \sin^2\phi$.
15. $\tan(45° + \theta) = \dfrac{1 + \tan\theta}{1 - \tan\theta}$.
16. $\sin(45° + \theta)\cos(45° + \theta) = \tfrac{1}{2}(\cos^2\theta - \sin^2\theta)$.
17. $\sin(\theta + 30°)\cos(\theta + 60°) = \tfrac{1}{4}(\cos^2\theta - 3\sin^2\theta)$.

18. Given $\sin\theta = \frac{3}{5}$, $\sin\phi = \frac{5}{13}$, θ and ϕ both acute. Find
(a) $\sin(\theta + \phi)$, (b) $\cos(\theta + \phi)$, (c) $\tan(\theta + \phi)$,
(d) $\cot(\theta + \phi)$, (e) $\sin(\theta - \phi)$, (f) $\cos(\theta - \phi)$,
(g) $\tan(\theta - \phi)$, (h) $\cot(\theta - \phi)$, (i) $\sin(\phi - \theta)$,
(j) $\cos(\phi - \theta)$, (k) $\tan(\phi - \theta)$, (l) $\cot(\phi - \theta)$.

19. Given $\sin\theta = \frac{8}{17}$, $\tan\phi = \frac{9}{40}$, θ in quadrant II, ϕ in quadrant III. Find
(a) $\sin(\theta + \phi)$, (b) $\cos(\theta + \phi)$, (c) $\tan(\theta + \phi)$,
(d) $\cot(\theta + \phi)$, (e) $\sin(\theta - \phi)$, (f) $\cos(\theta - \phi)$,
(g) $\tan(\theta - \phi)$, (h) $\cot(\theta - \phi)$.

20. Given $\cos\theta = -\frac{4}{5}$, $\sin\phi = \frac{7}{25}$, θ in quadrant II. Find all possible values of the following:
(a) $\sin(\theta + \phi)$, (b) $\cos(\theta + \phi)$, (c) $\tan(\theta + \phi)$,
(d) $\cot(\theta + \phi)$, (e) $\sin(\theta - \phi)$, (f) $\cos(\theta - \phi)$,
(g) $\tan(\theta - \phi)$, (h) $\cot(\theta - \phi)$.

21. Given $\tan\theta = \frac{8}{15}$, $\cot\phi = \frac{12}{5}$. Find all possible values of
(a) $\sin(\theta + \phi)$, (b) $\cos(\theta + \phi)$, (c) $\tan(\theta + \phi)$,
(d) $\cot(\theta + \phi)$, (e) $\sin(\theta - \phi)$, (f) $\cos(\theta - \phi)$,
(g) $\tan(\theta - \phi)$, (h) $\cot(\theta - \phi)$.

Prove:

22. $$\sin(\theta + \phi + \psi) = \sin\theta\cos\phi\cos\psi + \cos\theta\sin\phi\cos\psi + \cos\theta\cos\phi\sin\psi - \sin\theta\sin\phi\sin\psi.$$

23. $$\cos(\theta + \phi + \psi) = \cos\theta\cos\phi\cos\psi - \cos\theta\sin\phi\sin\psi - \sin\theta\cos\phi\sin\psi - \sin\theta\sin\phi\cos\psi.$$

24. $$\tan(\theta + \phi + \psi) = \frac{\tan\theta + \tan\phi + \tan\psi - \tan\theta\tan\phi\tan\psi}{1 - \tan\phi\tan\psi - \tan\psi\tan\theta - \tan\theta\tan\phi}.$$

25. $$\cot(\theta + \phi + \psi) = \frac{\cot\theta\cot\phi\cot\psi - \cot\theta - \cot\phi - \cot\psi}{\cot\phi\cot\psi + \cot\psi\cot\theta + \cot\theta\cot\phi - 1}$$

73. Functions of twice an angle.

If, in formulas (5), (3), (7), (9) of section 72, we substitute θ for ϕ, we obtain the following results:

$$\sin(\theta + \theta) = \sin\theta\cos\theta + \cos\theta\sin\theta,$$

or
$$\sin 2\theta = 2 \sin \theta \cos \theta; \tag{1}$$

$$\cos(\theta + \theta) = \cos \theta \cos \theta - \sin \theta \sin \theta,$$

or
$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta; \tag{2}$$

$$\tan(\theta + \theta) = \frac{\tan \theta + \tan \theta}{1 - \tan \theta \tan \theta}$$

or

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}; \tag{3}$$

$$\cot(\theta + \theta) = \frac{\cot \theta \cot \theta - 1}{\cot \theta + \cot \theta}$$

or

$$\cot 2\theta = \frac{\cot^2 \theta - 1}{2 \cot \theta}. \tag{4}$$

Two other useful formulas for $\cos 2\theta$ may be derived as follows: Remembering that

$$\sin^2 \theta = 1 - \cos^2 \theta, \qquad \cos^2 \theta = 1 - \sin^2 \theta,$$

and substituting these separately in (2), we get

$$\cos 2\theta = 2 \cos^2 \theta - 1, \tag{5}$$

and

$$\cos 2\theta = 1 - 2 \sin^2 \theta. \tag{6}$$

74. Functions of half an angle.

From the relation connecting sine and cosine, and the formula for the cosine of twice an angle, we have

$$\cos^2 \phi + \sin^2 \phi = 1, \tag{1}$$

$$\cos^2 \phi - \sin^2 \phi = \cos 2\phi. \tag{2}$$

Adding these two equations, we get

$$2 \cos^2 \phi = 1 + \cos 2\phi.$$

From this we get

$$\cos^2 \phi = \frac{1 + \cos 2\phi}{2}$$

or

$$\cos \phi = \pm\sqrt{\frac{1 + \cos 2\phi}{2}}.$$

If ϕ is replaced by $\frac{1}{2}\theta$, this becomes

$$\boldsymbol{\cos \tfrac{1}{2}\theta = \pm\sqrt{\frac{1 + \cos \theta}{2}}} \tag{3}$$

By subtracting (2) from (1) and proceeding as before, we obtain the formula

$$\sin \phi = \pm\sqrt{\frac{1 - \cos 2\phi}{2}}$$

which is equivalent to

$$\boldsymbol{\sin \tfrac{1}{2}\theta = \pm\sqrt{\frac{1 - \cos \theta}{2}}} \tag{4}$$

The sign to be used in the foregoing formulas depends upon the quadrant in which $\frac{1}{2}\theta$ lies.

Dividing (4) by (3), we get

$$\tan \tfrac{1}{2}\theta = \pm\sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} \tag{5}$$

Multiplying numerator and denominator of the right-hand side of this last equation by $\sqrt{1 - \cos \theta}$, we get

$$\tan \tfrac{1}{2}\theta = \frac{1 - \cos \theta}{\pm\sqrt{1 - \cos^2 \theta}}$$

or

$$\boldsymbol{\tan \tfrac{1}{2}\theta = \frac{1 - \cos \theta}{\sin \theta}}. \tag{6}$$

Here the ambiguous sign ($\pm$) is not needed. For the numerator of the fraction in (6) is always positive (or zero),

so that the sign of the right-hand member depends upon the denominator, namely, $\sin\theta$. Now $\sin\theta$ will be positive if θ is in either of the first two quadrants, and negative otherwise. But if θ is in quadrant I or quadrant II, $\frac{1}{2}\theta$ will be in quadrant I; if θ is in quadrant III or quadrant IV, $\frac{1}{2}\theta$ will be in quadrant II. However, $\tan\frac{1}{2}\theta$ will be positive if $\frac{1}{2}\theta$ is in quadrant I, negative if $\frac{1}{2}\theta$ is in quadrant II. Thus, $\sin\theta$ and $\tan\frac{1}{2}\theta$ will always have the same sign, and there is no ambiguity.

If we multiply both numerator and denominator of the fraction in (5) by $\sqrt{1+\cos\theta}$, and reduce, we get

$$\tan\tfrac{1}{2}\theta = \frac{\sin\theta}{1+\cos\theta}, \tag{7}$$

where again there is no ambiguity.

Similarly, we obtain the formulas

$$\cot\tfrac{1}{2}\theta = \pm\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} \tag{8}$$

$$\cot\tfrac{1}{2}\theta = \frac{\sin\theta}{1-\cos\theta}, \tag{9}$$

$$\cot\tfrac{1}{2}\theta = \frac{1+\cos\theta}{\sin\theta}. \tag{10}$$

EXERCISES VIII. D

1. Verify the formulas for $\sin 2\theta$, $\cos 2\theta$, $\tan 2\theta$, $\cot 2\theta$ by setting $\theta = 30°$.
2. Verify the formulas for $\sin 2\theta$, $\cos 2\theta$, $\cot 2\theta$ by setting $\theta = 45°$.
3. Find $\sin 120°$, $\cos 120°$, $\tan 120°$, $\cot 120°$ by using the functions of $60°$.
4. Verify the formulas for $\sin\frac{1}{2}\theta$, $\cos\frac{1}{2}\theta$, $\tan\frac{1}{2}\theta$, $\cot\frac{1}{2}\theta$ by setting $\theta = 60°$.
5. Find $\sin 15°$, $\cos 15°$, $\tan 15°$, $\cot 15°$ by setting $\theta = 30°$ in the formulas for the functions of $\frac{1}{2}\theta$.

6. Given $\cos\theta = \frac{24}{25}$, θ an acute angle. Find
(a) $\sin 2\theta$, (b) $\cos 2\theta$, (c) $\tan 2\theta$, (d) $\cot 2\theta$,
(e) $\sin \frac{1}{2}\theta$, (f) $\cos \frac{1}{2}\theta$, (g) $\tan \frac{1}{2}\theta$, (h) $\cot \frac{1}{2}\theta$.

7. Given $\sin\theta = \frac{40}{41}$. Find
(a) $\sin 2\theta$, (b) $\cos 2\theta$, (c) $\tan 2\theta$, (d) $\cot 2\theta$,
(e) $\sin \frac{1}{2}\theta$, (f) $\cos \frac{1}{2}\theta$, (g) $\tan \frac{1}{2}\theta$, (h) $\cot \frac{1}{2}\theta$.

8. Given $\tan\theta = -2$. Find
(a) $\sin 2\theta$, (b) $\cos 2\theta$, (c) $\tan 2\theta$, (d) $\cot 2\theta$,
(e) $\sin \frac{1}{2}\theta$, (f) $\cos \frac{1}{2}\theta$, (g) $\tan \frac{1}{2}\theta$, (h) $\cot \frac{1}{2}\theta$.

Prove the following identities:

9. $\tan(45^\circ + \frac{1}{2}\theta) = \dfrac{1 + \cos\theta + \sin\theta}{1 + \cos\theta - \sin\theta}$.

10. $\sin\theta = \dfrac{2\tan\frac{1}{2}\theta}{1 + \tan^2\frac{1}{2}\theta}$.

11. $\tan\frac{1}{2}\theta + \cot\frac{1}{2}\theta = 2\csc\theta$.

12. A picture of height 5 feet hangs on the wall, with its lower edge 4 feet from the floor. At a certain point on the floor, directly in front of the picture, the angle subtended by the picture (that is, by its vertical dimension of 5 feet) is equal to the angle of elevation of the lower edge of the picture. How far is this point from the wall?

Prove:

13. $\sin\frac{1}{2}\theta + \cos\frac{1}{2}\theta \quad \pm\sqrt{1 + \sin\theta}$.

14. $\sin\frac{1}{2}\theta \quad \cos\frac{1}{2}\theta = \pm\sqrt{1 - \sin\theta}$.

75. Sums and differences of functions.

By the addition and subtraction formulas for the sine and cosine, we have

$$\sin(x + y) = \sin x \cos y + \cos x \sin y, \tag{1}$$

$$\sin(x - y) = \sin x \cos y - \cos x \sin y, \tag{2}$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y, \tag{3}$$

$$\cos(x - y) = \cos x \cos y + \sin x \sin y. \tag{4}$$

Addition of (1) and (2) gives

$$\sin(x + y) + \sin(x - y) = 2 \sin x \cos y. \tag{5}$$

If we let

$$x + y = \theta, \qquad x - y = \phi, \tag{6}$$

and solve for x and y we find that

$$x = \tfrac{1}{2}(\theta + \phi), \qquad y = \tfrac{1}{2}(\theta - \phi). \tag{7}$$

Thus, (5) becomes

$$\sin \theta + \sin \phi = 2 \sin \tfrac{1}{2}(\theta + \phi) \cos \tfrac{1}{2}(\theta - \phi). \tag{8}$$

Subtracting (2) from (1) gives

$$\sin(x + y) - \sin(x - y) = 2 \cos x \sin y,$$

which, by the substitutions (6) and (7), becomes

$$\sin \theta - \sin \phi = 2 \cos \tfrac{1}{2}(\theta + \phi) \sin \tfrac{1}{2}(\theta - \phi). \tag{9}$$

From (3) and (4) we obtain, in a similar manner,

$$\cos \theta + \cos \phi = 2 \cos \tfrac{1}{2}(\theta + \phi) \cos \tfrac{1}{2}(\theta - \phi), \tag{10}$$

$$\cos \theta - \cos \phi = -2 \sin \tfrac{1}{2}(\theta + \phi) \sin \tfrac{1}{2}(\theta - \phi). \tag{11}$$

EXERCISES VIII. E

Represent as a product:

1. $\sin 40° + \sin 20°$.
2. $\cos 80° - \cos 20°$.
3. $\cos 60° + \cos 40°$.
4. $\sin 30° - \sin 80°$.
5. $\cos 38° + \cos 42°$.
6. $\sin 35° + \sin 25°$.
7. $\sin 40° + \sin 25°$.
8. $\cos 17° - \cos 36°$.
9. $\sin 32° + \cos 22°$.

SUGGESTION. $\cos 22° = \sin(90° - 22°)$.

10. $\cos 10° + \sin 17°$.
11. $\sin 44° + \cos 40°$.
12. $\sin 4\theta - \sin 2\theta$.
13. $\sin 3\theta + \sin \theta$.
14. $\cos 5\theta + \cos 9\theta$.
15. $\sin \frac{1}{2}\theta + \sin\theta$.
16. $\cos 7\theta - \cos 3\theta$.
17. $\cos 4\theta + \cos 3\theta$.

Prove:

18. $\sin\theta + \cos\theta = \sqrt{2}\cos(\theta - 45°)$.

SUGGESTION. $\cos\theta = \sin(90° - \theta)$.

19. $\dfrac{\sin\theta + \sin\phi}{\cos\theta - \cos\phi} = \cot\tfrac{1}{2}(\phi - \theta)$.

20. $\dfrac{\sin\theta - \sin\phi}{\sin\theta + \sin\phi} = \dfrac{\tan\frac{1}{2}(\theta - \phi)}{\tan\frac{1}{2}(\theta + \phi)}$.

21. $\dfrac{\sin 3\theta + \sin 5\theta}{\cos 3\theta - \cos 5\theta} = \cot\theta$.

22. $\dfrac{\sin 75° - \sin 15}{\cos 75° + \cos 15°} \quad \dfrac{\sqrt{3}}{3}$.

23. $\cos 20° + \cos 100° + \cos 140° = 0$.

24. $\sin\theta + \sin 3\theta + \sin 5\theta + \sin 7\theta = 4\cos\theta\cos 2\theta\sin 4\theta$.

25. $\cos\theta + \cos 3\theta + \cos 5\theta + \cos 7\theta = 4\cos\theta\cos 2\theta\cos 4\theta$.

MISCELLANEOUS EXERCISES VIII. F

Prove:

1. $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$.
2. $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$.
3. $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$.
4. $\cot 3\theta = \dfrac{\cot^3\theta - 3\cot\theta}{3\cot^2\theta - 1}$.
5. $\sin 4\theta = 4\sin\theta\cos\theta(1 - 2\sin^2\theta)$
6. $\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1$.
7. $\tan 4\theta = \dfrac{4\tan\theta(1 - \tan^2\theta)}{1 - 6\tan^2\theta + \tan^4\theta}$
8. $\cot 4\theta = \dfrac{\cot^4\theta - 6\cot^2\theta + 1}{4\cot\theta(\cot^2\theta - 1)}$
9. $\tan\theta + \tan\phi = \dfrac{\sin(\theta + \phi)}{\cos\theta\cos\phi}$.
10. $\tan\theta - \tan\phi = \dfrac{\sin(\theta - \phi)}{\cos\theta\cos\phi}$.
11. $\cot\theta + \cot\phi = \dfrac{\sin(\theta + \phi)}{\sin\theta\sin\phi}$.

12. $\cot\theta - \cot\phi = \dfrac{\sin(\phi-\theta)}{\sin\theta\sin\phi}$

13. $\dfrac{\sin\theta+\sin\phi}{\cos\theta+\cos\phi} = \tan\tfrac{1}{2}(\theta+\phi).$

14. $\dfrac{\cos\theta-\cos\phi}{\cos\theta+\cos\phi} = -\tan\tfrac{1}{2}(\theta+\phi)\tan\tfrac{1}{2}(\theta-\phi).$

15. $\dfrac{\cos(n-2)\theta-\cos n\theta}{\sin(n-2)\theta+\sin n\theta} = \tan\theta.$

16. $\sin^2\theta-\sin^2\phi = \sin(\theta+\phi)\sin(\theta-\phi).$

17. $\cos^2\theta-\cos^2\phi = -\sin(\theta+\phi)\sin(\theta-\phi).$

18. $\dfrac{\sin(\theta+\phi)}{\sin(\theta-\phi)} = \dfrac{\tan\theta+\tan\phi}{\tan\theta-\tan\phi} = \dfrac{\cot\phi+\cot\theta}{\cot\phi-\cot\theta}$

19. $\dfrac{\cos(\theta+\phi)}{\sin(\theta-\phi)} = \dfrac{1-\tan\theta\tan\phi}{\tan\theta-\tan\phi} = \dfrac{1-\cot\theta\cot\phi}{\cot\theta-\cot\phi}$

20. $\dfrac{3\sin\theta-\sin 3\theta}{3\cos\theta+\cos 3\theta} = \tan^3\theta.$

21. $\sin\theta+\sin 3\theta+\sin 5\theta = \dfrac{\sin^2 3\theta}{\sin\theta}.$

22. Given $\sin\theta = \frac{4}{5}$, $\cos\phi = \frac{12}{13}$, both angles acute. Find
(a) $\sin(\theta+\phi)$, (b) $\cos(\theta+\phi)$, (c) $\tan(\theta+\phi)$, (d) $\cot(\theta+\phi)$,
(e) $\sin(\theta-\phi)$, (f) $\cos(\theta-\phi)$, (g) $\tan(\theta-\phi)$, (h) $\cot(\theta-\phi)$,
(i) $\sin 2\theta$, (j) $\cos 2\theta$, (k) $\tan 2\theta$, (l) $\cot 2\theta$,
(m) $\sin\frac{1}{2}\theta$, (n) $\cos\frac{1}{2}\theta$, (o) $\tan\frac{1}{2}\theta$, (p) $\cot\frac{1}{2}\theta$,
(q) $\sin\frac{1}{2}\phi$, (r) $\cos\frac{1}{2}\phi$, (s) $\tan\frac{1}{2}\phi$, (t) $\cot\frac{1}{2}\phi$,
(u) $\sin\theta+\sin\phi$, (v) $\sin\theta-\sin\phi$,
(w) $\cos\theta+\cos\phi$, (x) $\cos\theta-\cos\phi$.

23. Given $\tan\theta = \frac{7}{24}$, $\cos\phi = -\frac{40}{41}$. Find all possible values for the expressions (a)–(x) in the preceding exercise.

24. Find $\sin 22\frac{1}{2}°$, $\cos 22\frac{1}{2}°$, $\tan 22\frac{1}{2}°$, $\cot 22\frac{1}{2}°$ by using the known functions of 45°.

25. Find sin 18°.

Solution. Let $\theta = 18°$; then $3\theta = 54° = 90° - 2\theta$.

$$\cos 3\theta = \cos(90° - 2\theta) = \sin 2\theta.$$

Using exercise 2 above, we get

$$4\cos^3\theta - 3\cos\theta = 2\sin\theta\cos\theta,$$

or $$\cos\theta(4\cos^2\theta - 2\sin\theta - 3) = 0.$$

Setting the first factor equal to zero, we get

$$\cos\theta = 0, \qquad \theta = 90° \text{ (not } 18°\text{)},$$

and this value must be discarded. From the second factor we get, after a slight reduction,

$$4\sin^2\theta + 2\sin\theta - 1 = 0.$$

This quadratic equation yields

$$\sin\theta = \frac{-1 \pm \sqrt{5}}{4}.$$

Since $\sin\theta$ must here be positive, we retain the upper sign only, and write

$$\sin 18° = \frac{-1 + \sqrt{5}}{4}$$

26. Find cos 18°, tan 18°, cot 18°
27. Find sin 36°, cos 36°, tan 36° cot 36°
28. Find sin 9°, cos 9°.
29. Find sin 3°, cos 3°.
30. Find sin 6°, cos 6°.
31. A flagpole 34 feet high stands on top of a tower 30 feet high. From a certain point in the same horizontal plane with the base of the tower, the angle subtended by the pole is equal to the angle of elevation of the top of the tower. Find the distance from this point to the base of the tower.
32. A tree stands on the edge of a small lake. A man stands on the opposite side of the lake, his eye being at a height h above the foot of the tree. He finds that the angle of elevation of the top of the tree is E and the angle of depression of its reflection in the water is D. Show that the height of the tree is

$$\frac{h\sin(D + E)}{\sin(D - E)}.$$

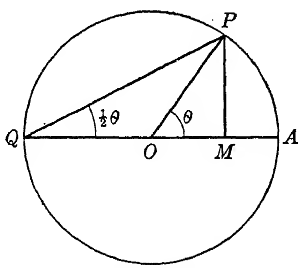


FIG. 64

33. The radius of the circle in Fig. 64 is 1. Consequently MP

$= \sin \theta$, $OM = \cos \theta$. Prove that $AQP = \frac{1}{2}\theta$, and show how to obtain the functions of $\frac{1}{2}\theta$ from the figure.

34. Draw a similar figure for the case in which θ is obtuse, and show that the same method applies.

Prove that if A, B, C are the angles of a triangle, then

35. $\sin A + \sin B + \sin C = 4 \cos \frac{1}{2}A \cos \frac{1}{2}B \cos \frac{1}{2}C$.
36. $\cos A + \cos B + \cos C = 1 + 4 \sin \frac{1}{2}A \sin \frac{1}{2}B \sin \frac{1}{2}C$.
37. $\tan A + \tan B + \tan C = \tan A \tan B \tan C$.
38. $\sin A + \sin B - \sin C = 4 \sin \frac{1}{2}A \sin \frac{1}{2}B \cos \frac{1}{2}C$.
39. $\cos A + \cos B - \cos C = -1 + 4 \cos \frac{1}{2}A \cos \frac{1}{2}B \sin \frac{1}{2}C$.
40. $\sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C$.
41. $\cos 2A + \cos 2B + \cos 2C = -1 - 4 \cos A \cos B \cos C$.
42. $\sin 2A + \sin 2B - \sin 2C = 4 \cos A \cos B \sin C$.
43. $\cos 2A + \cos 2B - \cos 2C = 1 - 4 \sin A \sin B \cos C$.
44. $\sin^2 A + \sin^2 B + \sin^2 C = 2(1 + \cos A \cos B \cos C)$.
45. $\cos^2 A + \cos^2 B + \cos^2 C = 1 - 2 \cos A \cos B \cos C$.
46. $\sin^2 A + \sin^2 B - \sin^2 C = 2 \sin A \sin B \cos C$.
47. $\cos^2 A + \cos^2 B - \cos^2 C = 1 - 2 \sin A \sin B \cos C$.
48. $\sin^2 \frac{1}{2}A + \sin^2 \frac{1}{2}B + \sin^2 \frac{1}{2}C = 1 - 2 \sin \frac{1}{2}A \sin \frac{1}{2}B \sin \frac{1}{2}C$.
49. $\sin^2 \frac{1}{2}A + \sin^2 \frac{1}{2}B - \sin^2 \frac{1}{2}C = 1 - 2 \cos \frac{1}{2}A \cos \frac{1}{2}B \sin \frac{1}{2}C$.
50. $\cot \frac{1}{2}A + \cot \frac{1}{2}B + \cot \frac{1}{2}C = \cot \frac{1}{2}A \cot \frac{1}{2}B \cot \frac{1}{2}C$.
51. $\tan \frac{1}{2}A \tan \frac{1}{2}B + \tan \frac{1}{2}B \tan \frac{1}{2}C + \tan \frac{1}{2}C \tan \frac{1}{2}A = 1$.
52. $\cot A \cot B + \cot B \cot C + \cot C \cot A = 1$.
53. $\sin(B + C - A) + \sin(C + A - B) + \sin(A + B - C)$
$$= 4 \sin A \sin B \sin C.$$
54. $\sin(B + 2C) + \sin(C + 2A) + \sin(A + 2B)$
$$= 4 \sin \tfrac{1}{2}(B - C) \sin \tfrac{1}{2}(C - A) \sin \tfrac{1}{2}(A - B).$$
55. $$\frac{\sin 2A + \sin 2B + \sin 2C}{\sin A + \sin B + \sin C} = 8 \sin \tfrac{1}{2}A \sin \tfrac{1}{2}B \sin \tfrac{1}{2}C.$$

56. Prove the law of tangents by using the law of sines and (8) and (9) of section 75.

SUGGESTION. From the law of sines we get

$$\frac{a - b}{a + b} = \frac{\sin A - \sin B}{\sin A + \sin B}.$$

★76. Reduction of $a \cos \theta \pm b \sin \theta$.

It is frequently desirable to reduce an expression of the form $a \cos \theta \pm b \sin \theta$ to the form

$r \sin(\theta \pm \phi)$ or $r \cos(\theta \pm \phi)$.

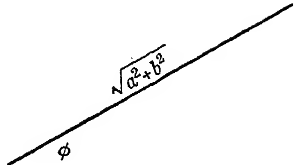


FIG. 65

These transformations adapt the expressions to logarithmic computations, and are often of advantage in solving trigonometric equations. They may be made in the following manner:

$$a \cos \theta + b \sin \theta$$
$$= \sqrt{a^2+b^2}\left(\frac{a}{\sqrt{a^2+b^2}} \cos \theta + \frac{b}{\sqrt{a^2+b^2}} \sin \theta\right).$$

Let us introduce an angle ϕ such that (see Fig. 65)

$$\sin \phi = - \qquad \cos \phi = - \quad b$$

Then,

$$a \cos \theta + b \sin \theta = \sqrt{a^2+b^2}\,(\cos \theta \cos \phi + \sin \theta \sin \phi)$$
$$= \sqrt{a^2+b^2} \cos(\theta - \phi).$$

Example.

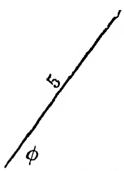


FIG. 66

Reduce $3 \cos \theta - 4 \sin \theta$ to the form $r \cos(\theta + \phi)$.

SOLUTION. Multiply and divide by $\sqrt{3^2+4^2} = 5$:

$$3 \cos \theta - 4 \sin \theta = 5(\tfrac{3}{5} \cos \theta - \tfrac{4}{5} \sin \theta).$$

If ϕ is an angle such that (see Fig. 66),

$$\cos \phi = \frac{\;}{5}, \qquad \sin \phi = \frac{4}{5},$$

then

$$3 \cos \theta - 4 \cos \theta = 5(\cos \theta \cos \phi - \sin \theta \sin \phi)$$
$$= 5 \cos(\theta + \phi).$$

From tables we find $\phi = 53°$ approximately. Therefore,

$$3 \cos \theta - 4 \sin \theta = 5 \cos(\theta + 53°).$$

EXERCISES VIII. G

1. Reduce $\sin\theta - \cos\theta$ to the form $r\sin(\theta - \phi)$, and find the angle ϕ.
2. Reduce $\sin\theta + 2\cos\theta$ to the form $r\sin(\theta + \phi)$, and find ϕ.

Reduce each of the following expressions to one of the forms $r\cos(\theta \pm \phi)$, $r\sin(\theta \pm \phi)$, and find the value of ϕ.

3. $12\cos\theta - 5\sin\theta$.
4. $3\sin\theta - 2\cos\theta$.
5. $\cos\theta + \sqrt{3}\sin\theta$.
6. $\frac{1}{2}\sin\theta + \dfrac{\sqrt{3}}{2}\cos\theta$.
7. $\cos\theta + \sin\theta$.
8. $0.4\cos\theta + 1.5\sin\theta$.
9. $0.3642\cos\theta - 1.2476\sin\theta$.

SUGGESTION. Use logarithms.

10. Given $3\sin\theta - 4\cos\theta = 2$. Reduce to the form $r\sin(\theta - \phi) = 2$, in which r and ϕ are known. Find $\sin(\theta - \phi)$, and, from tables, $\theta - \phi$. Finally, find a value of θ which satisfies the original equation.

CHAPTER IX

Radian Measure

77. Radian

One **radian** is the measure of an angle which, if its vertex is placed at the center of a circle, intercepts on the circumference an arc equal in length to the radius. It may be abbreviated 1 rad. or $1^{(r)}$. This unit of measurement of angle is important in deriving and in simplifying certain formulas in calculus and higher mathematics. Radian measure is sometimes called **circular measure** of angles.

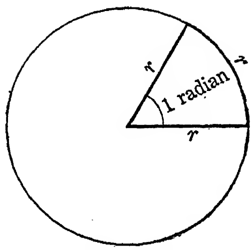


Fig. 67

78. Relation between radian and degree.

The relation between the radian and the degree may be found as follows: The circumference of a circle is 2π times the radius. Therefore, the number of radians in 360° is 2π. That is, $360° = 2\pi^{(r)}$. If we divide this equation by 2 we get

$$\mathbf{180° = \pi^{(r)} = 3.1416^{(r)}.} \tag{1}$$

This is a convenient relation to remember when reducing degrees to radians or radians to degrees.

Frequently used are the following angles:

$90° = \frac{\pi}{2}^{(r)}$ $\qquad 60° = \frac{\pi}{3}^{(r)}$ $\qquad 45° = \frac{\pi}{4}^{(r)}$ $\qquad 30° = \frac{\pi}{6}^{(r)}$

From (1) we get

$$1^\circ = \frac{\pi^{(r)}}{180} = 0.017453^{(r)},$$

also

$$1^{(r)} = \frac{180^\circ}{\pi} = 57.29578^\circ = 57^\circ\ 17'\ 44.8''.$$

Example 1.

Convert $37^\circ\ 43'\ 26''$ to radians.

Solution. $37^\circ\ 43'\ 26'' = 37.7239^\circ$
$= 37.7239 \times 0.017453^{(r)} = 0.6584^{(r)}.$

Example 2.

Convert 2.25 radians to degrees, minutes, and seconds.

Solution. $2.25^{(r)} = 2.25 \times 57.29578^\circ$
$= 128.9155^\circ = 128^\circ\ 54'\ 56''.$

If tables for converting degrees to radians (e.g., Table IV of the Macmillan Logarithmic and Trigonometric Tables) and radians to degrees (e.g., Table Va of the Macmillan Tables) are available, problems such as the foregoing are considerably simplified.

EXERCISES IX. A

1. Reduce the following angles to radians, giving the results in terms of π:

(a) 10°, (b) 35°, (c) 48°, (d) 70°,
(e) 150°, (f) 280°, (g) 18°, (h) 400°,
(i) $10^\circ\ 30'$, (j) $24^\circ\ 45'$, (k) $480^\circ\ 45'$, (l) $17^\circ\ 20'$.

2. Reduce the following angles to radians, giving the results in decimal form:

(a) 15°, (b) $10^\circ\ 17'$, (c) $10^\circ\ 17'\ 22''$,
(d) $18^\circ\ 24'\ 16''$, (e) $370^\circ\ 15'\ 8''$, (f) $142^\circ\ 25'\ 30''$,
(g) $67^\circ\ 43'\ 52''$, (h) $21^\circ\ 21'\ 21''$, (i) $2^\circ\ 3'\ 49''$.

3. Express the following angles in degrees. (When it is quite clear that radian measure is to be used, the symbol for radians

is commonly omitted. Thus, the angle π radians may be written simply π.)

(a) $\frac{\pi}{10}$, (b) $\frac{\pi}{12}$, (c) $\frac{\pi}{15}$, (d) $\frac{\pi}{18}$,
(e) $\frac{2\pi}{3}$, (f) $\frac{3\pi}{4}$, (g) $\frac{3\pi}{2}$, (h) $\frac{5\pi}{6}$,
(i) $\frac{\pi}{5}$, (j) $\frac{2\pi}{5}$, (k) $\frac{3\pi}{5}$, (l) $\frac{4\pi}{5}$,
(m) $\frac{3\pi}{10}$, (n) $\frac{7\pi}{15}$, (o) $\frac{5\pi}{12}$, (p) $\frac{7\pi}{9}$.

Express the following ang

(a) $\frac{\pi}{8}$, (b) $\frac{\pi}{50}$, (c) $\frac{\pi}{150}$, (d) $\frac{\pi}{7}$,
(e) $\frac{2\pi}{11}$, (f) $\frac{\pi}{40}$, (g) $\frac{5\pi}{24}$, (h) $\frac{\pi}{16}$,
(i) $\frac{\pi}{25}$, (j) $\frac{11\pi}{50}$ (k) $\frac{3\pi}{32}$, (l) $\frac{\pi}{48}$.

5. Reduce to degrees, minutes, and seconds:
(a) $\frac{1}{2}^{(r)}$, (b) $\frac{2}{3}^{(r)}$, (c) $\frac{2}{7}^{(r)}$, (d) $2\frac{5}{6}^{(r)}$,
(e) $3.2^{(r)}$, (f) $1.236^{(r)}$, (g) $0.1236^{(r)}$, (h) $0.1236\pi^{(r)}$.

6. One angle of a triangle is 25°, another angle is 1.3 radians. Find the third angle in degrees, and also in radians.

7. Find, in radians, the angle between the hands of a clock at (a) 2 o'clock, (b) 5 o'clock, (c) 7:30, (d) 5:15.

8. Through how many radians does the hour hand of a watch turn in (a) 5 hours? (b) $\frac{1}{2}$ hour? (c) 10 minutes? (d) 3 days? (e) between 8:00 a.m. and 5:30 p.m.?

9. Through how many radians does the earth turn in (a) 1 hour? (b) 1 minute? (c) 3 hours and 20 minutes? (d) 3 days? (e) between 8:00 a.m. and 5:30 p.m.?

10. An automobile wheel is 2 feet in diameter. Through how many radians does it turn while the automobile travels 1 mile?

11. Find the value of each of the following functions, using tables if necessary:

(a) $\sin\frac{\pi}{3}$, (b) $\cos\frac{2\pi}{3}$, (c) $\tan\frac{5\pi}{4}$,
(d) $\cot\left(-\frac{\pi}{6}\right)$, (e) $\sec\frac{3\pi}{4}$, (f) $\csc\frac{5\pi}{6}$,

(g) $\sin \frac{3\pi}{2}$. (h) $\cos \frac{2\pi}{9}$, (i) $\tan \frac{21\pi}{20}$,

(j) $\cot \frac{6\pi}{7}$; (k) $\sin \frac{12\pi}{11}$, (l) $\cos \frac{\pi}{13}$,

(m) $\sin 1^{(r)}$, (n) $\cos 2.3^{(r)}$, (o) $\tan(-5.2)^{(r)}$,

(p) $\cot 0.435^{(r)}$, (q) $\sin 0.01^{(r)}$, (r) $\cos 100^{(r)}$.

79. Relation between arc and angle.

Suppose that the arc CD in Fig. 68 subtends a central angle of θ radians, and that the arc AB subtends a central angle of 1 radian. Since central angles have the same ratio as their intercepted arcs, $\theta/1 = s/r$, or

$$\theta = \frac{s}{r}, \qquad s = r\theta. \tag{1}$$

That is,

$$arc = radius \times angle \text{ (in radians)}.$$

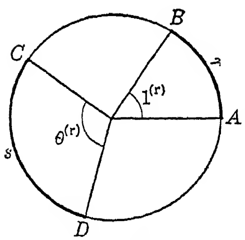


Fig. 68

It is readily seen that for a unit circle (that is, a circle whose radius is 1), a central angle expressed in radians is numerically equal to the intercepted arc expressed in linear units. For example, in a circle having a radius of 1 inch, a central angle of 2.3 radians will intercept an arc of 2.3 inches.

Example.

What is the length of the arc intercepted by a central angle of 95° in a circle whose radius is 12 feet?

Solution. First reduce the angle to radians:

$$\theta = 95 \times \frac{\pi}{180} = 1.6$$

From (1), $s = r\theta = 12 \times 1.66 = 19.9$ ft.

*80. Angular velocity.

If a wheel turns completely round thirty times in a second, we say that it is rotating at the rate of thirty revo-

lutions per second, abbreviated r.p.s. (Similarly, the expression "revolutions per minute" is abbreviated r.p.m.) A spoke of this wheel will turn through 360° in each rotation, or through $30 \times 360° = 10800°$ per second. Since the spoke turns through 2π radians in each rotation, in each second it turns through $30 \times 2\pi$ radians, or 60π radians. The wheel is said to have an **angular velocity** of 30 r.p.s., or 10800° per second, or 60π radians per second.

Suppose now that the wheel has a radius of 2 feet. When the wheel has turned through an angle of 1 radian, a point on the circumference will have moved through 2 feet. For any number of radians through which the spoke turns, a point on the circumference travels twice that number of feet. But the wheel turns through 60π radians per second. Hence, a point on the circumference moves through $60\pi \times 2$ feet per second, or it has a **linear velocity** of 120π feet per second.

In general, let us suppose that a line OP, of length r, is rotating about the point O with a constant angular velocity. If it turns through an angle θ in t units of time, the angular velocity ω is given by the formula

$$\omega = \frac{\theta}{t},$$

from which we get

$$= \omega t. \tag{1}$$

Since the length of OP is r, we find from (1) of the preceding section that the arc through which P moves while OP turns through θ radians is

$$s = r\theta = r\omega t. \tag{2}$$

But if v is the velocity of P in linear units per unit of time, we have $s = vt$, that is,

$$vt = r\omega t.$$

Dividing by t, we obtain the formula

$$v = r\omega. \tag{3}$$

Example.

A rotating wheel has a radius of 2 feet 6 inches. A point on the rim of the wheel moves 10 yards in 3 seconds. Find the angular velocity of the wheel.

SOLUTION. The linear velocity of the point on the rim is

$$\frac{10}{3} \text{ yd. per sec.} = \frac{30}{3} \text{ ft. per sec.} = 10 \text{ ft. per sec.}$$

(It should be noted that like quantities must be reduced to the same unit.) Substituting $v = 10$, $r = 2.5$ in (3), we get

$$10 = 2.5\omega, \qquad \omega = \frac{10}{2.5} = 4^{(r)} \text{ per sec.}$$

EXERCISES IX. B

1. A central angle in a circle of radius 10 inches intercepts an arc of 14 inches. How many radians are there in the angle?
2. A circle has a radius of 15 inches. Find, in radians, a central angle subtended by an arc of (a) 25 inches, (b) 1 inch, (c) 2 feet 6 inches.
3. An arc of 4 feet 3 inches subtends a central angle of 1.2 radians. Find the radius of the circle.
4. Find the length of the arc intercepted by an inscribed angle of 0.35 radian in a circle whose radius is 3 inches.
5. The angle between a tangent and a chord is $\frac{1}{4}$ radian. If the length of the arc subtended by the chord is 5 inches, what is the radius of the circle?
6. Find, in radians, the angle between the tangents to a circle at two points whose distance apart, measured on the circumference of the circle, is 350 feet, the radius of the circle being 800 feet.
7. Each of two tangents from an external point to a circle is 3 inches long. The smaller arc which they intercept is 2 radians. Find the radius of the circle.

8. A flywheel 1.5 feet in diameter has an angular velocity of 8 radians per second. Find the linear velocity of a point on the rim.
9. The wheel of an automobile is 2 feet in diameter. The automobile is traveling at the rate of 30 miles an hour. Find the angular velocity of the wheel in radians per minute.
10. A belt travels around two pulleys whose diameters are 10 inches and 4 feet respectively. The larger pulley makes 100 revolutions per minute. Find the angular velocity of the smaller pulley in radians per second.
11. An airplane propeller measures 8 feet from tip to tip. It rotates at the rate of 1800 r.p.m. (a) Find its angular velocity in radians per second. (b) Find the linear speed of a point on the tip of one of the blades, assuming that the airplane itself is not in motion.

*81. Area of sector and of segment.

A **sector** of a circle is a portion of the circle bounded by two radii and their intercepted arc. In plane geometry it is shown that the area of a sector is equal to one-half its arc times the radius of the circle. Thus, the area of the sector OAB in Fig. 69 is given by the formula $\frac{1}{2}rs$, s being the length of the arc AB. If the angle θ in this figure is expressed in radians, we have $s = r\theta$, and, substituting this in the expression $\frac{1}{2}rs$, we have

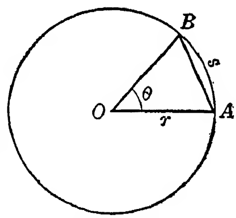


FIG. 69

$$\text{area of sector} = \tfrac{1}{2}r^2\theta \qquad (\theta \text{ in radians}). \qquad (1)$$

A **segment** of a circle is a portion of the circle bounded by an arc and its chord. The area of the sector bounded by arc AB and chord AB in Fig. 69 is obviously equal to the area of the sector AOB minus the area of the triangle AOB. But the area of the triangle is equal to $\frac{1}{2}r^2 \sin\theta$. (See section 51.) Thus,

$$\text{area of segment} = \tfrac{1}{2}r^2(\theta - \sin\theta) \qquad (\theta \text{ in radians}). \qquad (2)$$

EXERCISES IX. C

1. Find the area of a sector having an angle of 0.75 radian in a circle whose radius is 6 inches. Find the area of the corresponding segment.
2. The perimeter of a circular sector, whose angle is 1.5 radians, is 14 inches. Find the radius of the circle.
3. The area of a sector of a circle, whose radius is 15 inches, is 135 square inches. Find the angle of the sector.
4. The area of a sector of a circle is 705.6 square centimeters. If the angle of the sector is 0.45 radian, what is the radius of the circle?
5. The central angle subtended by the arc of a segment of a circle is 1.3 radians. The area of the segment is 17 square inches. Find the radius of the circle.
6. A chord of 0.75 foot subtends an arc of 0.75 radian. Find the area of the segment bounded by the chord and the arc.
7. A segment of height 3 inches (distance from center of chord to center of arc) has an arc of $\frac{1}{3}$ radian. Find the area of the segment.
8. The perimeter of a segment of a circle is 22 inches. The arc is 2 radians. What is the area of the segment?
9. A right circular cone is made by cutting out a sector, whose angle is 1.2 radians, from a circular piece of paper of radius 5 inches, and then placing the cut edges of the remaining portion together. Find (a) the lateral area and (b) the volume of the cone. (Lat. area = $\frac{1}{2}$ circumf. of base $\times$ slant ht., Vol. = $\frac{1}{3}$ area of base $\times$ alt.)
10. Find the area of a 35° sector in a circle whose diameter is 7 inches. Find the area of the corresponding segment.
11. A horizontal cylindrical tank has a diameter of 4 feet and a length of 10 feet. It is filled with liquid to a depth of 8 inches. How many gallons of liquid does it contain? (1 gal. = 231 cu. in.)

★82. Angles near 0° or 90°.

For angles near 0° or 90° (say between 0° and 3° or between 87° and 90°) interpolation by proportional parts may yield results which are considerably in error.

This difficulty may be remedied, to considerable extent, by using special tables for such angles (e.g., Table IIIa of the Macmillan Logarithmic and Trigonometric Tables). However, the difficulty may be met in another way, which is also useful for still further refinements even if such special tables are available.

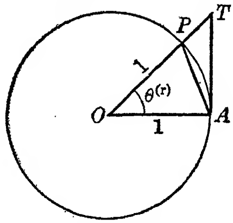


FIG. 70

In Fig. 70, AT is tangent to the unit circle with center at O, AP is a chord, angle θ is measured in radians. It is evident that, in area,

$$\text{triangle } AOP < \text{sector } AOP < \text{triangle } AOT. \qquad (1)$$

But by formula (7) of section 51,

$$\text{area triangle } AOP = \tfrac{1}{2} \sin \theta. \qquad (2)$$

By formula (1) of the preceding section,

$$\text{area sector } AOP = \tfrac{1}{2}\theta. \qquad (3)$$

Since $AT = \tan \theta$,

$$\text{area triangle } OAT = \tfrac{1}{2} \tan \theta. \qquad (4)$$

Substituting (2), (3), (4) in (1), and dividing through by $\frac{1}{2}$, we get

$$\sin \theta < \theta < \tan \theta. \qquad (5)$$

That is, if a positive acute angle is measured in radians, it will always be greater than its sine and less than its tangent.

If we divide (4) by $\sin \theta$, we find that

$$1 < \frac{}{\sin} < \sec \theta. \qquad (6)$$

Now, as the angle θ shrinks in size to 0, $\sec \theta$ approaches the value 1. It is evident, therefore, that as θ approaches 0, the

ratio $\theta/\sin\theta$ must also approach 1 as its value. This be written

$$\lim_{\theta \to 0} \frac{\theta}{\sin \theta} = 1. \tag{7}$$

Similarly, we may divide (5) by $\tan\theta$, getting

$$\cos \theta < {}_{\tan \theta} < 1. \tag{8}$$

Since $\cos 0 = 1$, it follows that

$$\lim_{\theta \to 0} \frac{\theta}{\tan \theta} = 1. \tag{9}$$

It may be noted that (7) and (9) are equivalent, respectively, to

$$\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1, \qquad \lim_{\theta \to 0} \frac{\tan \theta}{\theta} = 1. \tag{10}$$

These equations mean that

$$\sin \theta \approx \theta, \qquad \tan \theta \approx \theta \qquad (\theta \text{ small}), \tag{11}$$

where the symbol $\approx$ denotes " is approximately equal to." This may be verified by reference to tables. To illustrate,

$$\sin 2^\circ = 0.03490, \qquad \tan 2^\circ = 0.03492, \qquad 2^\circ = 0.03491^{(r)}.$$

If θ is near 90° $\left(\text{i.e., } \frac{\pi}{2}^{(r)}\right)$, we may write $\theta = \frac{\pi}{2} - \phi$, and ϕ will be a small angle. Consequently,

$$\cos \theta = \cos\left(\frac{\pi}{2} - \phi\right) = \sin \phi \approx \phi = \frac{\pi}{2} - \theta. \tag{12}$$

Similarly,
$$\cot \theta \approx \frac{\pi}{2} - \theta. \tag{13}$$

We may summarize as follows:

If θ *is near* 0,

$$\begin{aligned} \sin\theta &\approx \tan\theta \approx \theta^{(r)}, \\ \cot\theta &\approx \csc\theta \approx \frac{1}{\theta^{(r)}}, \end{aligned} \tag{14}$$

$\cos\theta$ and $\sec\theta$ may be found from tables, as usual.

If θ *is near* 90° (*i.e.*, $\frac{\pi}{2}^{(r)}$),

$$\begin{aligned} \cos\theta &\approx \cot\theta \approx \frac{\pi}{2} - \theta^{(r)}, \\ \tan\theta &\approx \sec\theta \approx \frac{1}{\frac{\pi}{2} - \theta^{(r)}}, \end{aligned} \tag{15}$$

$\sin\theta$ and $\csc\theta$ may be found from tables, as usual.

Example 1.

Find log tan 2′ 54″.

SOLUTION. $2'\,54'' = 0.048333° = (0.048333 \times 0.017453)^{(r)}$.

$$\begin{aligned} \log 0.048333 &= 8.68425 - 10 \\ \log 0.017453 &= \underline{8.24187 - 10} \\ \log \tan 2'\,54'' &= 6.92612 - 10 \end{aligned}$$

This agrees exactly with the value found in tables giving values for every second.

Example 2.

Find the angle subtended by a yardstick at a distance of 1 mile.

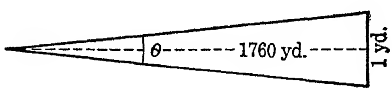


FIG. 71

SOLUTION. Strictly speaking, the yardstick would be the base of an isosceles triangle whose altitude is 1 mile, or 1760 yards. We could thus find (see Fig. 71)

$$\tan \tfrac{1}{2}\theta = \frac{0.5}{1760}$$

from which, since $\tan\frac{1}{2}\theta$ may be replaced by $\frac{1}{2}\theta$, θ is readily

obtainable. However, it makes no essential difference if we regard the yardstick as one side of a right triangle of which the other side is 1 mile. Indeed, probably the best way to regard the problem is to think of the yardstick as the arc, rather than the chord, of a circle of radius 1 mile. Any of these methods leads to the approximate equation

$$\theta = \frac{1}{1760} = 0.0005682^{(r)} = 1'\,57.2''.$$

A slowly changing function does not determine the angle very definitely. For example, if it is given that $\log \cos \theta = 9.99990 - 10$, reference to a five-place table giving the values of the logarithmic functions for every minute, shows that θ may have any value from $1°\,12'$ to $1°\,15'$ inclusive. Hence we should, if possible, avoid using $\cos \theta$ if θ is near 0, or $\sin \theta$ if θ is near 90°.

EXERCISES IX. D

Find the values of the following functions:

1. (a) $\sin 1°\,13'\,17''$,
(b) $\tan 1°\,13'\,17''$,
(c) $\cot 1°\,13'\,17''$.

2. (a) $\cos 89°\,2'\,20''$,
(b) $\cot 89°\,2'\,20''$,
(c) $\tan 89°\,2'\,20''$.

3. (a) $\log \sin 54'\,22''$,
(b) $\log \tan 54'\,22''$,
(c) $\log \cot 54'\,22''$.

4. (a) $\log \cos 89°\,20'\,54''$,
(b) $\log \cot 89°\,20'\,54''$,
(c) $\log \tan 89°\,20'\,54''$.

5. A railroad is inclined at an angle of 50′ with the horizontal. How many feet does it rise in a horizontal distance of 2 miles?

6. A highway rises 70 feet in a horizontal distance of 1 mile. What is its angle of inclination?

7. If the moon is at a distance of 238860 miles from the earth, and its diameter subtends an angle of 31′ 5″ at the earth, what is its diameter?

8. If the sun is 92,897,000 miles from the earth, and subtends an angle of 31′ 59″ at the earth, what is its diameter?

9. At Alpha Centauri, the nearest star to our sun, the distance from the earth to the sun (see preceding exercise) subtends an angle of 0.76″. Find the distance from the sun to the star.

10. The mean radius of the earth is approximately 3957 miles. It subtends an angle of 8.8″ at the sun. Find the distance from the earth to the sun.

11. If the mean radius of the earth (see preceding exercise) subtends an angle of 57′ 2.6″ at the moon, what is the distance from the earth to the moon?

Solve the following triangles:

12. $A = 1° 28.1'$,	$C = 90°$,	$a = 12.486$.
13. $C = 90°$,	$a = 0.76128$,	$b = 57.953$.
14. $A = 1° 13.2'$,	$B = 46° 21.4'$,	$a = 124.75$.
15. $a = 54321$,	$b = 28967$,	$c = 25422$.
16. $C = 56.9'$,	$a = 5.2389$,	$b = 1.9942$.
17. $B = 88° 15.3'$,	$C = 32° 19.7'$,	$a = 0.11654$.

*83. Mil.

A unit of angular measurement used in military science is the **mil**, which is $\frac{1}{1600}$ of a right angle, or $3' \, 22\frac{1}{2}''$. One degree is $17\frac{7}{9}$ mils. A mil is approximately equal to one thousandth of a radian (more accurately, 0.000982 radian). Practically, it is the angle subtended by a line of unit length at a distance of 1000 units.

If a line L units in length at a distance, or range, of R units, subtends an angle M (see Fig. 72), then the number of mils in M is given by the approximate formula

$$M \approx \tag{1}$$

From this we get

$$L \approx 0.001\, RM, \qquad R \approx \frac{1000\, L}{M} \tag{2}$$

The errors resulting from the use of formulas (1) and (2) will be less than 2 per cent provided the angle is not greater than 680 mils (about 38°).

In Fig. 72, L is the base of an isosceles triangle whose vertex angle is M. If, as in Fig. 73, the lengths L and R

are the sides of a right triangle having the acute angle M opposite side L, formulas (1) and (2) still hold. In this

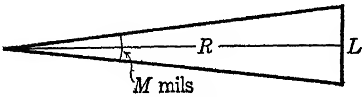


FIG. 72

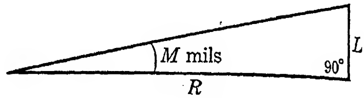


FIG. 73

case the error caused by using them will be less than 2 per cent if the angle is not greater than 340 mils (about 19°).

Example.

Find the angle subtended by an object 8 yards long at a distance of 2000 yards.

SOLUTION. Here $L = 8$, $R = 2000$, and from (1) we find

$$M \approx \frac{1000 \times 8}{20000} = 4 \text{ mils}.$$

EXERCISES IX. E

1. An object 20 feet long is 500 feet away. How many mils does it subtend if it is at right angles to the line of sight?
2. A tree 250 yards distant subtends an angle of 30 mils. How tall is it?
3. A boxcar which is known to be 42 feet long subtends an angle of 20 mils. If it is perpendicular to the line of vision, how far away is it?
4. A hill at a distance of 1560 meters subtends an angle of 40 mils. How high is it?
5. What angle does a pole 25 feet high subtend at a distance of 100 yards?
6. A balloon known to be 150 feet long is directly overhead and subtends an angle of 125 mils. How high is it?
7. A hill 50 meters high is 1500 meters away. At what angle with the horizontal must a gun be pointed in order for the projectile just to clear the top of the hill, if an allowance of 10 mils must be made for the fall of the projectile?
8. A tree 75 feet high is at a distance of 500 feet from a given point on the ground; 1500 feet farther away is a hill 350 feet

high. If a line is drawn from the point on the ground through the top of the tree, how far from the top of the hill will it strike?

9. A gun is 2500 yards from its target. A shot is fired and the projectile is observed to strike even with the target but 8 mils to the right. By how many yards did it miss the target?
10. Change into mils: $10°$, $15°$, $10'$, $10''$.
11. Change into degrees, minutes, and seconds: 10 mils, 50 mils, 100 mils.

CHAPTER X

Graphic Representations of the Trigonometric Functions

*84. Line representations of the trigonometric functions.

We shall now show how to represent the trigonometric functions by means of line segments. In so representing

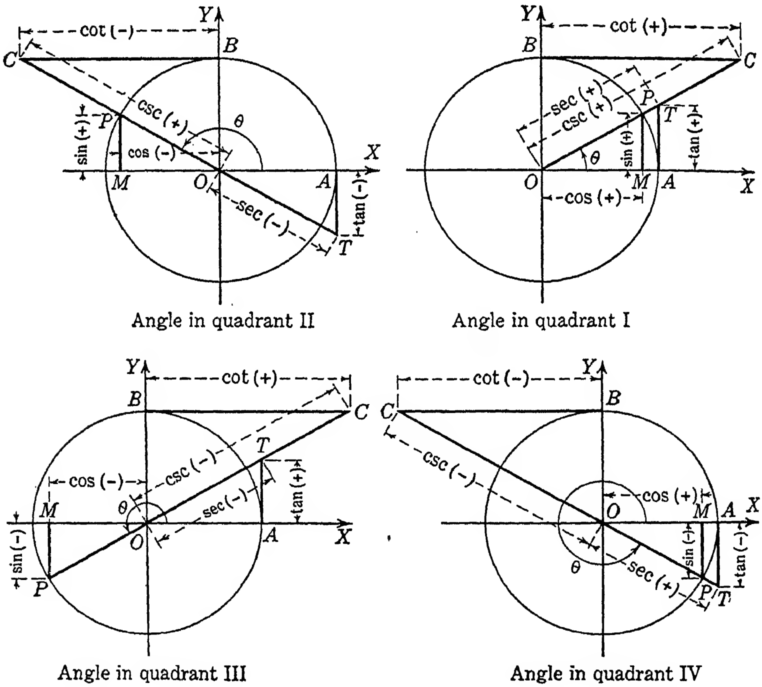


Fig. 74

the functions we shall make use of a **unit circle,** that is, a circle whose radius is 1.

The circles in Fig. 74 are unit circles. In this figure the

initial side of the angle θ is, as usual, in coincidence with the positive end of the x-axis; its terminal side is OP, P being the point in which the terminal side intersects the unit circle. Four different values of θ are shown, one in each of the four quadrants. In each case MP is drawn perpendicular to the x-axis, and the lines at A and B are tangent to the circle. (Points A and B are the intersections of the circle with the positive ends of the x- and y-axes respectively.)

Referring to the figure, we see that for θ in any quadrant,

$$\sin\theta = \frac{MP}{OP} = \frac{MP}{1} = MP,$$

$$\cos\theta = \frac{OM}{OP} = \frac{OM}{1} = OM.$$

The signs of these functions are determined by the directions of the segments MP and OP. The segment MP will be regarded as positive if the direction from M to P is upward, as negative if this direction is downward. The segment OM will be regarded as positive if the direction from O to M is to the right, as negative if this direction is to the left.

In order to complete this scheme of representing the functions, we must write the remaining functions as ratios in which the denominator is 1. This is accomplished by the selection of similar right triangles. Moreover, we wish to select the line segments which represent the functions so that they will have the proper signs.

To represent the tangent we note that

$$\tan\theta = \frac{MP}{OM} - \frac{AT}{OA} = \frac{AT}{1} = AT.$$

It is readily proved that the right triangles MOP and BOC are similar, and it follows that

$$\cot\theta = \frac{OM}{MP} = \frac{BC}{OB} = \frac{BC}{\ } = BC.$$

The conventions regarding signs, as stated above, will apply to the segments AT and BC.

The secant and the cosecant are measured along the terminal side of the angle. We shall specify that when they are measured in the same direction as the terminal line, that is, from the origin out, they are positive, and when measured in the reverse direction they are negative. (Cf. section 72.) Then, from similar triangles, we have

$$\sec\theta = \frac{OP}{OM} = \frac{OT}{OA} = \frac{OT}{1} = OT,$$

$$\csc\theta = \frac{OP}{MP} = \frac{OC}{OB} = \frac{OC}{1} = OC.$$

It should be noted that the functions are not lines. They are ratios, and therefore abstract numbers. The values of the functions are given by the measures of the lengths of the lines (i.e., line segments) in terms of the radius as a unit. The use of the circle explains why the trigonometric functions are sometimes called **circular functions.** It also explains the origin of the terms "tangent" and "secant."

Certain relations connecting the functions can be proved very readily from Fig. 74. For example,

$$\sin^2\theta + \cos^2\theta = 1, \qquad 1 + \tan^2\theta = \sec^2\theta,$$

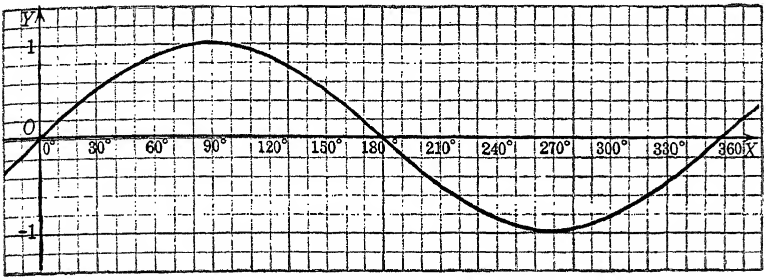
$$1 + \cot^2\theta = \csc^2\theta.$$

85. Graph of the sine.

A study of Fig. 74 shows that for an angle of 0° the line MP, representing the sine, disappears; that is, $\sin 0° = 0$. As the angle increases from 0°, the sine increases, until at 90° it reaches its maximum value of 1; as the angle increases further, the value of the sine decreases to 0 at 180°, and to -1 at 270°. It has now reached its minimum value, and as the angle increases beyond 270° the sine increases from -1 to 0, which value it reaches when the angle reaches 360°.

This variation in value of the sine is shown in Fig. 75,

which is the graph of $y = \sin x$. The values 1 and -1 are marked on the y-axis, and any convenient unit is chosen on the x-axis. The information of the preceding paragraph is supplemented by using tables to obtain values of y for a number of values of x, so that the points can be plotted



$y = \sin x$

FIG. 75

accurately. If a sufficient number of points are taken, a smooth curve can be drawn through them.

If tables are not conveniently at hand, the values of the sine for the angles 0°, 30°, 45°, 60°, 90°, 120°, and so on, can readily be calculated without tables. These values are listed in the accompanying table. From them the sine curve can often be plotted with sufficient accuracy.

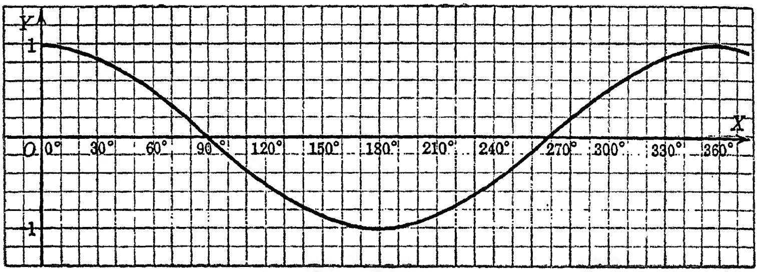
θ	$\sin\theta$	θ	$\sin\theta$
0°	0	180°	0
30°	$\frac{1}{2} = 0.50$	210°	$-\frac{1}{2} = -0.50$
45°	$\frac{\sqrt{2}}{2} = 0.71$	225°	$-\frac{\sqrt{2}}{2} = -0.71$
60°	$\frac{\sqrt{3}}{2} = 0.87$	240°	$-\frac{\sqrt{3}}{2} = -0.87$
90°	1	270°	-1
120°	$\frac{\sqrt{3}}{2} = 0.87$	300°	$-\frac{\sqrt{3}}{2} = -0.87$
135°	$\frac{\sqrt{2}}{2} = 0.71$	315°	$-\frac{\sqrt{2}}{2} = -0.71$
150°	$\frac{1}{2} = 0.50$	330°	$-\frac{1}{2} = -0.50$
180°	0	360°	0

These same angles are useful in constructing graphs of the other functions. (See following sections.)

If the angle increases beyond 360°, the sine runs through the same values again. Thus, the part of the graph between 0° and 360° is a complete pattern of the entire curve, which extends indefinitely both to the right and to the left. For this reason, 360° is called the **period** of the sine.

86. Graph of the cosine.

The cosine starts with its maximum value of 1 when the angle is 0°, decreases to 0 at 90°, to −1 at 180°, and then increases from this minimum value through 0 at 270° to 1



$y = \cos x$

Fig. 76

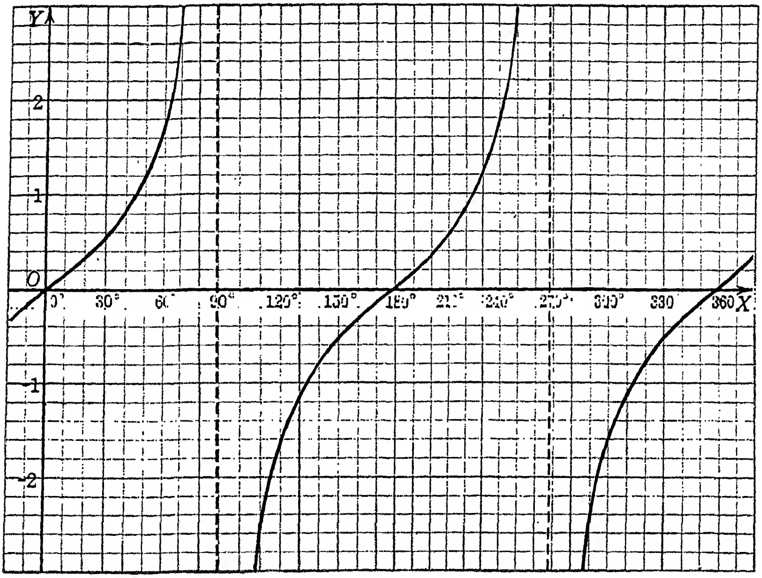
at 360°. The period of the cosine is also 360°. The graph of $y = \cos x$ is shown in Fig. 76.

87. Graphs of the tangent and the cotangent.

In Fig. 74 the value of the tangent is given by the length and the direction of the tangent line AT. Since this length is determined by the point of intersection of the tangent line at A with the terminal side of the angle, at 0° the tangent is 0. The tangent increases as the angle increases, until at 90° the terminal side is parallel to the tangent line, and there can be no point of intersection. That is, there is no value of the tangent for an angle of 90°. However, since the value of the tangent for an angle just less than 90° is

very great, and since the tangent is increasing as the angle increases, it is customary to say that the tangent approaches infinity (∞) as the angle approaches 90°. (See section 38.)

In the second quadrant the terminal line must be prolonged backward to intersect the tangent line. This means that AT extends downward, and that the tangent is negative. As the angle increases beyond 90°, the tangent, which



$y = \tan x$

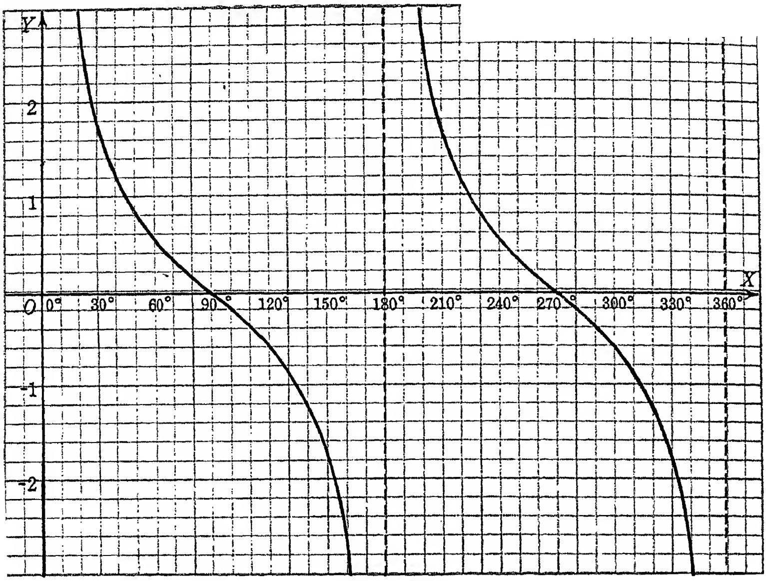
FIG. 77

has just extended indefinitely far in a positive direction, now begins at an indefinitely great distance in the negative direction.*

Thus, the tangent does not have a continuous change in value; there is a break at 90°. It increases from very large negative values, for values of the angle just greater than

* When θ approaches 90° from below (i.e., in the first quadrant), the limit of $\tan \theta$ is $+\infty$; when θ approaches 90° from above (i.e., in the second quadrant), the limit of $\tan \theta$ is $-\infty$.

90°, to 0 at 180°. As the angle increases through the third quadrant, the terminal line must be prolonged backward, and the values are the same as in the first quadrant. As the angle increases from 270° to 360°, the tangent repeats



$y = \cot x$

FIG. 78

its values of the second quadrant. The tangent thus passes through a complete cycle of values twice in one complete rotation of the line generating the angle. Its period is consequently 180°.

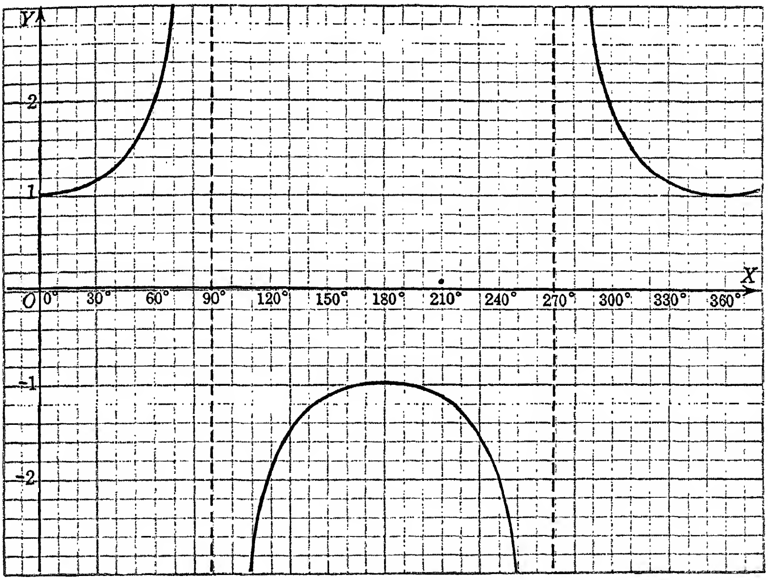
For a graph of $y = \tan x$ see Fig. 77.

In like manner, since the length and the direction of the cotangent line are determined by the intersection of the tangent line at B with the terminal side of the angle, the cotangent starts with very large values for very small positive values of the angle, and decreases to 0 at 90°. It continues to decrease through negative values in the second quadrant, these negative values becoming numerically greater and greater as the angle approaches 180°. As the

angle passes through 180°, the cotangent swings back to very large positive values, and decreases through 0 at 270° to very large negative values as the angle approaches 360°. (See Fig. 78.) Hence the cotangent also passes through a complete cycle of values twice in one complete rotation of the terminal line, and its period is 180°.

88. Graphs of the secant and the cosecant.

The secant starts with the value 1 at 0°, increases without bound as the angle approaches 90°, and jumps to very



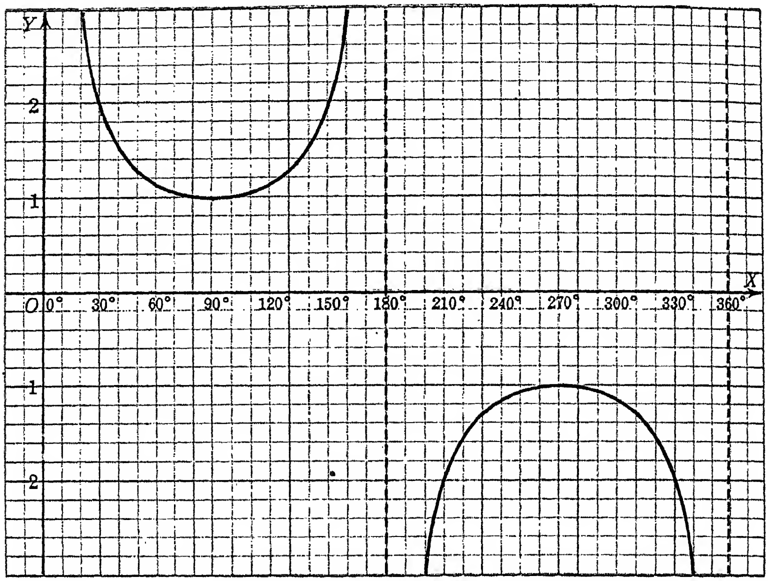
$y = \sec x$

FIG. 79

large negative values as the angle passes through 90°; it then increases to -1 at 180°, but decreases back through large negative values as the angle approaches 270°. As the angle passes through 270°, the secant changes sign and comes back to the value 1 at 360°. (See Fig. 79.) Its period is 360°.

The cosecant starts with very large values for small

values of the angle, decreases to 1 at 90°, and increases without bound as the angle approaches 180°. It then changes sign and rises from very large negative values to -1 as the angle increases to 270°, but recedes indefinitely



$y = \csc x$

FIG. 80

as the angle continues to 360°. (See Fig. 80.) Its period is 360°.

89. Use of radian measure in graphing.

It is sometimes desirable to use radian measure in constructing the graphs of the functions. In such cases the point on the x-axis which previously was marked 360° would be marked 2π radians, the point corresponding to 180° would be marked π, and so on. Here it is usual to take the same unit on each axis; thus, the point π would be $3.14+$ units from the origin.

If the radian is used as the unit of measure of angle, the

period of sine, cosine, secant, and cosecant is 2π; the period of tangent and cotangent is π.

★90. Geometric construction of the sine and cosine graphs.

By using a unit circle, we can construct the sine curve as indicated in Fig. 81. In this figure a unit circle is drawn

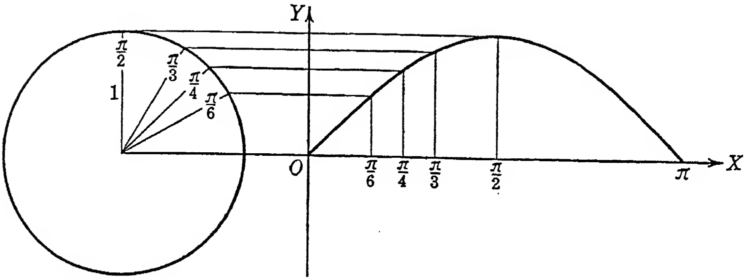


FIG. 81

at the left, and a horizontal line, to be used as the x-axis, is drawn through its center. On this line is marked an origin O, through which is drawn the y-axis. The segment from O to the point marked π is 3.1416 units long; that is, it is equal in length to the semicircumference. The distance from O to the point marked $\pi/6$ is equal to the arc of the circle from the point of its intersection with the x-axis to the point marked $\pi/6$, and so on. The method by which we obtain the ordinate corresponding to a given abscissa is evident from the figure.

The corresponding method of constructing the graph of the cosine curve is left as an exercise for the student.

EXERCISES X. A

Plot the following curves:

1. $y = 2 \sin x$.	2. $y = 2 \cos x$.	3. $y = \frac{1}{2} \sin x$.
4. $y = \sin 2x$.	5. $y = \sin \frac{1}{2}x$.	6. $y = \cos \frac{1}{2}x$.
7. $y = \cot \frac{1}{2}x$.	8. $y = \sin 3x$.	9. $y = \tan 2x$.
10. $y = \sin \pi x$.	11. $y = \cos \dfrac{\pi x}{2}$.	12. $y = \sin \dfrac{2\pi x}{3}$.

13. Plot $y = \sin x \cos x$.

SUGGESTION. $\sin x \cos x = \frac{1}{2} \sin 2x$.

14. In what points will a line one unit above the x-axis intersect the curve $y = \tan x$?

15. If the graphs of $y = \sin x$ and $y = \cos x$ are plotted on the same set of axes, for what values of x will they intersect?

16. Plot $y = \sin\left(x + \frac{\pi}{2}\right)$ and compare with $y = \cos x$.

17. Plot $y = \cos\left(\frac{\pi}{2} - x\right)$ and compare with $y = \sin x$.

18. Draw the graph of $y = \cos\left(x - \frac{\pi}{4}\right)$.

19. Draw the graph of $y = \sin(x - \frac{1}{2})$. Here radian measure is understood.

20. Given the equation $y = \sin x + \cos x$.

(a) Plot the curve by plotting the sine curve and the cosine curve separately and adding their ordinates geometrically (for example, by using dividers).

(b) Plot the curve by first reducing $\sin x + \cos x$ to the form $r \sin(x + \phi)$.

21. Draw the graph of $y = \sin x - \cos x$.

22. Plot $y = x + \sin x$, using radian measure.

23. Find the periods of the curves in exercises 1–12.

CHAPTER XI

Inverse Trigonometric Functions

91. Inverse trigonometric functions.

If $x = y^2$, then y is the positive or negative square root of x; in symbols, $y = \pm\sqrt{x}$. Similarly, if $x = \sin y$, then y is an angle whose sine is x; in abbreviated form we write

$$y = \arcsin x. \tag{1}$$

The right-hand member of this equation may be read "arc sine x" or "an angle whose sine is x," it being recalled that if a central angle of a unit circle is measured in radians, the intercepted arc is equal to the angle. The notation

$$y = \sin^{-1} x \tag{2}$$

is also used. The symbol $\sin^{-1} x$ may be read "inverse sine of x" or "antisine of x" or, to emphasize its meaning, "an angle whose sine is x." It should be carefully noted that the -1 is not an exponent. If we wish to have -1 as the exponent of a trigonometric function such as $\sin x$, we must write $(\sin x)^{-1}$, which means $1/\sin x$.

The function $\arcsin x$, or $\sin^{-1} x$, is called the **inverse sine function** of x. The other **inverse trigonometric functions** are

$$\begin{array}{lcl}
\operatorname{arccos} x & \text{or} & \cos^{-1} x, \\
\arctan x & \text{or} & \tan^{-1} x, \\
\operatorname{arccot} x & \text{or} & \cot^{-1} x, \\
\operatorname{arcsec} x & \text{or} & \sec^{-1} x, \\
\operatorname{arccsc} x & \text{or} & \csc^{-1} x.
\end{array}$$

92. Principal values.

An inverse trigonometric function, such as arcsin x, has infinitely many values corresponding to each value of x. Consider, for example, arcsin $\frac{1}{2}$. There are two angles less than 360° whose sine is $\frac{1}{2}$, namely 30° and 150°. Any angle obtained from either of these by adding or subtracting a multiple of 360° also has its sine equal to $\frac{1}{2}$. Therefore we may write

$$\arcsin \tfrac{1}{2} = 30° + n \cdot 360° \quad \text{or} \quad 150° + n \cdot 360°; \quad n = 0, \pm 1, \pm 2, \cdots, \tag{1}$$

or, if we use radian measure, which is usually more desirable in dealing with the inverse functions,

$$\arcsin \tfrac{1}{2} = \frac{\pi}{6} + 2n\pi \quad \text{or} \quad \frac{5\pi}{6} + 2n\pi; \quad n = 0, \pm 1, \pm 2, \tag{2}$$

The **principal value** of arcsin x, which will be denoted by **Arcsin** x or **Sin**^{-1}x, is that value between $-\pi/2$ and $\pi/2$ inclusive. Thus, the principal value of arcsin $\frac{1}{2}$ is $-\pi/6$. If the principal value of arcsin x is θ, then all possible values are contained in the two sets

$$\theta + 2n\pi, \qquad \pi - \theta + 2n\pi; \qquad n = 0, \pm 1, \pm 2, \cdots. \tag{3}$$

These two sets may be grouped together by the formula

$$n\pi + (-1)^n\theta; \qquad n = 0, \pm 1, \pm 2, \cdots. \tag{4}$$

The notation for the principal values of the other inverse trigonometric functions is like that for the inverse sine, namely, **Arccos** x or **Cos**^{-1}x, **Arctan** x or **Tan**^{-1}x, etc.

The principal values of the inverse functions are defined as follows. That is, the principal value is that value in the interval specified.

$$-1 \leqq x \leqq 1, \qquad -\frac{\pi}{2} \leqq \text{Arcsin } x \leqq \frac{\pi}{2},$$

$$-\infty < x < \infty, \qquad -\frac{\pi}{2} < \text{Arctan } x < \frac{\pi}{2},$$

$$-1 \leqq x \leqq 1, \qquad 0 \leqq \text{Arccos } x \leqq \pi,$$

$$-\infty < x < \infty, \qquad 0 < \text{Arccot } x < \pi,$$

$$x \geqq 1, \qquad 0 \leqq \text{Arcsec } x < \frac{\pi}{2},$$

$$x \leqq -1, \qquad -\pi \leqq \text{Arcsec } x < -\frac{\pi}{2},$$

$$x \geqq 1, \qquad 0 < \text{Arccsc } x < \frac{\pi}{2},$$

$$x \leqq -1, \qquad -\pi < \text{Arccsc } x \leqq -\frac{\pi}{2}.$$

NOTE. Other definitions of the principal values of the inverse trigonometric functions for negative values of x are sometimes given. However, the foregoing definitions are the most convenient from the standpoint of calculus.

If the principal value of an inverse trigonometric function is θ, then all values of the inverse sine or of the inverse cosecant are given by (3) or (4). All values of the inverse cosine or of the inverse secant are given by

$$2n\pi \pm \theta; \qquad n = 0, \pm 1, \pm 2, \cdot \cdot \cdot. \qquad (5$$

All values of the inverse tangent or of the inverse cotangent are given by

$$\theta + n\pi; \qquad n = 0, \pm 1, \pm 2, \cdot \cdot \cdot. \qquad (6)$$

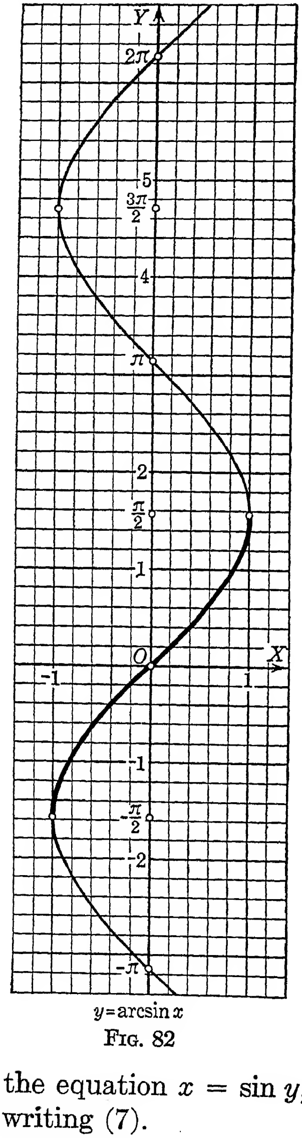
93. Graphs of the inverse trigonometric functions.

The graph of the equation

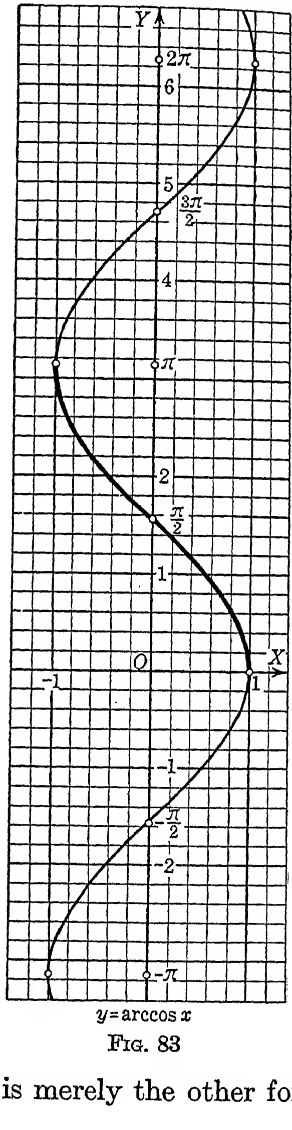
$$y = \arcsin x, \qquad (7)$$

in which y is expressed in radians, is given in Fig. 82. The principal values of the function are indicated by the heavier

part of the curve, which constitutes the **principal branch** of the curve. It is clear that this curve is also the graph



$y = \arcsin x$
FIG. 82

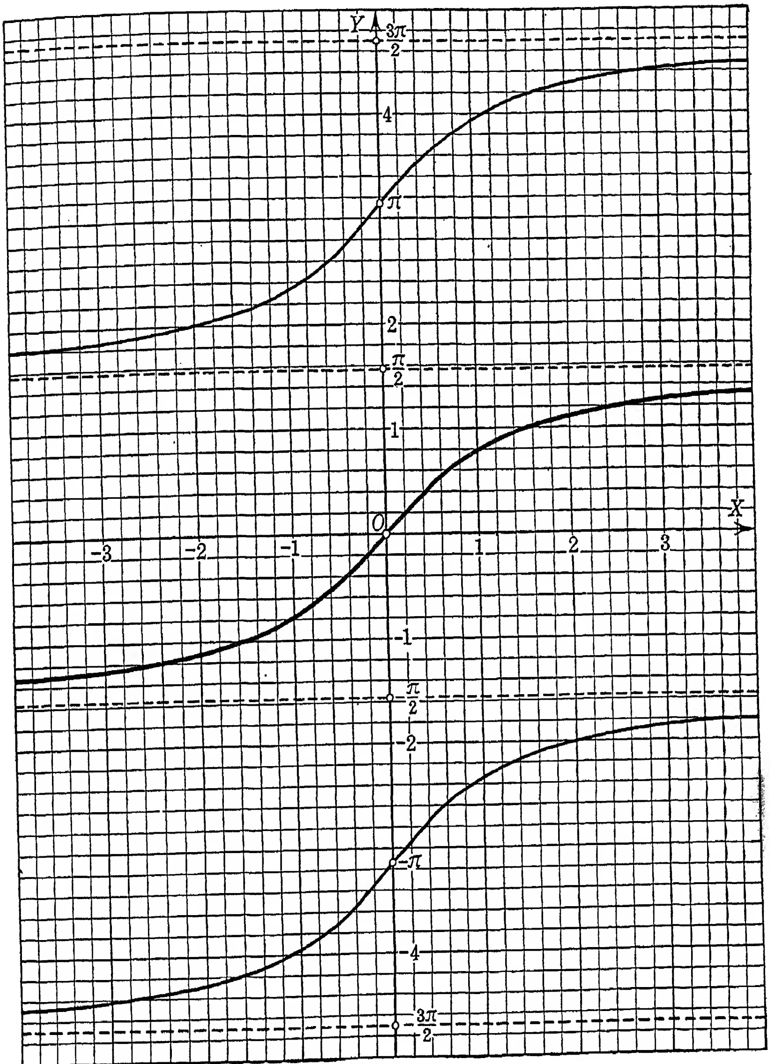


$y = \arccos x$
FIG. 83

of the equation $x = \sin y$, which is merely the other form of writing (7).

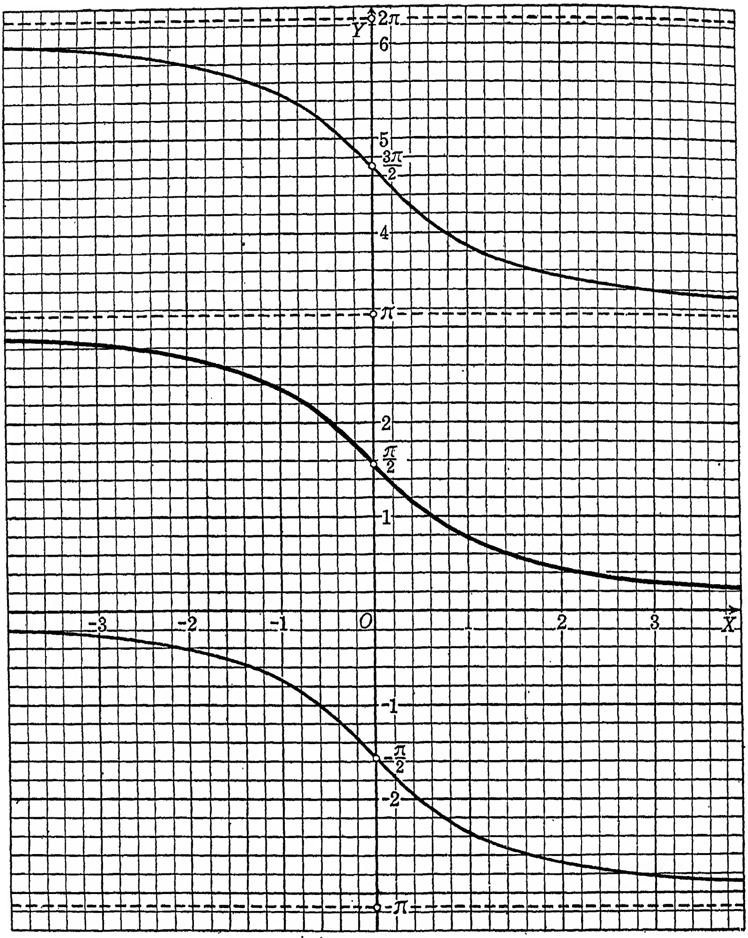
The graphs of the other inverse functions are shown in

Figs. 83–87. The principal branch in each case is indicated by the heavier part of the curve.



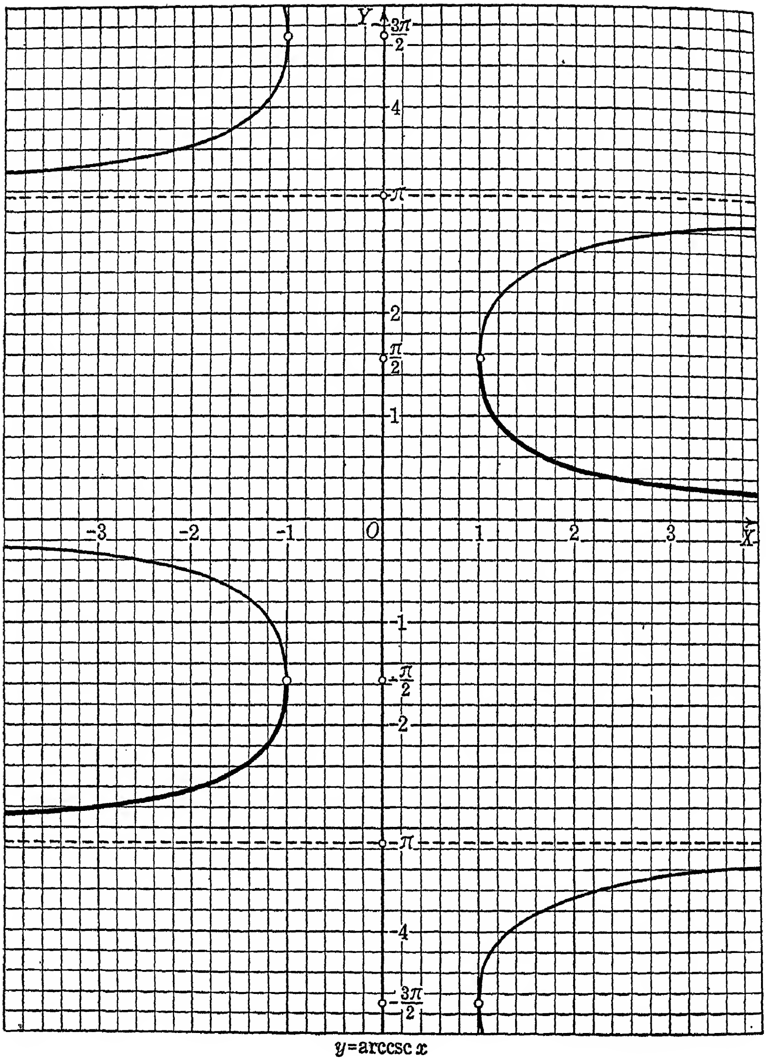
$y = \arctan x$

FIG. 84



$y = \operatorname{arccot} x$

FIG. 85



$y = \text{arccsc}\, x$

FIG. 87

EXERCISES XI. A

1. Find arcsin $\sqrt{3}$

Solution. Let $\theta = \arcsin \dfrac{\sqrt{3}}{2}\cdot$ Then $\sin\theta = \dfrac{\sqrt{3}}{2}$, and the principal value of θ is $60°$ or $\pi/3$. Therefore, by (4),

$,\pi$

2. $\arcsin \frac{1}{2}$.

3. $\arccos\left(-\dfrac{\sqrt{2}}{2}\right)$.

4. arcsin 0.

5. arccos 0.

6. $\operatorname{arccot} \dfrac{\sqrt{3}}{3}$

7. arctan 1.

8. $\operatorname{arccsc}\sqrt{2}$.

9. $\arctan(-\sqrt{3})$.

10. $\arcsin\left(-\dfrac{\sqrt{3}}{2}\right)\cdot$

Find, by using tables, the principal values, and also the general values of

11. arcsin 0.23770.
12. arccos 0.93590.
13. arctan 1.4910.
14. arcsin(−0.95510).
15. arccos(−0.01020).
16. arctan(−12.350).
17. $\arcsin \frac{2}{3}$.
18. $\arccos \frac{1}{8}$.
19. arctan 2.
20. Find $\cos(\arctan \frac{5}{3})$.

Solution. Let $\theta = \arctan \frac{5}{3}$. Then (see Fig. 88),

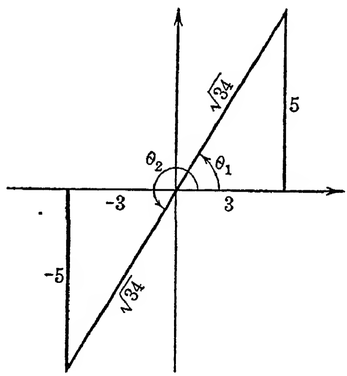


Fig. 88

$$\tan\theta = \frac{5}{3}, \qquad \cos\theta = \frac{3}{\pm\sqrt{34}} = \pm\frac{3\sqrt{34}}{34}\cdot$$

Find

21. $\tan(\text{Arcsin } \frac{3}{5})$.
22. $\sin(\text{Arccos } \frac{7}{25})$.
23. $\cos(\text{arccos } \frac{9}{13})$.
24. $\sin[\text{Arccos}(-\frac{15}{17})]$.
25. $\tan[\text{Arccos}(-\frac{15}{17})]$.
26. $\cot[\text{Arcsin}(-\frac{12}{13})]$.
27. $\sin(\text{arctan } \frac{20}{21})$.
28. $\cos(\text{arcsin } \frac{24}{25})$.
29. $\tan[\text{arccos}(-\frac{4}{5})]$.
30. $\sec(\text{arctan } 1.05)$.
31. $\cot[\text{arctan}(-3)]$.
32. $\sec(\text{arccot } 2)$.
33. $\sin(\text{arcsin } x)$.
34. $\cos(\text{arcsin } x)$.
35. $\tan(\text{arcsin } x)$.
36. $\sin(\text{arccos } x)$.
37. $\cot(\text{arccos } x)$.
38. $\tan(\text{arccos } x)$.
39. $\sin(\text{arctan } x)$.
40. $\cos(\text{arctan } x)$.
41. $\sec(\text{arctan } x)$.
42. $\tan(\text{arcsec } x)$.

43. Find the value of $\sin(\text{arccos } \frac{3}{5} + \text{arctan } \frac{12}{5})$.

SOLUTION. Let $\theta = \text{arccos } \frac{3}{5}$, $\phi = \text{arctan } \frac{12}{5}$. Then,

$$\begin{aligned}\sin(\text{arccos } \tfrac{3}{5} + \text{arctan } \tfrac{12}{5}) &= \sin(\theta + \phi)\\ &= \sin\theta\cos\phi + \cos\theta\sin\phi = (\pm\tfrac{4}{5})(\pm\tfrac{5}{13}) + \tfrac{3}{5}(\pm\tfrac{12}{13}).\end{aligned}$$

Using all possible combinations of signs, we find the following four distinct values for the above expression:

$$\begin{aligned}&\tfrac{4}{13} + \tfrac{36}{65} = \tfrac{56}{65}, &\qquad &\tfrac{4}{13} - \qquad = -\tfrac{16}{65},\\ -&\tfrac{4}{13} + \tfrac{36}{65} = \tfrac{16}{65}, &\qquad -&\tfrac{4}{13} - \qquad = -\tfrac{56}{65}.\end{aligned}$$

They may be expressed in the more compact form: $\pm\frac{56}{65}$, $\pm\frac{16}{65}$.

Find the values of

44. $\sin(\text{Arcsin } \frac{7}{25} + \text{Arccos } \frac{4}{5})$.
45. $\cos(\text{Arcsin } \frac{15}{17} + \text{Arccot } \frac{9}{40})$.
46. $\tan(\text{arctan } \frac{3}{4} + \text{arctan } \frac{8}{15})$.
47. $\sin(\text{arcsin } \frac{1}{3} + \text{arccos } \frac{1}{3})$.
48. $\cos[\text{arcsin } \frac{8}{17} + \text{arcsin}(-\frac{3}{5})]$.
49. $\cos\left(2 \text{ arcsin } \dfrac{\sqrt{5}}{3}\right)$.
50. $\sin(\frac{1}{2} \text{ arccos } \frac{7}{9})$.
51. $\tan(\text{arcsin } \frac{5}{13} + 2 \text{ arctan } \frac{4}{5})$.
52. $\tan[\text{arctan } \frac{3}{5} + \text{arcsin}(-\frac{3}{5})]$.
53. $\sin(\text{arctan } \frac{9}{40} - \text{arccot } \frac{21}{20})$.
54. $\cos[\text{arcsec } \frac{25}{7} - \text{arctan}(-\frac{15}{8})]$.

55. $\sin(2 \arcsin \frac{3}{5} + \frac{1}{2} \arccos \frac{1}{49})$.

56. $\cos\left(\frac{1}{2} \arcsin \cdot \sqrt{15} - 2 \arctan \frac{15}{8}\right)$.

57. $\sin(\arcsin \frac{4}{5} + \arctan \frac{12}{5} - \arccos \frac{8}{17})$.

58. Show that $\operatorname{Arctan} \frac{1}{2} + \operatorname{Arctan} \frac{1}{3} = \frac{\pi}{4}$.

Solution. Let $\theta = \operatorname{Arctan} \frac{1}{2}$, $\phi = \operatorname{Arctan} \frac{1}{3}$. Then we wish to prove that $\theta + \phi = \pi/4$.

$$\tan(\theta + \phi) = \frac{\tan \theta + \tan \phi}{1 - \tan \theta \tan \phi} = \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \cdot \frac{1}{3}} = 1.$$

From this we might have

$$+ \phi = \frac{\pi}{4} + n\pi; \qquad n = 0, \pm 1, \pm 2,$$

However, since we are dealing with principal values, θ and ϕ are in the interval from 0 to $\pi/2$. Therefore $\theta + \phi$ is in the interval from 0 to π, and we must have $\theta + \phi = \pi/4$.

Prove that

59. $\operatorname{Arcsin} \frac{3}{5} - \operatorname{Arctan} \frac{3}{5} = \operatorname{Arctan} \frac{3}{29}$.

60. $\operatorname{Arctan} \frac{1}{3} - \operatorname{Arctan} \frac{1}{4} = \operatorname{Arctan} \frac{1}{13}$.

61. $\operatorname{Arcsin} \frac{3}{5} + \operatorname{Arcsin} \frac{8}{17} = \operatorname{Arcsin} \frac{77}{85}$.

62. $\operatorname{Arccos} \frac{4}{5} + \operatorname{Arccos} \frac{12}{13} = \operatorname{Arccos} \frac{33}{65}$.

63. $\operatorname{Arccos} \frac{4}{5} + \operatorname{Arctan} \frac{3}{5} = \operatorname{Arctan} \frac{27}{11}$.

64. $2 \operatorname{Arctan} \frac{1}{3} + \operatorname{Arctan} \frac{1}{7} = \frac{\pi}{4}$.

65. $\operatorname{Arccos} \frac{63}{65} + 2 \operatorname{Arctan} \frac{1}{5} = \operatorname{Arcsin} \frac{3}{5}$.

66. $\operatorname{Arctan} \frac{1}{4} + \operatorname{Arctan} \frac{2}{9} = \frac{1}{2} \operatorname{Arccos} \frac{3}{5}$.

67. $\operatorname{Arctan} \frac{1}{2} + \operatorname{Arctan} \frac{1}{5} + \operatorname{Arctan} \frac{1}{8} = \frac{\pi}{4}$.

68. Prove that $\operatorname{Arctan} x + \operatorname{Arctan} y = \operatorname{Arctan} \frac{x + y}{1 - xy}$ provided the value of the left-hand side is between $-\pi/2$ and $\pi/2$.

Note. In general,

$$\arctan x + \arctan y = \arctan \frac{x + y}{1 - xy},$$

if it is understood that the particular values assigned to two of the inverse functions are arbitrary; the particular value of the third is determined when the values of the others are assigned.

Prove that

69. $\text{Arcsin}\, x + \text{Arccos}\, x = \frac{\pi}{2}$ for $-1 \leqq x \leqq 1$.

70. $\text{Arctan}\, x + \text{Arccot}\, x = \frac{\pi}{2}$ for all values of x.

71. $2\, \text{Arcsin}\, x = \text{Arccos}(1 - 2x^2)$ for $0 \leqq x \leqq 1$.

72. $\text{Arcsin}\, x = \pm \text{Arccos}\, \sqrt{1 - x^2}$, according as $x \gtrless 0$.

73. $\text{Arctan}\, x = \text{Arcsin}$ for all values of x.

74. $\text{Arctan}\, \dfrac{2x}{1 - x^2} = \text{Arcsin}\, \dfrac{2x}{1 + x^2}$ for $-1 < x < 1$.

75. $\text{Arctan}\, x + \text{Arccot}(x + 1) = \text{Arctan}(x^2 + x + 1)$ for all ues of x.

76. Find all possible values of $\arcsin(\cos\theta)$.

Solution. Let $\phi = \arcsin(\cos\theta)$. Then,

$$\sin\phi = \cos\theta = \sin\left(\frac{\pi}{2} - \theta\right).$$

Therefore,

$$\phi = \begin{array}{l} \dfrac{\pi}{2} - \theta + n \cdot 2\pi, \\ \pi - \left(\dfrac{\pi}{2} - \theta\right) + n \cdot 2\pi. \end{array}$$

These two sets of solutions may be expressed in the form

$$\phi = \frac{\pi}{2} \pm \theta + 2n\pi.$$

Find all possible values of the following expressions:

77. $\arcsin(\sin\theta)$.

78. $\arccos(\cos\theta)$.

79. $\arctan(\tan\theta)$.

80. $\arccos(\sin\theta)$.

CHAPTER XII

Trigonometric Equations

94. Trigonometric equations.

An equation which is satisfied by certain values only of the unknown quantity or quantities that it contains is called a **conditional equation.** Examples of conditional equations are $2x - 1 = 0$, which is satisfied by $x = \frac{1}{2}$ only; $x^2 + y^2 = 25$, which is satisfied by an infinite number of pairs of values of x and y, but certainly not by all pairs of values; $\sin\theta = \frac{1}{2}$, which is satisfied by $\theta = 30°$, $150°$, $390°$, $510°$, etc., but not by all values of θ.

An **identical equation,** or **identity,** is an equation which is satisfied by all values (with perhaps some exceptions *) of the unknown quantity or quantities which it contains. Examples of identities are

$$(x + 1)^2 = x^2 + 2x + 1,$$
$$\sin^2\theta + \cos^2\theta = 1,$$
$$\cos(\theta + \phi) = \cos\theta\cos\phi - \sin\theta\sin\phi.$$

The equations † which we shall consider in this chapter are conditional equations, identities having already been considered in various places throughout the book.

Trigonometric equations require, for a complete solution, general expressions such as (1) or (2) in section 92 of the preceding chapter. However, the equation is sometimes

* For example, the identity $\tan\theta = \sin\theta/\cos\theta$ is not defined for values of θ, such as $\pi/2$, which make the denominator of the right-hand side equal to zero.

† It is customary to omit the qualifying adjective, and to refer to a conditional equation merely as an "equation."

considered sufficiently solved if all positive values of the unknown quantity less than 360° are obtained, or if the principal value of an inverse function is obtained.

There is no general method of solving trigonometric equations. If the equation contains a single function of an angle, solve for this function by appropriate algebraic methods, and then find the corresponding values of the angle. If more than one function appears in the equation, the equation should ordinarily be transformed so that it contains only one function, or into a factored form so that each factor contains only one function.

When the equation involves functions of different angles, such as θ, 2θ, $\frac{1}{2}\theta$, $\theta + 45°$, it can sometimes be reduced to an equivalent equation which contains but a single function of a single angle, or to an equivalent equation which can be separated into factors each of which contains a single function of a single angle.

As in algebra, the test for each solution of an equation is to substitute it in the original equation to determine whether it satisfies the equation.

Some of the methods of solving trigonometric equations will be illustrated by examples.

Example 1.

Solve the equation $\sin\theta = \cos\theta$.

SOLUTION. Divide both sides by $\cos\theta$:*

$$\tan\theta = 1.$$

The principal value of θ is 45°. The two positive values of θ less than 360° are 45° and 225°. The complete solution is

$$\theta = 45° + n \cdot 180°, \quad \text{or} \quad \theta = \frac{\pi}{4} + n\pi; \quad n = 0, \pm 1, \pm 2,$$

* When both sides of an equation are divided by a quantity containing the unknown, this quantity should be set equal to zero to obtain possible solutions. If we set $\cos\theta = 0$, we get $\theta = 90°, 270°, \cdots$. However, these values are not solutions of the equation $\sin\theta = \cos\theta$.

This equation can also be solved by replacing $\cos\theta$ by $\pm\sqrt{1-\sin^2\theta}$ and squaring both sides:

$$\begin{aligned}
\sin^2\theta &= 1 - \sin^2\theta,\\
2\sin^2\theta &= 1,\\
\sin\theta &= \pm\frac{1}{\sqrt{2}},\\
\theta &= 45^\circ, 135^\circ, 225^\circ, 315^\circ, \cdots.
\end{aligned}$$

If this method is used, all the values obtained must be tested. It will be found that 135° and 315° do not satisfy the original equation. They are **extraneous solutions** introduced by squaring, and must be discarded.

Example 2.

Solve: $$\cos^2\theta + 2\sin\theta + 1 = 0.$$

SOLUTION. Replacing $\cos^2\theta$ by $1 - \sin^2\theta$, we get, after a slight simplification,

$$\sin^2\theta - 2\sin\theta - 2 = 0.$$

This is a quadratic equation in $\sin\theta$; solving it by the quadratic formula, we find

$$\begin{aligned}
\sin\theta \quad \frac{2\pm\sqrt{4+8}}{2} &= \frac{2\pm 2\sqrt{3}}{2} = 1\pm\sqrt{3}\\
&= 1 \pm 1.73205 = 2.73205 \text{ or } -0.73205.
\end{aligned}$$

The first value must be discarded, since the sine cannot be greater than 1; from the second we get two values of θ between 0° and 360°, viz.,

$$\begin{aligned}
\theta &= 180^\circ + 47^\circ\,3.5' = 227^\circ\;\;3.5',\\
\theta &= 360^\circ - 47^\circ\,3.5' = 312^\circ\,56.5'.
\end{aligned}$$

The general solution is given by

$$\theta = \begin{cases} 227^\circ\;\;3.5' + n\cdot 360^\circ,\\ 312^\circ\,56.5' + n\cdot 360^\circ; \end{cases} \qquad n \quad 0, \pm 1, \pm 2, \cdots.$$

Example 3.

Solve: $$2\sin^2\theta - \cos 2\theta = 0.$$

SOLUTION. Replace $\cos 2\theta$ by $1 - 2\sin^2\theta$, and combine like terms:

$$\begin{aligned} 4\sin^2\theta - 1 &= 0, \\ \sin\theta &= \pm\tfrac{1}{2}, \\ \theta &= 30^\circ, 150^\circ, 210^\circ, 330^\circ, \cdots. \end{aligned}$$

The general solution may be written in the form

$$\theta = n\cdot 180^\circ \pm 30^\circ = n\pi \pm \frac{\pi}{6}$$

Equations of the form $a\cos\theta \pm b\sin\theta = c$ can be solved by reducing the left side to one of the forms $r\sin(\theta \pm \phi)$, $r\cos(\theta \pm \phi)$. (See section 76.)

Example 4.

Solve: $$3\sin\theta - 4\cos\theta = 1.$$

SOLUTION. Divide both sides by $\sqrt{3^2 + (-4)^2} = 5$:

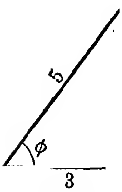


FIG. 89

$$\tfrac{3}{5}\sin\theta - \tfrac{4}{5}\cos\theta = \tfrac{1}{5} = 0.2. \tag{1}$$

If ϕ is an angle such that (see Fig. 89)

$$\cos\phi = \tfrac{3}{5}, \qquad \sin\phi = \tfrac{4}{5}, \tag{2}$$

then (3) takes the form

$$\sin\theta\cos\phi - \cos\theta\sin\phi = 0.2,$$

or
$$\sin(\theta - \phi) = 0.2.$$

But from (2), using tables, we find $\phi = 53^\circ\,8'$. Therefore

$$\begin{aligned} \sin(\theta - 53^\circ\,8') &= 0.2, \\ \theta - 53^\circ\,8' &= 11^\circ\,32', 168^\circ\,28', \cdots, \\ \theta &= 64^\circ\,40', 221^\circ\,36', \cdots. \end{aligned}$$

This method of solution is particularly valuable if the numbers involved are not simple, since it is adapted to the use of logarithms.

The equation could be solved by making the substitution $\cos\theta = \pm\sqrt{1 - \sin^2\theta}$ and following the method used for solving radical equations in algebra. (Cf. example 1, second method.) This, however, introduces extraneous solutions.

EXERCISES XII. A

Solve the following equations:

1. $2\cos^2\theta - \sin^2\theta = 2$.

2. $2\cos^2\theta + 3\sin\theta = 0$.

3. $\tan\theta + \cot\theta = 2$.

4. $\sin\theta = 2\cos\theta$.

5. $\sec\theta = 4\csc\theta$.

6. $\cos 2\theta + \sin\theta = 0$.

7. $\sin 2\theta + \cos\theta = 0$.

8. $\sin 2\theta = 3\sin^2\theta - \cos^2\theta$.

9. $\sin^2\theta = 1 - \sin 2\theta$.

10. $\tan^2\theta = \sin 2\theta$.

11. $\sin 2\theta + 2\cos 2\theta = 1$.

12. $4\sec^2 2\theta + \tan 2\theta = 7$.

13. $\sin 2\theta = \cos 3\theta$.

SOLUTION. $\sin 2\theta = \cos 3\theta = \sin(90° - 3\theta)$.

Now if $\sin\theta = \sin\phi$, it follows that either

$$\theta = \phi + n\cdot 360° \qquad \text{or} \qquad \theta = 180° - \phi + n\cdot 360°.$$

In the present case, therefore,

$$2\theta = 90° - 3\theta + n\cdot 360° \qquad \text{or} \qquad 2\theta = 180° - (90° - 3\theta) + n\cdot 360°.$$

The first equation yields

$$5\theta = 90° + n\cdot 360°, \qquad \theta = 18° + n\cdot 72°.$$

The second can be reduced to $\theta = 270° + n\cdot 360°$.

14. $\sin\theta = \cos(\theta + 15°)$.

15. $\sin(\theta + 10°) = \cos(\theta - 40°)$.

16. $\sin(15° - 2\theta) = \cos(7\theta + 10°)$.

17. $\tan 5\theta = \cot 3\theta$.

18. $\tan(\theta + 25°) = \cot 2\theta$.

19. $\tan(2\theta - 18°) = \cot(3\theta + 48°)$.

20. $\cos\theta + \cos 2\theta + \cos 3\theta = 0$.

21. $\csc 2\theta + \cot 2\theta = 2$.

22. $\sin 2\theta \cos 2\theta = -2 \sin \theta.$
23. $\sin \theta + \cos \theta = 1.$
24. $5 \cos \theta + 12 \sin \theta = 4.$
25. $3264 \sin \theta - 5728 \cos \theta = 6018.$
26. $0.1723 \cos \theta + 1.3284 \sin \theta = 0.8492.$
27. $\sqrt{3} \cos \theta - \sin \theta = \sqrt{2}.$
28. $\csc \theta = \cot \theta + \sqrt{3}.$
29. $2 \sin^2 \theta + \sin^2 2\theta = 2.$
30. $\tan^2 \theta + \cot^2 \theta = \frac{10}{3}.$
31. $\cos 3\theta - 2 \cos 2\theta + \cos \theta = 0.$
32. $\sin(\theta + 12°) + \sin(\theta - 8°) = \sin 20°.$
33. $\sin^4 \theta - \cos^4 \theta = \frac{7}{36}.$
34. $\sin^4 \theta + \cos^4 \theta = 1.$
35. $\sin 3\theta = \cos 2\theta - 1.$
36. $3 - 4 \cos^2 \theta = \cos 3\theta.$
37. $\sin(60° - \theta) - \sin(60° + \theta) = \dfrac{\sqrt{3}}{2}.$
38. $\tan(\theta + 15°) = 3 \tan(\theta - 15°).$
39. Solve the following simultaneous equations for r and θ in terms of x and y:

$$x = r \cos \theta,$$
$$y = r \sin \theta.$$

40. Solve the following simultaneous equations for r, θ, ϕ in terms of x, y, z, restricting r to positive values:

$$x = r \sin \theta \cos \phi,$$
$$y = r \sin \theta \sin \phi,$$
$$z = r \cos \theta.$$

Solve for θ and ϕ:

41. $\sin \theta - \sin \phi = 0.7038,$ $\cos \theta - \cos \phi = -0.7245.$

42. $\cos \theta + \cos \phi + \frac{1}{2} = 0,$ $\cos \frac{1}{2}\theta + \frac{1}{2} \cos \phi - \frac{1}{4} = 0.$

43. $\sin \theta = \tan \phi,$ $\cos \theta \cos \phi = \frac{1}{2}.$

44. $\sin \theta + \sin \phi = a,$ $\cos \theta + \cos \phi = b.$

45. Solve the equation $\cos x = x$ (x in radians).

SOLUTION. Draw the graphs of $y = \cos x$ and $y = x$. (See Fig. 90.) The value of x for which the curve and the line intersect is the solution of the equation. According to the graph, this value is approximately $x = 0.74$, about $42° 24'$.

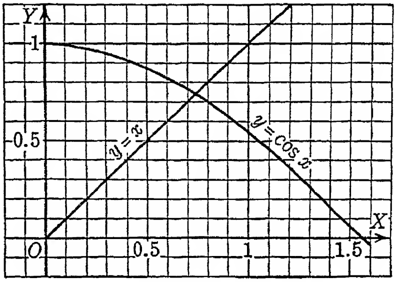


FIG. 90

A more accurate value may be obtained by writing the equation in the form $\cos x - x = 0$, and employing interpolation. Tabulating for several values of x, we get the results shown below.

	x	$\cos x$	$\cos x - x$
$42° 20'$	.73886	.73924	.00038
$42° 21'$	.73915	.73904	−.00011
$42° 22'$	.73944	.73885	−.00059
$42° 23'$	.73973	.73865	−.00108
$42° 24'$	.74002	.73846	−.00156

Since we want the value of $\cos x - x$ to be zero, the required value of x is between 0.73886 and 0.73915. Using the ordinary methods of interpolation, we have

$$\frac{x - 0.73886}{0.73915 - 0.73886} : \frac{0 - 0.00038}{-0.00011 - 0.00038};$$

or

$$\frac{x - 0.73886}{0.00029} \quad \frac{38}{49},$$

from which we get

$$x = 0.73886 + \tfrac{38}{49} \times 0.00029$$
$$= 0.73886 + 0.00022 = 0.73908.$$

By means of more extensive tables, the value correct to five decimal places is found to be 0.73909.

Solve the following equations, in which x is to be expressed in radians:

46. $\cos x = 2x.$

47. $\sin x = x - 1.$

48. $\sin x = \frac{1}{x}\cdot$

49. $\tan x = 1 - x.$

50. $\sin x = \log_{10} x.$

51. $\cos x = x^2.$

52. $\log_{10} x + x = 0.$

53. $x = 2 + \pi \sin x.$

54. $x = 1 + \frac{\pi}{6} \sin x.$

55. $x = \sin 2x.$

56. $3^x = 2 \cos x.$

57. $\sin x = 10^x.$

58. A horizontal cylindrical tank is 10 feet long and 4 feet in diameter. It contains 10 gallons of liquid. How deep is the liquid? (1 gal. = 231 cu. in.)

Some of the following equations are conditional, some are identical. Solve the former, prove the latter.

59. $\dfrac{\sin^2 \theta}{1 + \cos \theta} = 1 - \cos \theta.$

60. $\dfrac{\sin^2 \theta}{1 + \sin \theta} = 1 - \sin \theta.$

61. $\cos 2\theta + \sin 2\theta = (\cos \theta + \sin \theta)^2 + 2 \sin^2 \theta.$

62. $\cos 2\theta + \sin 2\theta = (\cos \theta + \sin \theta)^2 - 2 \sin^2 \theta.$

63. $\cot \frac{1}{2}\theta = \cot \theta(1 + \sec \theta).$

64. $\csc 2\theta + 2 \tan \theta = 3.$

65. $2 \csc 2\theta - \tan \theta = \cot \theta.$

CHAPTER XIII

★ Complex Numbers

95. Imaginary and complex numbers.

The **imaginary unit**, denoted by i, is a number having the property $i^2 = -1$. We postulate that it obeys all the laws of addition and multiplication assumed for real numbers.

Since $i^3 = i^2 \cdot i = -i$, $i^4 = (i^2)^2 = 1$, $i^5 = i^4 \cdot i = i$, it is seen that the successive integral powers of i run through the cycle i, -1, $-i$, 1.

A number of the form $a + bi$, in which a and b are real numbers, is called a **complex number.** The number a is called the **real part,** and bi is called the **imaginary part** of the complex number, b being the coefficient of the imaginary part. If $b \neq 0$, the complex number is called an **imaginary number.** If $b \neq 0$ and $a = 0$, the complex number reduces to the form bi, which is called a **pure imaginary number.** If both a and b are different from zero, the number is sometimes called a **mixed imaginary number.** If $b = 0$, the complex number reduces to the real number a.

Two complex numbers such as $a + bi$ and $a - bi$, which differ only in the signs of their imaginary parts, are called **conjugate** complex numbers. Either is said to be the conjugate of the other.

Two complex numbers are equal if and only if their real parts are equal and their imaginary parts are equal. In particular, $a + bi = 0$ if and only if $a = 0$ and $b = 0$.

96. Operations with complex numbers.

By definition, addition or subtraction of complex numbers is effected by adding or subtracting their real parts to

obtain the real part of their sum or difference, and by adding or subtracting their imaginary parts to obtain the imaginary part of their sum or difference. Thus,

$$(a + bi) + (c + di) = (a + c) + (b + d)i, \tag{1}$$
$$(a + bi) - (c + di) = (a - c) + (b - d)i. \tag{2}$$

We multiply complex numbers according to the laws of real numbers, simplifying results by making use of the relation $i^2 = -1$. Thus,

$$\begin{aligned}(a + bi)(c + di) &= ac + adi + bci + bdi^2 \\ &= (ac - bd) + (ad + bc)i. \end{aligned} \tag{3}$$

Division of complex numbers can be accomplished by writing the quotient in fractional form and multiplying both numerator and denominator by the conjugate of the denominator. Thus, to divide $a + bi$ by $c + di$ (c and d not both zero) we write

$$\begin{aligned}\frac{a + bi}{c + di} = \frac{a + bi}{c + di} \cdot \frac{c - di}{c - di} &= \frac{ac - adi + bci - bdi^2}{c^2 - d^2i^2} \\ &= \frac{(ac + bd) + (bc - ad)i}{c^2 + d^2} \end{aligned} \tag{4}$$

97. Geometric representation of complex numbers.

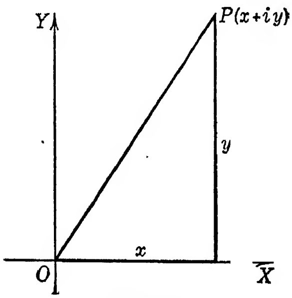


Fig. 91

The complex number $x + yi$ may be represented by the point whose abscissa is x and whose ordinate is y. (See Fig. 91.) When complex numbers are so represented, the horizontal axis is called the **axis of real numbers**, and the vertical axis is called the **axis of imaginary numbers**.

98. Geometric addition and subtraction of complex numbers.

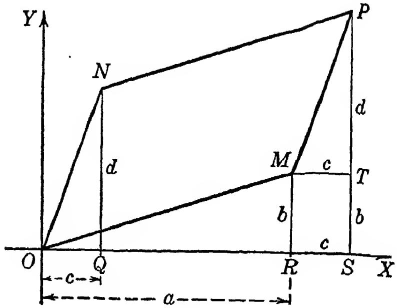


FIG. 92

Let the complex numbers $a + bi$ and $c + di$ be represented by the points M and N respectively, and their sum, $(a + c) + (b + d)i$, by the point P. (See Fig. 92.) Draw OM, ON, MP, NP. Drop NQ, MR, PS perpendicular to OX. Draw MT parallel to OX. Then,

$$MT = RS = OS - OR = (a + c) - a = c = OQ,$$
$$TP = SP - ST = (b + d) - b = d = QN.$$

Also, angle TPM is equal to angle QNO, and MP is parallel to ON. Quadrilateral $OMPN$ is a parallelogram, since two of its sides are both equal and parallel.

Thus, *to add two complex numbers geometrically, complete the parallelogram which has as adjacent sides the lines drawn from the origin to the points representing the two numbers. The fourth vertex of the parallelogram will be the point representing the sum of the two numbers.*

If we think of the complex numbers $a + bi$ and $c + di$ as represented by the vectors OM and ON in Fig. 92, the sum of the numbers will be the vector OP. (See section 67.)

To subtract $c + di$ from $a + bi$ geometrically, we may add $a + bi$ and $-c - di$.

EXERCISES XIII. A

Perform the indicated operations geometrically:

1. $(2 + 5i) + (6 + i)$.
2. $(3 + 4i) + (5 - 2i)$.
3. $(5 + 3i) - (3 - 2i)$.
4. $(-4 + 2i) + (3 + 5i)$.
5. $(3i) + (6 + 2i)$.
6. $(5i) + (6)$.

7. $(5) - (6 - 7i)$.
8. $(1 + 2i) + (3 + 6i)$.
9. $(-6 + i) + (7 + 2i)$.
10. $(3 + 6i) - (1 + 2i)$.
11. $(7 + 5i) + (7 - 5i)$.
12. $(7 + 5i) - (7 - 5i)$.
13. $(-5 - 5i) + (10 + 3i)$.
14. $(8 + 6i) - (4 + 6i)$.
15. $(-3 + 2i) + (3 - 7i)$.
16. $(5 + 7i) + (5 + 7i)$.
17. $(10 + 2i) + (-2 + 5i) + (3 - 4i)$.

SUGGESTION. Combine the first two numbers graphically, and then combine their sum with the third.

18. $(5 + 6i) + (6 - 2i) - (4 - 7i)$.
19. Given the complex numbers $10 - 4i$, $5 + 5i$, $1 - 6i$. Show that the same result is obtained by geometrically (a) adding the first and second and then adding their sum to the third, (b) adding the first and third and then adding their sum to the second, (c) adding the second and third and then adding their sum to the first.

99. Trigonometric form of complex numbers.

Let the complex number $x + yi$ be represented by the point P in Fig. 93. As usual, let $OP = r$ (a non-negative number), and denote the angle XOP by θ. Then,

$$x = r \cos \theta, \qquad y = r \sin \theta, \tag{1}$$

and the complex number may be written

$$r(\cos \theta + i \sin \theta), \tag{2}$$

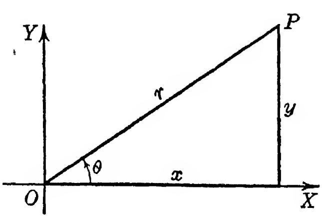


FIG. 93

which is called the **trigonometric** or **polar form** of the complex number, the form $x + yi$ being called the **rectangular form.** The expression $\cos \theta + i \sin \theta$ is sometimes abbreviated **cis θ.**

In the trigonometric form (2), r is called the **modulus** or the **absolute value** of the complex number, θ is called the **amplitude** or the **argument.** We have

$$r = \qquad\qquad \tan \theta = \frac{y}{x}. \tag{3}$$

Example 1.

Reduce $3 + 4i$ to trigonometric form.

SOLUTION.

$$r = \sqrt{3^2 + 4^2} = 5,$$

$$\tan\theta = \tfrac{4}{3} = 1.3333, \qquad \theta = 53.1°,$$

$$3 + 4i = 5(\cos 53.1° + i \sin 53.1°).$$

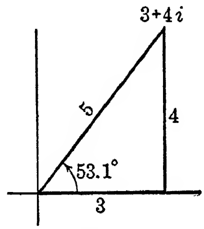


FIG. 94

Example 2.

Reduce $-1 + i\sqrt{3}$ to trigonometric form.

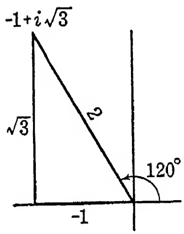


FIG. 95

SOLUTION.

$$r = \sqrt{1 + 3} = 2,$$

$$\tan\theta = \frac{\sqrt{3}}{-1} = -\sqrt{3}, \qquad \theta = 120°,$$

$$-1 + i\sqrt{3} = 2(\cos 120° + i \sin 120°).$$

EXERCISES XIII. B

Reduce to trigonometric form:

1. $-5 + 5i$.
2. $3 + 4i$.
3. $\sqrt{3} + i$.
4. $6 + 6i$.
5. $3 - 4i$.
6. $5 + 5i\sqrt{3}$.
7. $6i$.
8. -10.
9. $-8 - 15i$.
10. $12 - 5i$.
11. $2 + 3i$.
12. $12 + 5i$.
13. $5 - i$.
14. $-5i$.
15. $-7 - 7i$.
16. $6 - 6i$.
17. $6 - 8i$.
18. $-2\sqrt{3} + 2i$.
19. $-7 + 2i$.
20. $10 - 8i$.
21. $\frac{1}{2} + \frac{1}{3}i$.

Reduce to rectangular form:

22. $2(\cos 60° + i \sin 60°)$.
23. $5(\cos 45° + i \sin 45°)$.
24. $7(\cos 30° + i \sin 30°)$.
25. $3(\cos 225° + i \sin 225°)$.
26. $4(\cos 330° + i \sin 330°)$.
27. $10(\cos 90° + i \sin 90°)$.
28. $5(\cos 180° + i \sin 180°)$.
29. $4(\cos 270° + i \sin 270°)$.
30. $8(\cos 150° + i \sin 150°)$.
31. $\sqrt{2}(\cos 315° + i \sin 315°)$.
32. $\sqrt{3}(\cos 210° + i \sin 210°)$.
33. $10[\cos(-35°) + i \sin(-35°)]$.
34. $8(\cos 100° + i \sin 100°)$.
35. $5(\cos 200° + i \sin 200°)$.
36. $2(\cos 300° + i \sin 300°)$.
37. $10(\cos 400° + i \sin 400°)$.

100. Multiplication and division of complex numbers in trigonometric form.

A very interesting result is obtained if two complex numbers expressed in trigonometric form are multiplied together. Thus,

$$
\begin{aligned}
&r_1(\cos\theta_1 + i\sin\theta_1) \times r_2(\cos\theta_2 + i\sin\theta_2)\\
&\quad = r_1r_2[(\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2)\\
&\qquad\qquad + i(\sin\theta_1\cos\theta_2 + \cos\theta_1\sin\theta_2)]\\
&\quad = r_1r_2[\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)]. \qquad (1)
\end{aligned}
$$

Therefore, *the product of two complex numbers is a complex number whose modulus is the product of the moduli of the numbers, and whose amplitude is the sum of their amplitudes.*

It can readily be seen that this holds for the product of any finite number of complex numbers.

If one complex number is divided by another,* we have

$$
\begin{aligned}
\frac{r_1(\cos\theta_1 + i\sin\theta_1)}{r_2(\cos\theta_2 + i\sin\theta_2)} &= \frac{r_1(\cos\theta_1 + i\sin\theta_1)}{r_2(\cos\theta_2 + i\sin\theta_2)}\cdot\frac{\cos\theta_2 - i\sin\theta_2}{\cos\theta_2 - i\sin\theta_2}\\
&= \frac{r_1(\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2) + i(\sin\theta_1\cos\theta_2 - \cos\theta_1\sin\theta_2)}{r_2(\cos^2\theta_2 + \sin^2\theta_2)}\\
&= \frac{r_1}{r_2}[\cos(\theta_1-\theta_2) + i\sin(\theta_1-\theta_2)]. \qquad (2)
\end{aligned}
$$

In words, *the quotient of two complex numbers is a complex number whose modulus is the modulus of the dividend divided by the modulus of the divisor, and whose amplitude is the amplitude of the dividend minus the amplitude of the divisor.*

EXERCISES XIII. C

Perform the indicated operations, first reducing the numbers to trigonometric form (if necessary):

1. $3(\cos 40^\circ + i\sin 40^\circ)\cdot 5(\cos 70^\circ + i\sin 70^\circ)$.
2. $2(\cos 200^\circ + i\sin 200^\circ)\cdot 6(\cos 300^\circ + i\sin 300^\circ)$.

* The divisor cannot be zero.

3. $\left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)(2 + 2i)$. 4. $(-3 + 3i)(3 - i\sqrt{3})$.

5. $6(\cos 70° + i \sin 70°) \cdot 2(\cos 40° + i \sin 40°)$.

6. $10(\cos 20° + i \sin 20°) \cdot 5(\cos 70° + i \sin 70°)$.

7. $(3 + 3i\sqrt{3}) \div (\sqrt{3} - i)$. 8. $(-5 + 5i\sqrt{3}) \div (3 + 3i)$.

9. $(6 - 6i) \div (-2 + 2i\sqrt{3})$. 10. $(1 + i) \div (1 + i\sqrt{3})$.

101. Powers of complex numbers.

Raising to a power is a special case of multiplication, and it follows, by a repeated application of (1) of section 100, that

$$[r(\cos \theta + i \sin \theta)]^n = r^n(\cos n\theta + i \sin n\theta),$$

where n is a positive integer. The foregoing relation is known as **De Moivre's theorem.***

Example.

Find the value of $(1 + i)^5$.

SOLUTION. Plot the complex number $1 + i$ (Fig. 96). The absolute value is $\sqrt{2}$ and the amplitude is 45°.

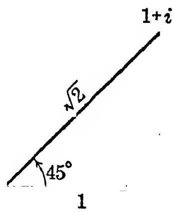


FIG. 96

$$(1 + i)^5 = [\sqrt{2}(\cos 45° + i \sin 45°)]^5$$
$$= 4\sqrt{2}(\cos 5 \cdot 45° + i \sin 5 \cdot 45°)$$
$$= 4\sqrt{2}(\cos 225° + i \sin 225°) = -4(1 + i).$$

102. Roots of complex numbers.

To prove De Moivre's theorem for the case in which the exponent is the reciprocal of a positive integer, take the expression

$$[r(\cos \theta + i \sin \theta)]^{1/n} = r^{1/n}(\cos \theta + i \sin \theta)^{1/n}. \quad (1)$$

* A formal proof of the theorem can be effected by the process of mathematical induction. For an explanation of this process, see the author's *College Algebra*, Chapter X.

Let $\theta = n\phi$. Then the right side of (1) reduces to

$$r^{1/n}(\cos n\phi + i \sin n\phi)^{1/n} = r^{1/n}[(\cos \phi + i \sin \phi)^n]^{1/n}$$
$$= r^{1/n}(\cos \phi + i \sin \phi),$$

or

$$[r(\cos \theta + i \sin \theta)]^{1/n} = r^{1/n}\left(\cos \frac{\theta}{n} + i \sin \frac{\theta}{n}\right). \qquad (2)$$

Since for any whole number k,

$$\cos(\theta + k \cdot 360°) = \cos \theta, \qquad \sin(\theta + k \cdot 360°) = \sin \theta,$$

we have

$$[r(\cos \theta + i \sin \theta)]^{1/n}$$
$$= [r\{\cos(\theta + k \cdot 360°) + i \sin(\theta + k \cdot 360°)\}]^{1/n}$$
$$= r^{1/n}\left(\cos \frac{\theta + k \cdot 360°}{n} + i \sin \frac{\theta + k \cdot 360°}{n}\right). \qquad (3)$$

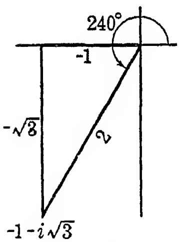


Fig. 97

By giving values to k from 0 to $n - 1$ inclusive, we obtain n distinct roots of the number $r(\cos \theta + i \sin \theta)$.

Example.

Find the fourth roots of $-1 - i\sqrt{3}$.

Solution. Plot the number $-1 - i\sqrt{3}$ (Fig. 97) and note that

$$-1 - i\sqrt{3} = 2(\cos 240° + i \sin 240°),$$

$$(-1 - i\sqrt{3})^{\frac{1}{4}} = 2^{\frac{1}{4}}\left(\cos \frac{240° + k \cdot 360°}{4} + i \sin \frac{240° + k \cdot 360°}{4}\right)$$
$$= \sqrt[4]{2}\cos(60° + k \cdot 90°) + i \sin(60° + k \cdot 90°).$$

Giving k successively the values 0, 1, 2, 3, we find for the four distinct fourth roots of $-1 - i\sqrt{3}$:

$\sqrt[4]{2}(\cos 60° + i \sin 60°)$

$$= \sqrt[4]{2}\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = -\frac{1}{2}\sqrt[4]{2} + \frac{i}{2}\sqrt[4]{18},$$

$\sqrt[4]{2}(\cos 150° + i \sin 150°)$

$$= \sqrt[4]{2}\left(-\frac{\sqrt{3}}{2} + \frac{i}{2}\right) = -\frac{1}{2}\sqrt[4]{18} + \frac{i}{2}\sqrt[4]{2},$$

$\sqrt[4]{2}(\cos 240° + i \sin 240°)$

$$= \sqrt[4]{2}\left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) = -\frac{1}{2}\sqrt[4]{2} - \frac{i}{2}\sqrt[4]{18},$$

$\sqrt[4]{2}(\cos 330° + i \sin 330°)$

$$= \sqrt[4]{2}\left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right) = \frac{1}{2}\sqrt[4]{18} - \frac{i}{2}\sqrt[4]{2}.$$

In Fig. 98, P represents the complex number $2(\cos 240° + i \sin 240°)$; P_1, P_2, P_3, P_4 represent the four roots whose amplitudes are 60°, 150°, 240°, 330°, respectively.

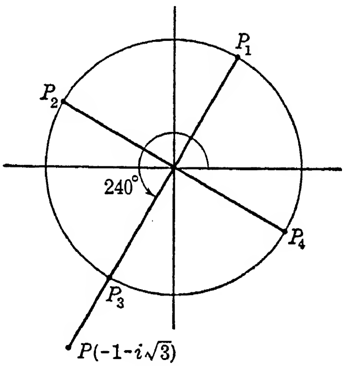


FIG. 98

Note that the roots can be found geometrically as follows: Draw a circle with center at the origin and with radius equal to the numerical fourth root of the absolute value of the number whose fourth roots are to be found, that is, a radius equal to $\sqrt[4]{2}$. Take one-fourth of the amplitude of the original number ($\frac{1}{4} \times 240° = 60°$). This locates the point P_1 on the circle. The four roots all lie on the circle and are spaced at equal intervals of 90°. Thus we can find P_2, P_3, P_4.

In general, the nth roots of the complex number $r(\cos \theta + i \sin \theta)$ can be found as follows: Draw a circle whose center is the origin and whose radius is the numerical nth root of r; divide the angle θ by n, the index of the root. Now divide the circumference of the circle, from θ/n to $\theta/n + 360°$, into n equal parts. The n points of division will be the required roots.

EXERCISES XIII. D

Use De Moivre's theorem to raise to the indicated powers:

1. $[7(\cos 18° + i \sin 18°)]^3$. **2.** $[\sqrt{3}(\cos 20° + i \sin 20°)]^8$.

3. $(1 + i)^{10}$. **4.** $(\sqrt{3} + i)^7$.

5. $(5 - 5i)^4$.

6. $[\sqrt{2}(\cos 100° + i \sin 100°)]^{10}$

7. $(\cos 22° + i \sin 22°)^8$.

8. $\left(\frac{1}{2} + i\right.$

9. $\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)^3$.

10. $[2(\cos 10° + i \sin 10°)]^{-3}$.

11. $[10(\cos 70° + i \sin 70°)]^{-6}$.

12. $(1 + i)^{-10}$.

Find all of the

13. Square roots of $9(\cos 80° + i \sin 80°)$.
14. Square roots of $4(\cos 100° + i \sin 100°)$.
15. Cube roots of $27(\cos 27° + i \sin 27°)$.
16. Square roots of $1 + i\sqrt{3}$.
17. Cube roots of $1 + i\sqrt{3}$.
18. Cube roots of $-\sqrt{3} + i$.
19. Cube roots of 1.

    SUGGESTION. $1 = \cos 0° + i \sin 0°$.

20. Fifth roots of -1.
21. Sixth roots of $-8i$.
22. Cube roots of $-2 + 3i$.
23. Fifth roots of $-4 - 4i$.
24. Seventh roots of $\sqrt{2}(1 - i)$.

Obtain all of the roots of the following equations:

25. $x^5 - 1 = 0$.
26. $x^3 + 1 = 0$.
27. $x^4 + 1 = 0$.
28. $x^5 + 32 = 0$.
29. $x^4 - 16i = 0$.
30. $x^7 - 1 = 0$.
31. $x^4 + x^3 + x^2 + x + 1 = 0$.

    SUGGESTION. Multiply by $x - 1$, solve the resulting equation, and discard the extraneous root $x = 1$.

32. $x^4 - x^3 + x^2 - x + 1 = 0$.

SPHERICAL TRIGONOMETRY

CHAPTER XIV

Introduction to Spherical Trigonometry

103. Definitions and propositions from solid geometry.

The intersection of a plane with a sphere is a circle. If the plane passes through the center of the sphere, the intersection is a **great circle**; otherwise the intersection is a **small circle.** Obviously the radius of a great circle is equal to the radius of the sphere, while the radius of a small circle is less than the radius of the sphere.

A line through the center of the sphere perpendicular to the plane of a circle is called the **axis** of the circle. This axis pierces the sphere in two points, which are called the **poles** of the circle.

The shortest distance in space between two points on a sphere is the straight line joining them, but this line does not lie on the surface of the sphere. The shortest path on the sphere between the two points is the arc (not greater than a semicircle) of a great circle joining the points. The **distance** (on the sphere) between the two points is defined to be the length of this arc. This distance is usually expressed in angular units, and is equal to the angle which the arc subtends at the center of the sphere. However, it can be converted into linear units if the radius of the sphere is known.

104. Spherical triangles.

A **spherical triangle** is that part of the surface of a sphere bounded by three arcs of great circles.* Like a plane tri-

* That part of the surface of a sphere bounded by the arcs of two great circles is called a **lune.**

angle, it is composed of six parts—three sides and three angles. We shall ordinarily designate the angles by A, B, C, and the opposite sides by a, b, c, respectively.

To each spherical triangle there corresponds a trihedral angle whose vertex is at the center of the sphere. A spherical triangle, with the corresponding trihedral angle, is illustrated in Fig. 99. In this figure, O is the center of the sphere. The sides of the spherical triangle are measured by the corresponding face angles of the trihedral angle. Thus, a is measured by BOC, b is measured by AOC, c is measured by AOB.

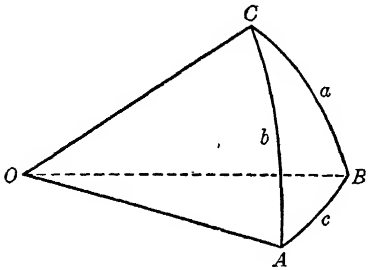


FIG. 99

The angles of the spherical triangle are measured by the corresponding dihedral angles of the trihedral angle. For example, angle A is measured by the dihedral angle whose edge is OA, namely B–OA–C.

This follows if the angle A of the spherical triangle is defined as the angle between the tangents at A to the arcs AB and AC, since the angle between these tangents is the plane angle of the dihedral angle.

It is possible to have spherical triangles with one or more sides or angles greater than 180°. However, we shall consider only triangles for which each side and each angle is less than 180°.* For such triangles, the sum of the sides is less than 360°, and the sum of the angles is between 180° and 540°; that is,

$$a + b + c < 360^\circ, \tag{1}$$

$$180^\circ < A + B + C < 540^\circ. \tag{2}$$

* Note that even with this restriction it is possible to have a spherical triangle with two, or even three, right angles. A spherical triangle having a right angle is called a **right spherical triangle,** one with two right angles is said to be **birectangular,** while one with three right angles is called **trirectangular.**

The amount by which the sum of the angles of a spherical triangle exceeds 180° is called the **spherical excess** of the triangle. That is, if E denotes the spherical excess, then

$$E = A + B + C - 180°. \tag{3}$$

The sum of any two sides is greater than the third side, and their difference is less than the third side.

If two sides are equal, the angles opposite are equal.

If two angles are equal, the sides opposite are equal.

If two sides are unequal, the angles opposite are unequal, and the greater angle is opposite the greater side.

If two angles are unequal, the sides opposite are unequal, and the greater side is opposite the greater angle.

105. Spherical polygons.

A **spherical polygon** is that part of the surface of a sphere bounded by three or more arcs of great circles. To every spherical polygon there corresponds a polyhedral angle whose vertex is at the center of the sphere. The sides of the polygon are measured by the corresponding face angles of the polyhedral angle, and the angles of the polygon are measured by the corresponding dihedral angles of the polyhedral angle.

A spherical polygon of n sides can be divided into $n - 2$ triangles by drawing diagonals from one vertex. The sum of the excesses of these triangles is equal to the sum of the angles of the polygon less $(n - 2) \cdot 180°$. This difference may be called the **spherical excess** of the polygon.

106. Polar triangles.

With the vertices of a spherical triangle ABC as poles, construct three great circles. The great circles whose poles are B and C will intersect in two diametrically opposite

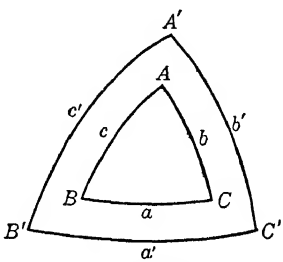


FIG. 100

points. Denote by A' that point of intersection which is on the same side of BC as is A. Determine B' and C' similarly. Then $A'B'C'$ is the **polar triangle** of ABC. (See Fig. 100.) Conversely, ABC is the polar triangle of $A'B'C'$.

Each angle of a spherical triangle is the supplement of the corresponding side in the polar triangle. That is,

$$A + a' = 180°, \qquad B + b' = 180°, \qquad C + c' = 180°,$$
$$A' + a = 180°, \qquad B' + b = 180°, \qquad C' + c = 180°.$$

107. Areas.

The area of the surface of a sphere of radius R is $4\pi R^2$.

The area of a spherical triangle on a given sphere is proportional to its spherical excess. Since the area of a trirectangular triangle (whose excess is $270° - 180° = 90°$) is one-eighth of the surface of the sphere, that is, $\frac{1}{8} \cdot 4\pi R^2 = \frac{1}{2}\pi R^2$, we have for the area of a triangle ABC,

$$\frac{\text{area}}{\frac{1}{2}\pi R^2} = \frac{E}{90},$$

or,

$$\text{area} = \frac{\pi R^2 E}{180} \tag{1}$$

This formula applies to any spherical polygon provided the excess of the polygon is defined as in section 105.

A **spherical degree** is a unit of surface measurement on a sphere equal to half a lune whose angle is 1°. (For definition of lune see footnote, page 197.) The area, in spherical degrees, of a spherical triangle, or of any spherical polygon, is equal to its spherical excess.*

* When the three sides of a spherical triangle are known, the excess can be determined by **L'Huilier's formula**, given here without derivation:

$$\tan \tfrac{1}{4}E = \sqrt{\tan \tfrac{1}{2}s \tan \tfrac{1}{2}(s - a) \tan \tfrac{1}{2}(s - b) \tan \tfrac{1}{2}(s - c)},$$

in which $s = \frac{1}{2}(a + b + c)$.

CHAPTER XV

Solution of Right Spherical Triangles

108. Formulas for solving right spherical triangles.

In Fig. 101 is represented a right spherical triangle, ABC, with the right angle at C (this will be the usual notation) and with sides a and b each less than 90°. Also shown is the

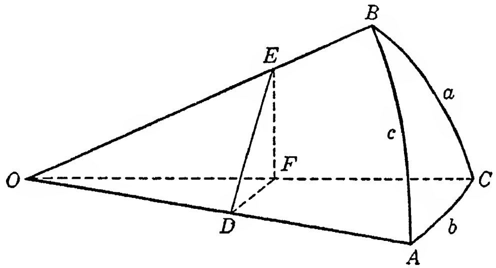


Fig. 101

trihedral angle O–ABC associated with the triangle, O being the center of the sphere.

Through any point E on the edge OB, pass a plane DEF perpendicular to the edge OA and intersecting this edge in D. Then DE and DF will be perpendicular to OA.

From the various constructions it follows that the plane triangles ODF, ODE, OFE, DFE are right triangles, the vertex of the right angle being named as the middle letter.

In triangle DFE, angle D is equal to angle A of the spherical triangle, and each of the other plane right triangles has an angle equal to one of the sides of the spherical triangle.

Making use of these facts, we have

$$\sin a = \sin FOE = \frac{FE}{OE}, \qquad \sin c = \sin DOE = \frac{DE}{OE},$$

$$\frac{\sin a}{\sin c} = \frac{FE}{DE} = \sin A. \tag{1}$$

Also,

$$\tan b = \tan DOF = \frac{DF}{OD}, \qquad \tan c = \tan DOE = \frac{DE}{OD},$$

$$\frac{\tan b}{\tan c} = \frac{DF}{DE} = \cos A. \tag{2}$$

Similarly,

$$\tan a = \tan FOE = \frac{FE}{OF}, \qquad \sin b = \sin DOF = \frac{DF}{OF},$$

$$\frac{\tan a}{\sin b} = \frac{FE}{DF} = \tan A. \tag{3}$$

Finally,

$$\cos a = \cos FOE = \frac{OF}{OE}, \qquad \cos b = \cos DOF = \frac{OD}{OF},$$

$$\cos a \cos b = \frac{OD}{OE} = \cos c. \tag{4}$$

If the plane DEF had been constructed perpendicular to OB instead of to OA, we should have been led to results similar to (1), (2), (3), which can be obtained from these formulas by interchanging A and B, a and b. They are

$$\frac{\sin b}{\sin c} = \sin B, \qquad \frac{\tan a}{\tan c} = \cos B, \qquad \frac{\tan b}{\sin a} = \tan B. \tag{5}$$

Note that when this interchange is applied to (4) the formula reverts into itself.

From the foregoing formulas it can further be proved that

$$\cos a \sin B = \cos A, \qquad \cos b \sin A = \cos B, \tag{6}$$

$$\cot A \cot B = \cos a \cos b = \cos c. \tag{7}$$

Collecting these numbered results, and clearing of frac-

tions when necessary, we have the following ten formulas for the solution of right spherical triangles:

$$\sin a = \sin c \sin A, \quad (8) \qquad \sin b = \sin c \sin B, \quad (9)$$
$$\tan a = \sin b \tan A, \quad (10) \qquad \tan b = \sin a \tan B, \quad (11)$$
$$\tan a = \tan c \cos B, \quad (12) \qquad \tan b = \tan c \cos A, \quad (13)$$
$$\cos c = \cos a \cos b, \quad (14) \qquad \cos c = \cot A \cot B, \quad (15)$$
$$\cos A = \cos a \sin B, \quad (16) \qquad \cos B = \cos b \sin A. \quad (17)$$

They have been derived for the case in which each part of the spherical triangle ABC (except the right angle C) is less than 90°. However, it can be proved that they hold for parts equal to or greater than 90°.

109. Napier's rules.

The foregoing ten formulas may, by a clever device due to Napier, be put into a form which is easily remembered.

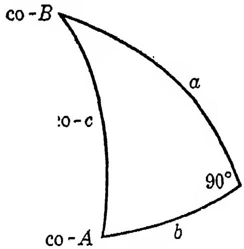


FIG. 102

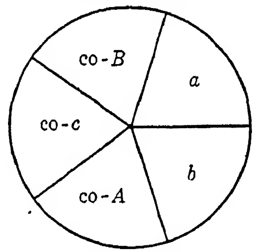


FIG. 103

In the schematic triangle of Fig. 102 we have replaced A by the symbol co-A, meaning "complement of A," and similarly for B and c.* Note that angle C is omitted. The five parts may also be arranged in a circle, as in Fig. 103, and are consequently often referred to as **circular parts.**

If in either of these diagrams any part is called the **middle part,** the two parts next to it are called the **adjacent parts,** and the other two are called the **opposite parts.** For example, if a is the middle part, then b and co-B are the adjacent parts, co-c and co-A are the opposite parts. **Napier's rules** are:

* It should be understood that Fig. 102 does not represent a triangle.

I. The sine of any middle part is equal to the product of the tangents of the adjacent parts.

II. The sine of any middle part is equal to the product of the cosines of the opposite parts.

As an illustration, let us take the part a. Rule I gives

$$\sin a = \tan b \tan \text{co-}B = \tan b \cot B,$$

which is formula (11). Rule II gives

$$\sin a = \cos \text{co-}c \cos \text{co-}A = \sin c \sin A,$$

which is (8).

By applying Napier's rules to each of the five parts of the diagram of Fig. 102 or that of Fig. 103, we obtain all ten of the formulas (8) to (17).

As a further mnemonic scheme we observe that the vowel **i** occurs in " sine " and " middle," the vowel **a** predominates in " tangents " and " adjacent " of Rule I, while the vowel **o** predominates in " cosines " and " opposite " of Rule II.

110. Solution of right spherical triangles.

If any two parts of a right spherical triangle (in addition to the right angle C) are given, the remaining parts can be found. However, it should be noted that sometimes no solution exists. (See example 2 later in this section.)

The quadrant in which any required part terminates may be determined by noting the signs of the functions involved. However, if the unknown part is determined from its sine, there are two possibilities for this part, the tabular value and its supplement, and consequently there are two solutions, subject however to the restrictions of the following theorems:

THEOREM I. *In a right spherical triangle, any side and the opposite angle terminate in the same quadrant.*

From equation (16), namely

$$\cos A = \cos a \sin B,$$

it is seen, since $\sin B$ is positive, that $\cos a$ and $\cos A$ must

have the same sign. That is, a and A terminate in the same quadrant. The same result can be proved for b and B.

THEOREM II. *If any two of the three parts a, b, c, terminate in the same quadrant, the third terminates in the first quadrant; if any two terminate in different quadrants, the third terminates in the second quadrant.*

The proof follows directly from equation (14),

$$\cos c = \cos a \cos b.$$

For if any two of the functions $\cos a$, $\cos b$, $\cos c$ have like signs, the third is positive; if any two have unlike signs, the third is negative.

The solution of a right spherical triangle can always be checked by the formula involving the three computed parts.

Example 1.

In a right spherical triangle ($C = 90°$), $A = 69° 50.8'$, $c = 72° 15.4'$; find B, a, b.

SOLUTION.	A	$69° 50.8'$
	c	$72° 15.4'$
$\sin a = \sin c \sin A$,	$\log \sin c$	$9.97884 - 10$
$\log \sin a = \log \sin c + \log \sin A$.	$\log \sin A$	$9.97256 - 10$
	$\log \sin a$	$9.95140 - 10$
		$63° 23.8'$ *
$\cos A = \tan b \cot c$,	$\log \cos A$	$9.53723 - 10$
$\log \tan b = \log \cos A - \log \cot c$.	$\log \cot c$	$9.50511 - 10$
	$\log \tan b$	0.03212
	b	**$47° 7.0'$**
$\cos c = \cot A \cot B$,	$\log \cos c$	$9.48395 - 10$
$\log \cot B = \log \cos c \quad \log \cot A$.	$\log \cot A$	$9.56467 - 10$
	$\log \cot B$	$9.91928 - 10$
	B	**$50° 17.7'$**
CHECK.† $\sin a = \tan b \cot B$,	$\log \tan b$	0.03212
$\log \sin a = \log \tan b + \log \cot B$.	$\log \cot B$	$9.91928 - 10$
	$\log \sin a$	$9.95140 - 10$

* The supplementary value is not admissible, since, by Theorem I, a and A must terminate in the same quadrant.

† This check verifies the consistency of the logarithms, but does not prove that the angular quantities are correct.

Example 2.

Solve the spherical triangle $C = 90°$, $A = 120°$, $a = 100°$.

SOLUTION.	A	$120°$	
		$100°$	
$\sin b = \tan a \cot A$,	$\log \tan a$	0.75368	(neg)*
$\log \sin b = \log \tan a + \log \cot A$.	$\log \cot A$	$9.76144 - 10$	(neg)
	$\log \sin b$	0.51512	
No solution.		impossible	

Example 3.

Given $C = 90°$, B $36° 42.2'$, $b = 30° 17.5'$; find the remaining parts.

SOLUTION.	B	$36° 42.2'$
		$30° 17.5'$
$\sin a = \tan b \cot B$,	$\log \tan b$	$9.76654 - 10$
$\log \sin a = \log \tan b + \log \cot B$.	$\log \cot B$	0.12757
	$\log \sin a$	$9.89411 - 10$
	a	$\mathbf{51° 35.6'}$ or $\mathbf{128° 24.4'}$
$\sin b = \sin c \sin B$,	$\log \sin b$	$9.70278 - 10$
$\log \sin c = \log \sin b - \log \sin B$.	$\log \sin B$	$9.77646 - 10$
	$\log \sin$	$9.92632 - 10$
		$\mathbf{57° 33.6'}$ or $\mathbf{122° 26.4'}$
$\cos B = \cos b \sin A$,	$\log \cos B$	$9.90403 - 10$
$\log \sin A$	$\log \cos b$	$9.93624 - 10$
$= \log \cos B - \log \cos b$.	$\log \sin A$	$9.96779 - 10$
		$\mathbf{68° 12.2'}$ or $\mathbf{111° 47.8'}$
CHECK. $\sin a = \sin c \sin A$,	$\log \sin c$	$9.92632 - 10$
$\log \sin a = \log \sin c + \log \sin A$.	$\log \sin A$	$9.96779 - 10$
	$\log \sin a$	$9.89411 - 10$

By Theorems I and II, the obtained values are grouped into the following two solutions:

$$A = 68° 12.2', \qquad a = 51° 35.6', \qquad c = 57° 33.6';$$
$$A' = 111° 47.8', \qquad a' = 128° 24.4', \qquad c' = 122° 26.4'.$$

* The notation (neg) indicates that the function is negative.

EXERCISES XV. A

Find the remaining parts of the following triangles, in each of which $C = 90°$:

✓1. $A = 80° 10.5'$, $c = 110° 46.3'$.
2. $B = 130° 30.0'$, $a = 114° 23.8'$.
✓3. $B = 36° 42.5'$, $c = 112° 25.0'$
4. $A = 136° 5.2'$, $a = 110° 18.6'$.
✓5. $A = 75° 15.0'$, $B = 133° 8.0'$.
6. $a = 66° 59.5'$, $b = 156° 34.3'$.
✓7. $B = 154° 44.3'$, $b = 156° 3.0'$.
8. $A = 116° 32.4'$, $b = 50° 25.6'$.
✓9. $B = 112° 19.7'$, $a = 77° 35.3'$.
10. $a = 39° 46.3'$, $b = 62° 30.6'$.
11. $a = 130° 12.9'$, $c = 73° 58.0'$.
12. $A = 19° 15.3'$, $B = 85° 33.0'$.
13. $b = 26° 28.7'$, $c = 61° 25.1'$.
14. $A = 132° 15.6'$, $B = 47° 44.4'$.
15. $a = 98° 8.1'$, $c = 77° 41.9'$.
16. $B = 124° 14.8'$, $b = 147° 15.2'$.
17. $A = 25° 16.6'$, $a = 18° 54.3'$.
18. $A = 69° 2.4'$, $a = 62° 12.8'$.
19. $A = 75° 21.9'$, $b = 14° 59.6'$.
20. $B = 83° 56.7'$, $b = 77° 21.8'$.
21. Three concurrent edges of a cube are OP, OQ, OR. Find the dihedral angle between the plane PQR and one of the faces of the cube.
22. Show that if $B = C = 90°$, then $b = c = 90°$, and that A and a are indeterminate, but $A = a$.
23. Show that if $c = C = 90°$, then either $A = a = 90°$, and B and b are indeterminate, but $B = b$; or else $B = b = 90°$, and A and a are indeterminate, but $A = a$.
24. Show that if C is a right angle and if $b = c$ (and consequently each is a right angle), then $B = 90°$, and that A and a are indeterminate, but $A = a$.

111. Quadrantal triangles.

A **quadrantal triangle** is a spherical triangle having a side equal to 90°. The polar triangle of a quadrantal triangle is

a right triangle, which can be solved by the methods explained in the preceding section. The parts of the quadrantal triangle can then be obtained.

For example, suppose we have given $c = 90°$, $b = 50°$, $A = 70°$. We know that

$$C' = 180° - c = 90°, \qquad B' = 180° - b = 130°,$$
$$a' = 180° - A = 110°.$$

We then find A', b', c', from which the values of a, B, C are readily obtained.

EXERCISES XV. B

Solve the following quadrantal triangles ($c = 90°$):

1. $a = 70° \; 7.8'$, $b = 52° \; 36.7'$.
2. $C = 135° \; 33.7'$, $a = 31° \; 30.7'$.
3. $A = 118° \; 46.4'$, $C = 100° \; 7.8'$.
4. $B = 55° \; 47.1'$, $C = 105° \; 9.5'$.
5. $A = 102° \; 38.3'$, $a = 96° \; 3.3'$.
6. $A = 73° \; 45.4'$, $b = 123° \; 36.1'$.
7. $a = 106° \; 38.6'$, $b = 36° \; 49.7'$.
8. $A = 122° \; 39.7'$, $a = 116° \; 52.5'$.
9. $B = 63° \; 4.6'$, $b = 69° \; 29.7'$.
10. $a = 60° \; 39.8'$, $b = 65° \; 52.4'$.

112. Isosceles spherical triangles.

The great circle drawn from the vertex of an isosceles spherical triangle to the midpoint of the opposite side divides the triangle into two symmetric right triangles. The solution of an isosceles spherical triangle can thus be reduced to the solution of a right spherical triangle.

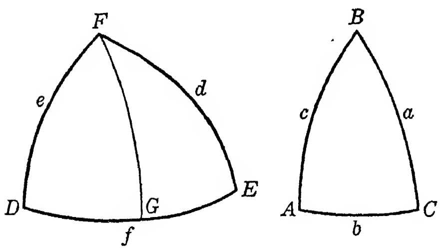


FIG. 104

Example.

Find the remaining parts of an isosceles spherical triangle in which the equal angles are $D = E = 80° \; 27'$ and the side included by these equal angles is $f = 76° \; 42'$. (See Fig. 104.)

Solution. Draw a perpendicular, FG, from the vertex F to the base DE. This divides the triangle into two symmetric right spherical triangles DFG and GFE. For clarity, the first of these has been redrawn at the right in Fig. 104, and has been relettered, so that A, B, C replace D, F, G, respectively. Then, in the triangle ABC, we have $C = 90°$, $b = \frac{1}{2}f$. The logarithmic work follows.

$$\cos B = \cos b \sin A,$$
$$\log \cos B = \log \cos b + \log \sin A.$$

	A	$80° \ 27'$
	b	$38° \ 21'$
log cos	b	$9.89445 - 10$
log sin	A	$9.99394 - 10$
log cos	B	$9.88839 - 10$
	B	$39° \ 20.5'$

$$\cos A = \tan b \cot c,$$
$$\log \cot c = \log \cos A - \log \tan b.$$

log cos	A	$9.21987 - 10$
log tan	b	$9.89827 - 10$
log cot	c	$9.32160 - 10$
	c	$78° \ 9'$

Returning to the isosceles triangle, we have

$$F = 2B = 2 \times 39° \ 20.5' = 78° \ 41',$$
$$d = e = c = 78° \ 9'.$$

EXERCISES XV. C

Solve the following triangles:

✓1. $A = C = 69° \ 2.3'$, $b = 93° \ 16.4'$.
2. $B = C = 52° \ 36.7'$, $b = 73° \ 58.0'$.
✓3. $B = 112° \ 47.8'$, $a = c = 99° \ 9.6'$.
4. $a = c = 77° \ 7.7'$, $b = 37° \ 30.4'$.
✓5. $A = 153° \ 48.2'$, $a = 145° \ 3.8'$, $B = C$.
6. $A = C = 77° \ 40.5'$, $b = 52° \ 1.8'$.
✓7. $A = B = 95° \ 5.1'$, $C = 100° \ 10.8'$.
8. $A = 58° \ 58.8'$, $b = c = 63° \ 47.8'$.
9. $A = 62° \ 1.5'$, $a = c = 71° \ 59.3'$.
10. $B = 72° \ 48.8'$, $b = 64° \ 50.6'$, $a = c$.
11. $a = b = c = 10°$.
12. $a = b = c = 80°$.
13. $a = b = c = 100°$.
14. $A = B = C = 80°$.
15. $A = B = C = 100°$.
16. $A = B = C = 170°$.

17. Show that if each side of a spherical triangle is 60° each angle is arccos $\frac{1}{3}$.

18. Show that if each angle of a spherical triangle is 120° each side is arccos $(-\frac{1}{3})$.

19. Show that if each side of a spherical triangle is 30° each angle is arccos $(2\sqrt{3} - 3)$.

20. Prove that in an equilateral spherical triangle

$$\cos A = \frac{\cos a}{1 + \cos a}.$$

21. Prove that in an equiangular spherical triangle

$$\cos a = \frac{\cos A}{1 - \cos A}$$

22. In an isosceles spherical triangle the base is 63° 8.8′ and the equal sides are 40° 4.4′. Find the perpendicular from the vertex to the base, also the perpendicular from one end of the base to the opposite side.

CHAPTER XVI

Solution of Oblique Spherical Triangles

113. Oblique spherical triangles.

If no angle of a spherical triangle is a right angle the triangle is **oblique.** For the solution of oblique spherical triangles, certain formulas, analogous to those of Chapter VII are needed, and we shall proceed to develop them.

114. Law of sines.

Let ABC be any spherical triangle. Through the vertex C draw the arc of a great circle perpendicular to the

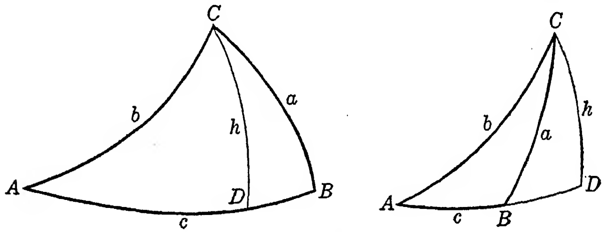


FIG. 105

side c (produced if necessary) at the point D. (See Fig. 105.) Designate the length of this perpendicular CD by h.

The foregoing construction yields two right spherical triangles, ADC and BDC. By Napier's rules we find

$$\sin h = \sin a \sin B, \qquad \sin h = \sin b \sin A. \tag{1}$$

Equating the two values of $\sin h$, and dividing by $\sin A \sin B$, we get

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B}. \tag{2}$$

Similarly, by drawing an arc through the vertex B perpendicular to the side b, we can prove the relation

$$\frac{\sin a}{\sin A} = \frac{\sin c}{\sin C}. \tag{3}$$

Combining (2) and (3), we obtain the **law of sines** for spherical triangles,

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}. \tag{4}$$

That is, *the sines of the sides of a spherical triangle and the sines of the corresponding opposite angles are in proportion.*

√115. Law of cosines for sides.

In Fig. 106, in which the construction is the same as that in Fig. 105, denote arc AD by m. Applying Napier's rules

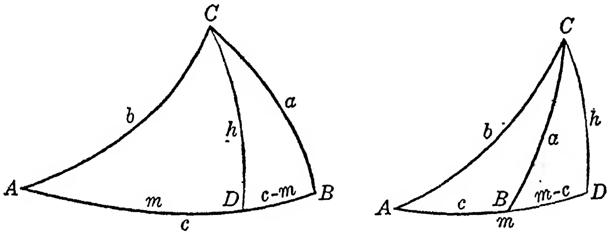


FIG. 106

to the right triangle BDC, we find, from either part of the figure, since $\cos(m - c) = \cos(c - m)$,

$$\begin{aligned} \cos a &= \cos h \cos(c - m) \\ &= \cos h(\cos c \cos m + \sin c \sin m). \end{aligned} \tag{1}$$

From the right triangle ADC, we find

$$\cos b = \cos h \cos m, \qquad \text{or} \qquad \cos m = \frac{\cos b}{\cos h}; \tag{2}$$

and

$$\sin m = \tan h \cot A, \tag{3}$$

$$\sin h = \sin b \sin A. \tag{4}$$

Substituting (2) and (3) in (1), we get

$$\cos a = \cos h\left(\cos c \frac{\cos b}{\cos h} + \sin c \tan h \cot A\right)$$
$$= \cos c \cos b + \sin c \sin h \cot A,$$

or, substituting the value of $\sin h$ from (4),

$$\cos a = \cos c \cos b + \sin c \sin b \cos A.$$

Rearranging this formula, and writing the two others obtainable from it by a cyclic change of letters,* we have

$$\cos a = \cos b \cos c + \sin b \sin c \cos A, \tag{5}$$
$$\cos b = \cos c \cos a + \sin c \sin a \cos B, \tag{6}$$
$$\cos c = \cos a \cos b + \sin a \sin b \cos C. \tag{7}$$

These formulas are known as the **law of cosines for sides.**

116. Law of cosines for angles.

Applying formula (5) to $A'B'C'$, the polar triangle of ABC, we get

$$\cos a' = \cos b' \cos c' + \sin b' \sin c' \cos A'. \tag{1}$$

If we now make use of the relations between the parts of a triangle and the parts of its polar triangle, $a' = 180° - A$, etc. (see section 106), and of the formulas

$$\cos(180° - \theta) = -\cos \theta, \qquad \sin(180° - \theta) = \sin \theta,$$

(1) reduces to

$$\cos A = -\cos B \cos C + \sin B \sin C \cos a. \tag{2}$$

Similarly,

$$\cos B = -\cos C \cos A + \sin C \sin A \cos b, \tag{3}$$
$$\cos C = -\cos A \cos B + \sin A \sin B \cos c. \tag{4}$$

* See section 54.

The three foregoing formulas constitute the **law of cosines for angles.**

The law of cosines, either for sides or for angles, together with the relations between the parts of a triangle and the parts of its polar triangle, is sufficient for solving any spherical triangle if three parts are given, since it is always possible to find a form of the law which involves the three given parts and a single unknown part. For example, if the given parts are A, B, a, we could use (2) to find C, then (3) and (4) to find b and c respectively. However, the law of cosines is not adapted to the use of logarithms, and as problems of spherical trigonometry ordinarily require accurate results, it is desirable to derive other formulas with which logarithms can be used.

√ 117. Law of tangents.

The law of sines for spherical triangles may be written in the form

$$\frac{\sin A}{\sin B} = \frac{\sin a}{\sin b}. \tag{1}$$

By composition and division,*

$$\frac{\sin A - \sin B}{\sin A + \sin B} = \frac{\sin a - \sin b}{\sin a + \sin b} \tag{2}$$

Applying formulas (9) and (8) of section 75 (page 132) to the numerator and denominator of the fraction on the left, we reduce it to the form

$$\frac{2 \cos \frac{1}{2}(A + B) \sin \frac{1}{2}(A - B)}{2 \sin \frac{1}{2}(A + B) \cos \frac{1}{2}(A - B)} = \frac{\tan \frac{1}{2}(A - B)}{\tan \frac{1}{2}(A + B)}. \tag{3}$$

The right side of (2) may be similarly reduced, and we get the **law of tangents** for spherical triangles,

$$\checkmark \quad \frac{\tan \frac{1}{2}(A - B)}{\tan \frac{1}{2}(A + B)} = \frac{\tan \frac{1}{2}(a - b)}{\tan \frac{1}{2}(a + b)}. \tag{4}$$

* See the author's *College Algebra*, p. 128.

✓118. Half-angle formulas.

We shall now develop the half-angle formulas for spherical trigonometry.

From formula (5) of section 74 (page 129), we have *

$$\tan \tfrac{1}{2}A = \sqrt{\frac{1 - \cos A}{1 + \cos A}} \tag{1}$$

Solving equation (5) of the law of cosines (section 115) for cos A, we find

$$\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c}.$$

Subtracting each side from 1, we get

$$\begin{aligned} 1 - \cos A &= 1 - \frac{\cos a - \cos b \cos c}{\sin b \sin c} \\ &= \frac{\sin b \sin c - \cos a + \cos b \cos c}{\sin b \sin c} \\ &= \frac{\cos(b - c) - \cos a}{\sin b \sin c}. \end{aligned} \tag{2}$$

Similarly, we find

$$1 + \cos A = \frac{\cos a - \cos(b + c)}{\sin b \sin c}. \tag{3}$$

Substituting (2) and (3) in (1), we get

$$\tan \tfrac{1}{2}A = \sqrt{\frac{\cos(b - c) - \cos a}{\cos a - \cos(b + c)}}. \tag{4}$$

By formula (11) of section 75 (page 132),

$$\cos(b - c) - \cos a = -2 \sin \tfrac{1}{2}(b - c + a) \sin \tfrac{1}{2}(b - c - a), \tag{5}$$

$$\cos a - \cos(b + c) = -2 \sin \tfrac{1}{2}(a + b + c) \sin \tfrac{1}{2}(a - b - c). \tag{6}$$

* Only the positive sign is used with the radical, since, by the restriction imposed in section 104, $A < 180°$, and consequently $\tfrac{1}{2}A < 90°$.

If we let *

$$s = \tfrac{1}{2}(a + b + c), \tag{7}$$

then it can easily be shown that

$$\begin{aligned} b + c - a &= 2(s - a),\\ a + c - b &= 2(s - b),\\ a + b - c &= 2(s - c). \end{aligned} \tag{8}$$

By means of (5), (6), (7), we can reduce (4) to the form

$$\tan \tfrac{1}{2}A = \sqrt{\frac{\sin(s - b)\sin(s - c)}{\sin s \sin(s - a)}} \tag{9}$$

and, if †

$$\tan r = \sqrt{\frac{\sin(s - a)\sin(s - b)\sin(s - c)}{\sin s}} \tag{10}$$

(10) reduces to the simpler form

$$\tan \tfrac{1}{2}A = \frac{\tan r}{\sin(s - a)}. \tag{11}$$

Similarly,

$$\tan \tfrac{1}{2}B = \frac{\tan r}{\sin(s - b)}, \tag{12}$$

$$\tan \tfrac{1}{2}C = \frac{\tan r}{\sin(s - c)}. \tag{13}$$

These may be termed the **half-angle formulas.**

119. Half-side formulas.

If we solve formula (2) of section 116 for $\cos a$ and proceed somewhat as above, we can derive the **half-side formulas:**

$$\tan \tfrac{1}{2}a = \tan R \cos(S - A), \tag{1}$$

$$\tan \tfrac{1}{2}b = \tan R \cos(S - B), \tag{2}$$

$$\tan \tfrac{1}{2}c = \tan R \cos(S - C), \tag{3}$$

in which ‡

* Cf. section 64.

† It can be shown that r is the radius of the small circle inscribed in the spherical triangle ABC.

‡ It can be shown that R is the radius of the small circle circumscribed about the spherical triangle ABC.

$$\tan R = \sqrt{\frac{-\cos S}{\cos(S-A)\cos(S-B)\cos(S-C)}} \tag{4}$$

and
$$S = \tfrac{1}{2}(A+B+C). \tag{5}$$

This is left as an exercise.

120. Napier's analogies.

Dividing (11) of section 118 by (12) of the same section, we get

$$\frac{\tan \frac{1}{2}A}{\tan \frac{1}{2}B} = \frac{\sin(s-b)}{\sin(s-a)}, \tag{1}$$

and by composition and division,

$$\frac{\tan \frac{1}{2}A - \tan \frac{1}{2}B}{\tan \frac{1}{2}A + \tan \frac{1}{2}B} = \frac{\sin(s-b) - \sin(s-a)}{\sin(s-b) + \sin(s-a)},$$

which reduces as follows:

$$\frac{\dfrac{\sin \frac{1}{2}A}{\cos \frac{1}{2}A} - \dfrac{\sin \frac{1}{2}B}{\cos \frac{1}{2}B}}{\dfrac{\sin \frac{1}{2}A}{\cos \frac{1}{2}A} + \dfrac{\sin \frac{1}{2}B}{\cos \frac{1}{2}B}} = \frac{2\cos \frac{1}{2}(2s-a-b)\sin \frac{1}{2}(a-b)}{2\sin \frac{1}{2}(2s-a-b)\cos \frac{1}{2}(a-b)}$$

$$\frac{\sin \frac{1}{2}A \cos \frac{1}{2}B - \cos \frac{1}{2}A \sin \frac{1}{2}B}{\sin \frac{1}{2}A \cos \frac{1}{2}B + \cos \frac{1}{2}A \sin \frac{1}{2}B} = \frac{\tan \frac{1}{2}(a-b)}{\tan \frac{1}{2}c}$$

$$\frac{\sin \frac{1}{2}(A-B)}{\sin \frac{1}{2}(A+B)} = \frac{\tan \frac{1}{2}(a-b)}{\tan \frac{1}{2}c}. \tag{2}$$

Multiplying (9) of section 118 by the corresponding formula for $\tan \frac{1}{2}B$ gives

$$\tan \tfrac{1}{2}A \tan \tfrac{1}{2}B = \frac{\sin(s-c)}{\sin s}. \tag{3}$$

Writing the left side in the form $\tan \frac{1}{2}A/\cot \frac{1}{2}B$ and taking steps quite similar to those taken in proving formula (2) of

the present section, we can reduce (3) to the form *

$$\frac{\cos \frac{1}{2}(A - B)}{\cos \frac{1}{2}(A + B)} = \frac{\tan \frac{1}{2}(a + b)}{\tan \frac{1}{2}c}. \tag{4}$$

This is left as an exercise.

It is also left as an exercise to prove, from (2) and (4), by the use of polar triangles, the following formulas:

$$\frac{\sin \frac{1}{2}(a - b)}{\sin \frac{1}{2}(a + b)} = \frac{\tan \frac{1}{2}(A - B)}{\cot \frac{1}{2}C}. \tag{5}$$

$$\frac{\cos \frac{1}{2}(a - b)}{\cos \frac{1}{2}(a + b)} = \frac{\tan \frac{1}{2}(A + B)}{\cot \frac{1}{2}C}. \tag{6}$$

By applying cyclic changes to the letters in formulas (2), (4), (5), (6) we obtain eight more formulas, or a total of twelve. These twelve formulas are called **Napier's analogies.**†

121. The six cases.

Problems in the solution of oblique spherical triangles may be classified into the following six cases:

✓*Case I. Three sides given.* Torns.

Case II. Three angles given.

Case III. Two sides and the included angle given.

Case IV. Two angles and the included side given.

Case V. Two sides and the angle opposite one of them given.

Case VI. Two angles and the side opposite one of them given.

Cases I and II, III and IV, V and VI, are essentially equivalent (in pairs) because of the relations between the parts of a triangle and the parts of its polar triangle. For example, if the three sides of a triangle are given, the three angles of the polar triangle can be found at once, so that

* Formula (4) can also be derived by using the law of tangents and (2).

† The word "analogy" is used in the now obsolete sense of "proportion."

Case I for the given triangle is Case II for the polar triangle.

The six cases can be solved by the application of the half-angle and half-side formulas, Napier's analogies, and the law of sines, as will be illustrated in subsequent sections.

122. Clearing up certain ambiguities.

When Napier's analogies are used, the quadrant in which any part terminates can always be determined by noting the signs of the functions involved. However, when the law of sines is used, two values are found for the required part. Whether one or both of these values are admissible may be determined by the principle established in solid geometry that the three sides and the three angles are in the same order of magnitude (e.g., if $A > B > C$, then $a > b > c$) or by the following theorems:

Theorem I. *Half the sum of any two sides is in the same quadrant as half the sum of the opposite angles.*

This theorem is easily proved by using Napier's analogy (4), namely,

$$\frac{\cos \frac{1}{2}(A - B)}{\cos \frac{1}{2}(A + B)} = \frac{\tan \frac{1}{2}(a + b)}{\tan \frac{1}{2}c}.$$

Since each part of a triangle is less than 180°, each of the quantities $\frac{1}{2}(A - B)$ and $\frac{1}{2}c$ is less than 90°. Consequently, $\cos \frac{1}{2}(A - B)$ and $\tan \frac{1}{2}(a - b)$ are both positive. Therefore, $\cos \frac{1}{2}(A + B)$ and $\tan \frac{1}{2}(a + b)$ are of the same sign, and $\frac{1}{2}(A + B)$ and $\frac{1}{2}(a + b)$ are either both in the first quadrant or both in the second quadrant.

Corollary. *If two sides are supplementary the angles opposite are supplementary, and conversely.*

Theorem II. *A side which differs from* 90° *more than another side does, terminates in the same quadrant as its opposite angle.*

Suppose, for example, that a differs from 90° more than b does.

From the law of cosines for sides (formula (5) of section 115), we have

$$\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c}.$$

From the hypothesis regarding a and b it follows that $\cos a$ is numerically greater than $\cos b$. Moreover, since $\cos c$ is numerically not greater than 1, $\cos a$ is also greater than $\cos b \cos c$. Hence the numerator of the above fraction has the same sign as $\cos a$. The denominator is positive, and consequently $\cos a$ and $\cos A$ have the same sign. Therefore a terminates in the same quadrant as A.

THEOREM III. *An angle which differs from* 90° *more than another angle does, terminates in the same quadrant as its opposite side.*

This theorem can be proved by using the law of cosines for angles. The proof is left as an exercise.

EXERCISES XVI. A

In the following sets of exercises, A, B, C, are the angles and a, b, c, the sides of spherical triangles.

1. Given $a = 100°$, $b = 95°$, $c = 75°$. State whether the following angles are acute or obtuse: (a) $\frac{1}{2}(A + B)$, (b) $\frac{1}{2}(A + C)$, (c) $\frac{1}{2}(B + C)$.
2. Given $A = 60°$, $B = 100°$, $C = 120°$. State whether the following quantities are acute or obtuse: (a) $\frac{1}{2}(a + b)$, (b) $\frac{1}{2}(a + c)$, (c) $\frac{1}{2}(b + c)$.
3. If $a = 100°$ and $b = 95°$, is A acute or obtuse?
4. Given $a = 100°$, $b = 75°$. Is B acute or obtuse?
5. Given $A = 132°$, $B = 62°$, $C = 42°$. State whether the following sides are acute or obtuse: a, c.
6. Given $A = 76°$, $B = 102°$, $c = 75°$. Which of the following quantities are acute and which obtuse? $\frac{1}{2}(a + b)$, a, $\frac{1}{2}(A + C)$.
7. Given $a = 82°$, $b = 98°$, $c = 99°$. Which of the following angles are acute and which obtuse? $\frac{1}{2}(A + B)$, $\frac{1}{2}(A + C)$, $\frac{1}{2}(B + C)$, A, B, C.

123. Delambre's or Gauss's formulas.

Methods of checking solutions will be given in the model solutions. However, one of the following formulas, known as **Delambre's or Gauss's formulas**, always affords a good check, since each formula involves all six parts of the triangle. The formulas are given without proof.

$$\frac{\sin \frac{1}{2}(a-b)}{\sin \frac{1}{2}c} = \frac{\sin \frac{1}{2}(A-B)}{\cos \frac{1}{2}C}, \tag{1}$$

$$\frac{\sin \frac{1}{2}(a+b)}{\sin \frac{1}{2}c} = \frac{\cos \frac{1}{2}(A-B)}{\sin \frac{1}{2}C}, \tag{2}$$

$$\frac{\cos \frac{1}{2}(a-b)}{\cos \frac{1}{2}c} = \frac{\sin \frac{1}{2}(A+B)}{\cos \frac{1}{2}C}, \tag{3}$$

$$\frac{\cos \frac{1}{2}(a+b)}{\cos \frac{1}{2}c} = \frac{\cos \frac{1}{2}(A+B)}{\sin \frac{1}{2}C}. \tag{4}$$

EXERCISE

Deduce Napier's analogies from the foregoing formulas.

124. Solution of Case I.

When we have the *three sides given*, the solution can be effected by the half-angle formulas and checked by the law of sines.

Example.

Solve the triangle $a = 56° \, 17.2'$, $b = 110° \, 4.7'$, $c = 71° \, 29.3'$.

SOLUTION. $s = \frac{1}{2}(a + b + c)$.

	56° 17.2′
	110° 4.7′
	71° 29.3′
$2s$	237° 51.2′
s	118° 55.6′
$s - a$	62° 38.4′
$s - b$	8° 50.9′
$s - c$	47° 26.3′
CHECK.	118° 55.6′

tan r

$$\log \tan r = \tfrac{1}{2}[\log \sin(s-a) + \log \sin(s-b) + \log \sin(s-c) + \text{colog} \sin s].$$

$$\tan\tfrac{1}{2}A = \frac{\tan r}{\sin(s-a)},$$

$$\log \tan\tfrac{1}{2}A = \log \tan r - \log \sin(s-a),$$

etc.

$\log \sin(s-a)$	$9.94848 - 10$
$\log \sin(s-b)$	$9.18701 - 10$
$\log \sin(s-c)$	$9.86720 - 10$
colog $\sin s$	0.05787
$\log \tan^2 r$	$9.06056 - 10$
$\log \tan r$	$9.53028 - 10$
$\log \tan\frac{1}{2}A$	$9.58180 - 10$
$\log \tan\frac{1}{2}B$	$0.34327 - 10$
$\log \tan\frac{1}{2}C$	$9.66308 - 10$
	$20° 53.7'$
$\frac{1}{2}B$	$65° 35.9'$
$\frac{1}{2}C$	$24° 43.1'$
A	**$41° 47.4'$**
B	**$131° 11.8'$**
C	**$49° 26.2'$**

CHECK. $$\frac{\sin A}{\sin a} \quad \frac{\sin B}{\sin b} \quad \frac{\sin C}{\sin c} = x,$$

$$\log x = \log \sin A - \log \sin a, \text{ etc.}$$

$\log \sin A$	$9.82374 - 10$	$\log \sin B$	$9.87648 - 10$
$\log \sin a$	$9.92004 - 10$	$\log \sin b$	$9.97277 - 10$
$\log x$	$9.90370 - 10$	$\log x$	$9.90371 - 10$

$\log \sin C$	$9.88063 - 10$
$\log \sin c$	$9.97692 - 10$
$\log x$	$9.90371 - 10$

125. Solution of Case II.

When we have the *three angles given* the solution can be effected by the half-side formulas and checked by the law of sines.

The computational setup is the same as for Case I.

EXERCISES XVI. B

Solve the following triangles:

1.	$a = 125^\circ\ 40.2'$,	$b = 53^\circ\ 56.2'$,	$c = 98^\circ\ 51.3'$.
2.	$a = 63^\circ\ 24.4'$,	$b = 74^\circ\ 45.2'$,	$c = 136^\circ\ 42.8'$
3.	$a = 53^\circ\ 42.0'$,	$b = 118^\circ\ 39.5'$,	$c = 130^\circ\ 38.3'$
4.	$a = 158^\circ\ 33.7'$,	$b = 123^\circ\ 13.5'$,	$c = 64^\circ\ 36.9'$.
5.	$a = 84^\circ\ 35.2'$,	$b = 65^\circ\ 34.4'$,	$c = 103^\circ\ 24.2'$
6.	$A = 105^\circ\ 14.1'$,	$B = 55^\circ\ 31.4'$,	$C = 88^\circ\ 51.1'$.
7.	$A = 43^\circ\ 40.4'$,	$B = 136^\circ\ 41.5'$,	$C = 65^\circ\ 16.7'$.
8.	$A = 63^\circ\ 24.4'$,	$B = 74^\circ\ 45.2'$,	$C = 136^\circ\ 42.8'$
9.	$A = 128^\circ\ 17.1'$,	$B = 50^\circ\ 2.5'$,	$C = 114^\circ\ 40.6'$
10.	$A = 81^\circ\ 52.5'$,	$B = 97^\circ\ 31.1'$,	$C = 111^\circ\ 3.7'$.
11.	$a = 51^\circ\ 43.3'$,	$b = 38^\circ\ 2.4'$,	$c = 75^\circ\ 11.5'$.
12.	$a = 146^\circ\ 48.7'$,	$b = 71^\circ\ 28.1'$,	$c = 129^\circ\ 16.3'$
13.	$A = 83^\circ\ 54.0'$,	$B = 102^\circ\ 6.4'$,	$C = 93^\circ\ 2.0'$.
14.	$A = 143^\circ\ 35.0'$,	$B = 104^\circ\ 16.2'$,	$C = 112^\circ\ 15.2'$
15.	$a = 170^\circ\ 30.8'$,	$b = 85^\circ\ 50.4'$,	$c = 108^\circ\ 5.3'$.
16.	$a = 69^\circ\ 8.7'$,	$b = 131^\circ\ 3.9'$,	$c = 141^\circ\ 33.2'$
17.	$A = 128^\circ\ 15.6'$,	$B = 120^\circ\ 28.2'$,	$C = 103^\circ\ 39.8'$
18.	$A = 59^\circ\ 4.4'$,	$B = 94^\circ\ 23.2'$,	$C = 120^\circ\ 4.8'$.
19.	$A = 45^\circ\ 24.6'$,	$B = 71^\circ\ 46.4'$,	$C = 100^\circ\ 3.0'$.
20.	$a = 105^\circ\ 27.3'$,	$b = 83^\circ\ 14.7'$,	$c = 96^\circ\ 53.2'$.

126. Solution of Case III.

In this case we have *two sides and the included angle given*. Suppose, for example, that these are a, b, C. We find $\frac{1}{2}(A + B)$ and $\frac{1}{2}(A - B)$ from Napier's analogies (6) and (5) respectively (section 120). Angles A and B are then readily found. Side c may then be found by either of Napier's analogies (2) or (4). The solution may be checked

by the law of sines. It is desirable to check angles A and B as soon as they have been found, since they are used in finding c.

Example.

Solve the triangle $b = 113° \ 17.3'$, $c = 95° \ 2.5'$, $A = 72° \ 51.6'$.

SOLUTION.

$$\tan \tfrac{1}{2}(B + C) = \frac{\cos \tfrac{1}{2}(b - c)}{\cos \tfrac{1}{2}(b + c)} \cot \tfrac{1}{2}A,$$

$$\tan \tfrac{1}{2}(B - C) = \frac{\sin \tfrac{1}{2}(b - c)}{\sin \tfrac{1}{2}(b + c)} \cot \tfrac{1}{2}A,$$

$$\log \tan \tfrac{1}{2}(B + C) = \log \cos \tfrac{1}{2}(b - c) + \operatorname{colog} \cos \tfrac{1}{2}(b + c) + \log \cot \tfrac{1}{2}A,$$

$$\log \tan \tfrac{1}{2}(B - C) = \log \sin \tfrac{1}{2}(b - c) + \operatorname{colog} \sin \tfrac{1}{2}(b + c) + \log \cot \tfrac{1}{2}A.$$

b	$113° \ 17.3'$
c	$95° \ 2.5'$
A	$72° \ 51.6'$
$b + c$	$208° \ 19.8'$
$b - c$	$18° \ 14.8'$
$\frac{1}{2}(b + c)$	$104° \ 9.9'$
$\frac{1}{2}(b - c)$	$9° \ 7.4'$
$\frac{1}{2}A$	$36° \ 25.8'$
$\log \cos \frac{1}{2}(b - c)$	$9.99447 - 10$
$\operatorname{colog} \cos \frac{1}{2}(b + c)$	0.61134 (neg) *
$\log \cot \frac{1}{2}A$	0.13190
$\log \sin \frac{1}{2}(b - c)$	$9.20020 - 10$
$\operatorname{colog} \sin \frac{1}{2}(b + c)$	0.01341
$\log \tan \frac{1}{2}(B + C)$	0.73771 (neg) *
$\log \tan \frac{1}{2}(B - C)$	$9.34551 - 10$
$\frac{1}{2}(B + C)$	$100° \ 22.0'$
$\frac{1}{2}(B - C)$	$12° \ 29.6'$
B	**$112° \ 51.6'$**
C	**$87° \ 52.4'$**

* The notation (neg) indicates that the corresponding function is negative. Thus, in finding $\frac{1}{2}(B + C)$, we must deduct the value found in the tables

$$\tan \tfrac{1}{2}a = \frac{\sin \frac{1}{2}(B + C)}{\sin \frac{1}{2}(B - C)} \tan \tfrac{1}{2}(b - c),$$

$$\log \tan \tfrac{1}{2}a = \log \sin \tfrac{1}{2}(B + C) + \text{colog} \sin \tfrac{1}{2}(B - C) + \log \tan \tfrac{1}{2}(b - c).$$

$\log \sin \frac{1}{2}(B + C)$	$9.99285 - 10$
$\text{colog} \sin \frac{1}{2}(B - C)$	0.66489
$\log \tan \frac{1}{2}(b - c)$	$9.20572 - 10$
$\log \tan \frac{1}{2} a$	$9.86346 - 10$
$\frac{1}{2}a$	$36°\ 8.3'$
a	**$72°\ 16.6'$**

CHECK. $$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c} = x,$$

$$\log x = \log \sin A - \log \sin a, \text{ etc.}$$

$\log \sin A$	$9.98027 - 10$	$\log \sin B$	$9.96447 - 10$
$\log \sin a$	$9.97888 - 10$	$\log \sin b$	$9.96309 - 10$
$\log x$	0.00139	$\log x$	0.00138

$\log \sin C$	$9.99970 - 10$
$\log \sin c$	$9.99832 - 10$
$\log x$	0.00138

127. Solution of Case IV.

The solution of this case, in which we have *two angles and the included side given,* is very similar to the solution of Case III. Using the appropriate analogies of Napier, we find half the sum and half the difference of the required sides. The sides themselves can then be found immediately. The unknown angle is found by using another of Napier's analogies, and the results may be checked by the law of sines, the two sides being checked as soon as they are found.

from 180°, since $\tan \frac{1}{2}(B + C)$ is negative. That is,

$$\tfrac{1}{2}(B + C) = 180° - 79°\,38.0' = 100°\,22.0'.$$

This could also be determined by Theorem I of section 122.

Example.

Solve the triangle $A = 93° 14.8'$, $C = 71° 23.2'$, $b = 112° 19.8'$.

SOLUTION.

$$\tan \tfrac{1}{2}(a + c) = \frac{\cos \frac{1}{2}(A - C)}{\cos \frac{1}{2}(A + C)} \tan \tfrac{1}{2}b,$$

$$\tan \tfrac{1}{2}(a - c) = \frac{\sin \frac{1}{2}(A - C)}{\sin \frac{1}{2}(A + C)} \tan \tfrac{1}{2}b,$$

$$\log \tan \tfrac{1}{2}(a + c) = \log \cos \tfrac{1}{2}(A - C) + \operatorname{colog} \cos \tfrac{1}{2}(A + C) + \log \tan \tfrac{1}{2}b,$$

$$\log \tan \tfrac{1}{2}(a - c) = \log \sin \tfrac{1}{2}(A - C) + \operatorname{colog} \sin \tfrac{1}{2}(A + C) + \log \tan \tfrac{1}{2}b.$$

	93° 14.8′
	71° 23.2′
	112° 19.8′
$A + C$	164° 38.0′
$A - C$	21° 51.6′
$\frac{1}{2}(A + C)$	82° 19.0′
$\frac{1}{2}(A - C)$	10° 55.8′
$\frac{1}{2}b$	56° 9.9′
$\log \cos \frac{1}{2}(A - C)$	9.99205 − 10
$\operatorname{colog} \cos \frac{1}{2}(A + C)$	0.87388
$\log \tan \frac{1}{2}b$	0.17371
$\log \sin \frac{1}{2}(A - C)$	9.27786 − 10
$\operatorname{colog} \sin \frac{1}{2}(A + C)$	0.00392
$\log \tan \frac{1}{2}(a + c)$	1.03964
$\log \tan \frac{1}{2}(a - c)$	9.45549 − 10
$\frac{1}{2}(a + c)$	84° 47.1′
$\frac{1}{2}(a - c)$	15° 55.8′
	100° 42.9′
	68° 51.3′

$$\cot \tfrac{1}{2}B = \frac{\sin \frac{1}{2}(a + c)}{\sin \frac{1}{2}(a - c)} \tan \tfrac{1}{2}(A - C),$$

$$\log \cot \tfrac{1}{2}B = \log \sin \tfrac{1}{2}(a + c) + \operatorname{colog} \sin \tfrac{1}{2}(a - c) + \log \tan \tfrac{1}{2}(A - C).$$

$\log \sin \frac{1}{2}(a + c)$	$9.99820 - 10$
$\text{colog} \sin \frac{1}{2}(a - c)$	0.56152
$\log \tan \frac{1}{2}(A - C)$	$9.28581 - 10$
$\log \cot \frac{1}{2}B$	$9.84553 - 10$
$\frac{1}{2}B$	$54° \, 58.9'$
B	**$109° \, 57.8'$**

CHECK. $$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c} = x,$$

$$\log x = \log \sin A - \log \sin a, \text{ etc.}$$

$\log \sin A$	$9.99930 - 10$	$\log \sin B$	$9.97309 - 10$
$\log \sin \ a$	$9.99236 - 10$	$\log \sin \ b$	$9.96615 - 10$
$\log x$	0.00694	$\log x$	0.00694

$\log \sin C$	$9.97667 - 10$
$\log \sin \ c$	$9.96972 - 10$
$\log x$	0.00695

EXERCISES XVI. C

Solve the following triangles:

1.	$a = 56° \, 19.7'$,	$b = 20° \, 16.7'$,	$C = 114° \, 20.3'$.
2.	$b = 47° \, 29.3'$,	$c = 50° \, 6.3'$,	$A = 129° \, 58.5'$.
3.	$a = 145° \, 58.2'$,	$b = 62° \, 50.6'$,	$C = 134° \, 52.0'$.
4.	$b = 120° \, 30.5'$,	$c = 70° \, 20.3'$,	$A = 50° \, 10.2'$.
5.	$a = 95° \, 12.9'$,	$b = 53° \, 10.1'$,	$C = 49° \, 11.3'$.
6.	$A = 128° \, 36.8'$,	$B = 106° \, 45.2'$,	$c = 87° \, 40.3'$.
7.	$A = 77° \, 59.6'$,	$B = 40° \, 59.8'$,	$c = 108° \, 0.5'$.
8.	$B = 108° \, 28.9'$,	$C = 38° \, 11.5'$,	$a = 52° \, 29.0'$.
9.	$A = 127° \, 19.6'$,	$C = 108° \, 41.5'$,	$b = 125° \, 22.5'$.
10.	$A = 142° \, 30.8'$,	$B = 68° \, 47.7'$,	$c = 135° \, 34.7'$.
11.	$b = 99° \, 40.8'$,	$c = 100° \, 49.5'$,	$A = 65° \, 33.2'$.
12.	$a = 41° \, 5.1'$,	$b = 44° \, 25.4'$,	$C = 37° \, 29.2'$.
13.	$A = 176° \, 16.6'$,	$C = 3° \, 18.2'$,	$b = 27° \, 1.1'$.
14.	$B = 64° \, 48.9'$,	$C = 40° \, 23.3'$,	$a = 108° \, 39.2'$.
15.	$a = 88° \, 37.7'$,	$b = 125° \, 18.3'$,	$C = 102° \, 16.6'$.
16.	$a = 67° \, 12.6'$,	$c = 135° \, 0.9'$,	$B = 74° \, 45.2'$.
17.	$A = 34° \, 29.5'$,	$B = 36° \, 6.8'$,	$c = 85° \, 59.0'$.

18. $A = 78° 30.8'$, $B = 91° 28.2'$, $c = 51° 22.4'$.
19. $a = 132° 46.7'$, $b = 59° 50.1'$, $C = 56° 28.4'$.
20. $b = 28° 20.3'$, $c = 112° 1.9'$, $A = 79° 28.6'$.

128. Solution of Case V.

Case V, in which we have *two sides and the angle opposite one of them given*, presents the same peculiarities as the corresponding case in plane trigonometry. Suppose that the given parts are a, b, A. Angle B can be determined by the law of sines,

$$\sin B = \frac{\sin b \sin A}{\sin a}. \tag{1}$$

If the ratio on the right of this equation is greater than 1 (in other words, if $\log \sin B > 0$), no solution exists.

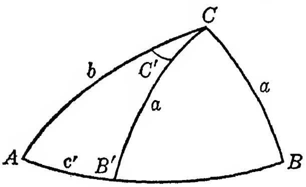


FIG. 107

If this ratio is equal to 1, B is 90° and the resulting right triangle is a unique solution.

If the ratio is less than 1, we find two values for B, the tabular value and its supplement. In this event there may be two solutions (see Fig. 107). The number of solutions may be determined by the principles of section 122.

The remaining angle, and likewise the required side, can be found by using appropriate forms of Napier's analogies.

Checking is perhaps best done by means of one of Delambre's formulas. Suppose, for example, that we rewrite (1) of section 123 in the form

$$\frac{\sin \frac{1}{2}(a - b) \cos \frac{1}{2}C}{\sin \frac{1}{2}(A - B) \sin \frac{1}{2}c} = 1. \tag{2}$$

Then, the logarithm of the left side should be equal to zero (since $\log 1 = 0$) if the work is correct.

Example.

Solve the triangle $a = 100° \, 48.2'$, $b = 70° \, 11.4'$, $B = 71° \, 9.6'$.

SOLUTION.

$$\sin A = \frac{\sin a \sin B}{\sin b},$$

$$\log \sin A = \log \sin a + \log \sin B + \operatorname{colog} \sin b.$$

$$\begin{array}{r|l} a & 100° \, 48.2' \\ b & 70° \, 11.4' \\ B & 71° \quad 9.6' \end{array}$$

$$\begin{array}{rl} \log \sin a & 9.99223 - 10 \\ \log \sin B & 9.97608 - 10 \\ \operatorname{colog} \sin b & 0.02649 \\ \log \sin A & 9.99480 - 10 \\ & \mathbf{81° \, 9.0'}, \; A' = \mathbf{98° \, 51.0'} \end{array}$$

$$\cot \tfrac{1}{2}C = \frac{\sin \frac{1}{2}(a + b)}{\sin \frac{1}{2}(a - b)} \tan \tfrac{1}{2}(A - B),$$

$$\log \cot \tfrac{1}{2}C = \log \sin \tfrac{1}{2}(a + b) + \operatorname{colog} \sin \tfrac{1}{2}(a - b) + \log \tan \tfrac{1}{2}(A - B).$$

$$\begin{array}{rl} a + b & 170° \, 59.6' \\ a - b & 30° \, 36.8' \\ A + B & 152° \, 18.6', \; A' + B = 170° \quad 0.6' \\ A - B & 9° \, 59.4', \; A' - B = 27° \, 41.4' \\ \frac{1}{2}(a + b) & 85° \, 29.8' \\ \frac{1}{2}(a - b) & 15° \, 18.4' \\ \frac{1}{2}(A + B) & 76° \quad 9.3', \; \frac{1}{2}(A' + B) = 85° \quad 0.3' \\ \frac{1}{2}(A - B) & 4° \, 59.7', \; \frac{1}{2}(A' - B) = 13° \, 50.7' \\ \hline \log \tan \frac{1}{2}(A - B) & 8.94151 \quad 10 \\ \log \sin \frac{1}{2}(a + b) & 9.99866 - 10 \\ \operatorname{colog} \sin \frac{1}{2}(a - b) & 0.57842 \\ \log \tan \frac{1}{2}(A' - B) & 9.39174 - 10 \\ \hline \log \cot \frac{1}{2}C & 9.51859 - 10 \\ \log \cot \frac{1}{2}C' & 9.96882 - 10 \\ \frac{1}{2}C & 71° \, 44.0' \\ \frac{1}{2}C' & 47° \quad 3.3' \\ C & \mathbf{143° \, 28.0'} \\ C' & \mathbf{94° \quad 6.6'} \end{array}$$

$$\tan \tfrac{1}{2}c = \frac{\sin \frac{1}{2}(A + B)}{\sin \frac{1}{2}(A - B)} \tan \tfrac{1}{2}(a - b),$$

$$\log \tan \tfrac{1}{2}c = \log \sin \tfrac{1}{2}(A + B) + \text{colog} \sin \tfrac{1}{2}(A - B) + \log\tan \tfrac{1}{2}(A - B).$$

$\log \sin \frac{1}{2}(A + B)$	$9.98720 - 10$
$\text{colog} \sin \frac{1}{2}(A - B)$	1.06014
$\log \tan \frac{1}{2}(a - b)$	$9.43727 - 10$
$\log \sin \frac{1}{2}(A' + B)$	$9.99835 - 10$
$\text{colog} \sin \frac{1}{2}(A' - B)$	0.62106
$\log \tan \frac{1}{2}c$	0.48461
$\log \tan \frac{1}{2}c'$	0.05668
$\frac{1}{2}c$	$71° 51.6'$
$\frac{1}{2}c'$	$48° 43.7'$
c	$143° 43.2'$
c'	$97° 27.4'$

CHECK. 1st solution.

$$\frac{\sin \frac{1}{2}(a - b) \cos \frac{1}{2}C}{\sin \frac{1}{2}(A - B) \sin \frac{1}{2}c} = 1$$

$$\log \sin \tfrac{1}{2}(a - b) + \log \cos \tfrac{1}{2}C + \text{colog} \sin \tfrac{1}{2}(A - B) + \text{colog} \sin \tfrac{1}{2}c = 0.$$

$\log \sin \frac{1}{2}(a - b)$	$9.42158 - 10$
$\log \cos \frac{1}{2}C$	$9.49615 - 10$
$\text{colog} \sin \frac{1}{2}(A - B)$	1.06014
$\text{colog} \sin \frac{1}{2}c$	0.02214
	0.00001

129. Solution of Case VI.

Case VI, *two angles and the side opposite one of them given*, is so similar to Case V that we shall not give a detailed discussion. A model solution, however, will be given.

Example.

Solve the triangle $A = 121° 17.7'$, $B = 29° 7.7'$, $a = 136° 12.0'$.

SOLUTION. $$\sin b = \frac{\sin a \sin B}{\sin A},$$

$$\log \sin b = \log \sin a + \log \sin B + \text{colog} \sin A.$$

A	$121^\circ\ 17.7'$
B	$29^\circ\ 7.7'$
	$136^\circ\ 12.0'$
log sin a	$9.84020 - 10$
log sin B	$9.68732 - 10$
colog sin A	0.06829
log sin b	$9.59581 - 10$
b	**$23^\circ\ 13.3'$**, $b' = 156^\circ\ 46.7'$ *

$$\tan \tfrac{1}{2}c = \frac{\sin \tfrac{1}{2}(A + B)}{\sin \tfrac{1}{2}(A - B)} \tan \tfrac{1}{2}(a - b),$$

$$\log \tan \tfrac{1}{2}c = \log \sin \tfrac{1}{2}(A + B) + \operatorname{colog} \sin \tfrac{1}{2}(A - B) + \log \tan \tfrac{1}{2}(a - b).$$

$A + B$	$150^\circ\ 25.4'$
$A - B$	$92^\circ\ 10.0'$
$a + b$	$159^\circ\ 25.3'$
$a - b$	$112^\circ\ 58.7'$
$\frac{1}{2}(A + B)$	$75^\circ\ 12.7'$
$\frac{1}{2}(A - B)$	$46^\circ\ 5.0'$
$\frac{1}{2}(a + b)$	$79^\circ\ 42.6'$
$\frac{1}{2}(a - b)$	$56^\circ\ 29.4'$
log sin $\frac{1}{2}(A + B)$	$9.98537 -$
colog sin $\frac{1}{2}(A - B)$	0.14246
log tan $\frac{1}{2}(a - b)$	0.17905
log tan $\frac{1}{2}c$	0.30688
$\frac{1}{2}c$	$63^\circ\ 44.5'$
c	**$127^\circ\ 29.0'$**

$$\cot \tfrac{1}{2}C = \frac{\sin \tfrac{1}{2}(a + b)}{\sin \tfrac{1}{2}(a - b)} \tan \tfrac{1}{2}(A - B),$$

$$\log \cot \tfrac{1}{2}C = \log \sin \tfrac{1}{2}(a + b) + \operatorname{colog} \sin \tfrac{1}{2}(a - b) + \log \tan \tfrac{1}{2}(A - B).$$

log sin $\frac{1}{2}(a + b)$	$9.99296 - 10$
colog sin $\frac{1}{2}(a - b)$	0.07894
log tan $\frac{1}{2}(A - B)$	0.01643
log cot $\frac{1}{2}C$	0.08833
$\frac{1}{2}C$	$39^\circ\ 12.8'$
C	**$78^\circ\ 25.6'$**

* Not admissible; for $A > B$, and therefore a must be greater than b.

CHECK. $$\frac{\sin \frac{1}{2}(a - b) \cos \frac{1}{2}C}{\sin \frac{1}{2}(A - B) \sin \frac{1}{2}c} = 1,$$

$$\log \sin \tfrac{1}{2}(a - b) + \log \cos \tfrac{1}{2}C + \operatorname{colog} \sin \tfrac{1}{2}(A - B) + \operatorname{colog} \sin \tfrac{1}{2}c = 0.$$

$\log \sin \frac{1}{2}(a - b)$	$9.92106 - 10$
$\log \cos \frac{1}{2}C$	$9.88919 - 10$
$\operatorname{colog} \sin \frac{1}{2}(A - B)$	0.14246
$\operatorname{colog} \sin \frac{1}{2}c$	0.04730
	0.00001

EXERCISES XVI. D

Solve the following triangles:

1.	$a = 44° 48.3'$,	$b = 17° 36.7'$,	$A = 63° 24.8'$.
2.	$a = 56° 30.0'$,	$b = 31° 20.0'$,	$A = 105° 11.2'$.
3.	$a = 52° 45.3'$,	$b = 71° 12.7'$,	$A = 46° 22.2'$.
4.	$b = 68° 52.8'$,	$c = 56° 49.8'$,	$C = 45° 15.2'$.
5.	$a = 30° 38.1'$,	$c = 31° 29.8'$,	$A = 87° 53.3'$.
6.	$A = 109° 20.2'$,	$B = 134° 16.4'$,	$a = 148° 48.7'$.
7.	$A = 143° 17.4'$,	$B = 70° 18.4'$,	$a = 160° 40.6'$.
8.	$A = 61° 37.9'$,	$B = 139° 54.6'$,	$b = 150° 17.4'$.
9.	$A = 70° 15.2'$,	$B = 119° 43.8'$,	$b = 80° 24.4'$.
10.	$B = 24° 30.5'$,	$C = 61° 29.5'$,	$c = 34° 0.5'$.
11.	$a = 80° 5.3'$,	$b = 82° 4.0'$,	$A = 83° 34.2'$.
12.	$a = 134° 15.9'$,	$b = 150° 57.1'$,	$B = 144° 22.7'$.
13.	$A = 79° 37.3'$,	$C = 145° 52.2'$,	$c = 150° 42.7'$.
14.	$A = 60° 20.2'$,	$B = 17° 12.9'$,	$b = 43° 50.5'$.
15.	$a = 148° 34.4'$,	$b = 142° 11.6'$,	$A = 153° 17.6'$.
16.	$a = 40° 20.4'$,	$b = 20° 18.2'$,	$A = 60° 44.4'$.
17.	$A = 117° 54.4'$,	$B = 45° 8.6'$,	$a = 76° 37.5'$.
18.	$b = 119° 19.9'$,	$c = 160° 2.3'$,	$C = 139° 9.1'$.
19.	$A = 104° 40.0'$,	$B = 80° 13.6'$,	$a = 126° 50.4'$.
20.	$a = 40° 5.4'$,	$b = 118° 22.1'$,	$A = 29° 42.6'$.

130. Summary of methods.

The methods of solving oblique spherical triangles are epitomized below.

Case I. Three sides given.	Use **half-angle formulas.** Check by law of sines.
Case II. Three angles given.	Use **half-side formulas.** Check by law of sines.
Case III. Two sides and the included angle given.	Find half the sum and half the difference of the required angles by using appropriate forms of **Napier's analogies.** The required angles are then readily found. Find required side by another of **Napier's analogies.** Check by law of sines.
Case IV. Two angles and the included side given.	Find half the sum and half the difference of the required sides by using appropriate forms of **Napier's analogies.** The required sides are then readily found. Find required angle by another of **Napier's analogies.** Check by law of sines.
Case V. Two sides and the angle opposite one of them given.	Use **law of sines** to find an angle. Find remaining angle and required side by appropriate forms of **Napier's analogies.** Note number of solutions. Check by one of Delambre's formulas.
Case VI. Two angles and the side opposite one of them given.	Use **law of sines** to find a side. Find remaining side and required angle by appropriate forms of **Napier's analogies.** Note number of solutions. Check by one of Delambre's formulas.

MISCELLANEOUS EXERCISES XVI. E

Solve the following triangles:

1. $a = 18° 29.3'$, $b = 30° 37.1'$, $C = 52° 51.8'$.

2. $a = 114° 43.3'$, $b = 136° 19.6'$, $c = 43° 18.5'$.

3. $A = 33° 15.1'$, $B = 31° 34.6'$, $C = 161° 25.3'$.
4. $A = 80° 2.3'$, $a = 118° 20.3'$, $b = 69° 56.3'$.
5. $B = 140° 43.2'$, $C = 100° 4.6'$, $a = 60° 43.6'$.
6. $a = 76° 40.4'$, $b = 54° 21.3'$, $c = 36° 8.7'$.
7. $a = 148° 34.4'$, $b = 142° 11.6'$, $A = 153° 17.6'$.
8. $A = 40° 20.4'$, $a = 60° 44.4'$, $b = 20° 18.2'$.
9. $a = 103° 44.7'$, $b = 64° 12.3'$, $C = 98° 33.8'$.
10. $A = 30° 51.2'$, $B = 71° 36.0'$, $C = 90°$.
11. $A = 100° 51.3'$, $B = 80° 47.6'$, $C = 74° 3.3'$.
12. $A = 150° 47.0'$, $C = 98° 22.7'$, $c = 90°$.
13. $A = 64° 34.3'$, $B = 119° 54.6'$, $C = 63° 20.2'$.
14. $A = 104° 30.7'$, $B = 62° 52.1'$, $c = 56° 6.4'$.
15. $A = 117° 54.4'$, $B = 45° 8.6'$, $a = 76° 37.5'$.
16. $C = 50° 10.2'$, $b = 69° 34.9'$, $c = 120° 30.5'$.
17. $C = 50° 10.2'$, $b = 120° 30.5'$, $c = 69° 34.9'$.
18. $A = 92° 47.4'$, $B = 73° 1.3'$, $c = 26° 6.9'$.
19. $a = 80° 39.1'$, $b = 75° 12.3'$, $c = 141° 5.6'$.
20. $A = 61° 37.9'$, $C = 139° 54.6'$, $c = 150° 17.4'$.
21. $A = 53° 15.5'$, $C = 68° 58.5'$, $b = 67° 12.6'$.
22. $A = 99° 34.1'$, $B = 67° 46.7'$, $C = 91° 56.8'$.
23. $a = 41° 19.3'$, $b = 112° 36.2'$, $c = 78° 9.6'$.
24. $a = 58° 49.6'$, $b = 75° 12.1'$, $C = 102° 58.0'$.
25. $A = 104° 30.7'$, $B = 62° 52.1'$, $c = 56° 6.4'$.
26. $A = 32° 40.2'$, $B = 122° 11.1'$, $C = 42° 36.2'$.
27. $A = 104° 40.0'$, $B = 80° 13.6'$, $a = 126° 50.4'$.
28. $A = 65° 33.2'$, $b = 99° 40.8'$, $c = 100° 49.5'$.
29. $A = 113° 30.0'$, $B = 125° 31.6'$, $a = 66° 44.7'$.
30. $B = 10° 10.2'$, $C = 90°$, $b = 10° 10.2'$.
31. Find the perimeter and the area of the spherical triangle in which $A = 65° 50'$, $b = 63° 17'$, $c = 107° 23'$, the radius of the sphere being 5 inches.
32. A triangle whose sides are 100°, 50°, and 60° lies on a sphere of radius 10 inches. Find the difference between the area of this triangle and that of an equilateral triangle having the same perimeter.
33. A triangle whose angles are 100°, 50°, and 60° lies on a sphere of radius 10 inches. Find the difference between the perimeter of this triangle and that of an equiangular triangle having the same area.

CHAPTER XVII

Applications of Spherical Trigonometry

131. Terrestrial sphere.

In long distance measurements on the surface of the earth, and in navigation, the earth is treated as a sphere having a radius of 3959 miles. This is called the **terrestrial sphere.**

It rotates about a diameter, called its **axis,** which pierces the sphere in the **north pole** P and the **south pole** P'. (See Fig. 108.)

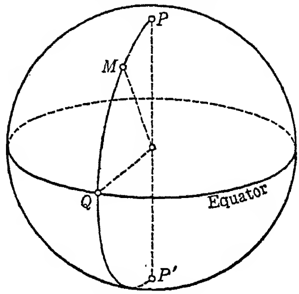


FIG. 108

The **equator** is the great circle whose plane is perpendicular to the axis.

A **meridian** is a great circle passing through the poles, for example, PMQ.

The **latitude** of a point M is the angular distance of the point from the equator, and will be considered positive if the point is north of the equator, negative if the point is south of the equator. It is measured by the arc QM of the meridian through the point. The **colatitude** is 90° minus the latitude.* It is the angular distance from the north pole and is measured by the arc MP.

The meridian through Greenwich is called the **prime meridian.** The **longitude** of a point is the angle between the prime meridian and the meridian through the point. It is measured by the number of degrees in the arc intercepted

* If the point is south of the equator, say 30° south, its latitude is $-30°$ and its colatitude is $90° - (-30°) = 120°$.

at the equator by these two meridians.* If for example, in Fig. 109, PGG' is the prime meridian and PAA' is the meridian through the point A, these meridians cutting the equator in G' and A' respectively, then the longitude of A is measured by the number of degrees in the arc $G'A'$. Longitude will be considered positive if the point is west of the prime meridian and negative if the point is east.

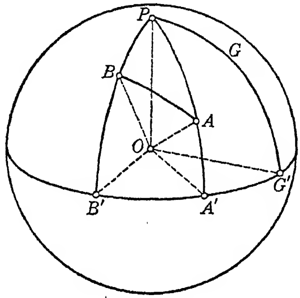


Fig. 109

The **distance** between two points A and B is the length of the arc AB (not greater than a semicircumference) of a great circle passing through A and B. This distance may be expressed in angular measure or in linear measure. To convert from angular units to linear units, we note that a **nautical mile** is the length of one minute of arc of a great circle on the terrestrial sphere. This is about 1.1516 **statute miles** of 5280 feet each, or 6080 feet.†

The **bearing** of point B from point A is the angle which the arc AB makes with the meridian through A (angle PAB in Fig. 109).‡

132. Terrestrial triangle.

To find the distance between A and B, and their bearings from each other, we consider the **terrestrial triangle** ABP, whose vertices are the two points and the north pole. If the latitude and longitude of the points are given, we can find arcs AP and BP, also angle APB, immedi-

* It is also frequently expressed in hours, minutes, and seconds of time (cf. section 133), 1 hour being equivalent to 1/24 of 360°, or 15° of arc, 1 minute of time consequently being equivalent to 15 minutes of arc, and 1 second of time to 15 seconds of arc.

† The United States nautical mile is 6080.27 feet, the British nautical mile is 6080 feet.

‡ In the United States Navy bearings are measured from 0° to 360°, from north through east. According to this convention, the bearing of B from A in Fig. 109 would be found by subtracting angle PAB from 360°.

ately, so that we have a problem under Case III, namely, two sides and the included angle given.

Example.

Find the distance between New York (40° 43′ N, 74° 0′ W) and Liverpool (53° 24′ N, 3° 4′ W) and the bearing of each of these places from the other.

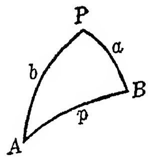


FIG. 110

SOLUTION. Represent New York by A and Liverpool by B (Fig. 110). Then,

$$b = AP = \text{colatitude } A = 90° - 40° 43' = 49° 17',$$

$$a = BP = \text{colatitude } B = 90° - 53° 24' = 36° 36',$$

$$P = \text{difference in longitude} = 74° 0' - 3° 4' = 70° 56'.$$

$$\tan \tfrac{1}{2}(B + A) = \frac{\cos \frac{1}{2}(b - a)}{\cos \frac{1}{2}(b + a)} \cot \tfrac{1}{2}P,$$

$$\tan \tfrac{1}{2}(B - A) = \frac{\sin \frac{1}{2}(b - a)}{\sin \frac{1}{2}(b + a)} \cot \tfrac{1}{2}P,$$

$$\log \tan \tfrac{1}{2}(B + A) = \log \cos \tfrac{1}{2}(b - a) + \text{colog} \cos \tfrac{1}{2}(b + a) + \log \cot \tfrac{1}{2}P,$$

$$\log \tan \tfrac{1}{2}(B - A) = \log \sin \tfrac{1}{2}(b - a) + \text{colog} \sin \tfrac{1}{2}(b + a) = \log \cot \tfrac{1}{2}P.$$

$b + a$	85° 53′
$b - a$	12° 41′
$\frac{1}{2}(b + a)$	42° 56.5′
$\frac{1}{2}(b - a)$	6° 20.5′
$\frac{1}{2}P$	35° 28′
$\log \cos \frac{1}{2}(b - a)$	9.99734 − 10
$\text{colog} \cos \frac{1}{2}(b + a)$	0.13546
$\log \cot \frac{1}{2}P$	0.14727
$\log \sin \frac{1}{2}(b - a)$	9.04319 − 10
$\text{colog} \sin \frac{1}{2}(b + a)$	0.16669
$\log \tan \frac{1}{2}(B + A)$	0.28007
$\log \tan \frac{1}{2}(B - A)$	9.35715 − 10
$\frac{1}{2}(B + A)$	62° 19′
$\frac{1}{2}(B - A)$	12° 49′
B	75° 8′
A	49° 30′

$$\tan \tfrac{1}{2}p = \frac{\sin \frac{1}{2}(B + A)}{\sin \frac{1}{2}(B - A)} \tan \tfrac{1}{2}(b - a).$$

$$\log \tan \tfrac{1}{2}p = \log \sin \tfrac{1}{2}(B + A) + \operatorname{colog} \sin \tfrac{1}{2}(B - A) + \log \tan \tfrac{1}{2}(b - a).$$

$$\begin{array}{rl}
\log \sin \frac{1}{2}(B + A) & 9.94720 - 10 \\
\operatorname{colog} \sin \frac{1}{2}(B - A) & 0.65398 \\
\log \tan \frac{1}{2}(b - a) & 9.04586 - 10 \\
\hline
\log \tan \frac{1}{2}p & 9.64704 - 10 \\
 & 23^\circ\ 55' \\
p & 47^\circ\ 50' = 2870'
\end{array}$$

Distance = 2870 nautical miles.
Bearing of Liverpool from New York = $A = N\ 49^\circ\ 30'$ E.
Bearing of New York from Liverpool = $B = N\ 75^\circ\ 8'\ W$.

The solution should be checked by the law of sines.

EXERCISES XVII. A

Find the distances between the following places, also the bearing of each from the other. Latitudes and longitudes are given at the end of the set of exercises.

1. New York and San Francisco.
2. New York and Paris.
3. New York and Cape of Good Hope.
4. San Francisco and Sydney.
5. San Francisco and Rio de Janeiro.
6. New York and Rio de Janeiro.
7. Rio de Janeiro and Sydney.
8. Moscow and San Francisco.
9. How close to the north pole does the great circle path of the preceding exercise pass?
10. A ship sailed due east from New York to a point on the meridian of 10° W near Portugal. Find the distance it would have saved if it had sailed along the arc of a great circle.
11. A ship sails from New York to Cape of Good Hope along the arc of a great circle. Find its course (i.e., direction) (a) when it crosses the equator, (b) when it crosses the meridian of 10° W. (Use results of exercise 3.)
12. Find the area of the triangle whose vertices are New York,

San Francisco, and Rio de Janeiro. (Use results of exercises 1, 5, 6.)

13. An airplane flies from New York to Chicago in 3 hours and 45 minutes. What is its average rate of speed in statute miles per hour?

14. An airplane flew from Chicago to San Francisco at an average speed of 180 statute miles per hour. How long did the flight take?

	Latitude	Longitude
Cape of Good Hope	34° 21′ S	18° 30′ E
Chicago	41° 50′ N	87° 37′ W
Moscow	55° 45′ N	37° 34′ E
New York	40° 43′ N	74° 0′ W
Paris	48° 50′ N	2° 20′ E
Rio de Janeiro	22° 54′ S	43° 10′ W
San Francisco	37° 47′ N	122° 26′ W
Sydney	33° 52′ S	151° 12′ E

133. Celestial sphere.

A sphere, concentric with the earth, and having a radius of indefinite length, is called the **celestial sphere.** (See

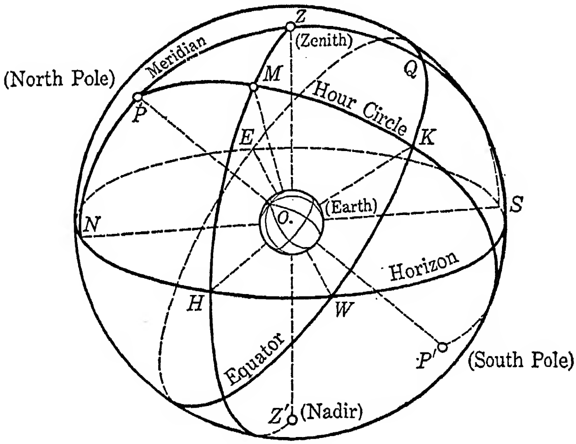


Fig. 111

Fig. 111, in which the earth is located at the point O.) With any point on this sphere is associated a direction, and thus

the angular distance (although not a linear distance) between any two points on it may be considered.

The points where the axis of the earth intersects the celestial sphere are the **north** and **south celestial poles**, P and P', respectively.

The plane of the equator of the earth cuts the celestial sphere in the **celestial equator,** EQW.

Great circles, such as PMP', passing through the celestial poles are called **hour circles.** The hour circle of the observer, the great circle $NPZQS$ in the figure, is called the observer's **celestial meridian.**

The point Z on the celestial sphere vertically above the observer is called the **zenith** of the observer. The diametrically opposite point, Z', is called the **nadir.**

The **horizon** of the observer is the great circle $NESW$ having the zenith and nadir as poles. On the horizon the cardinal points (north, south, east, west) are marked by the respective initial letters.

The **declination** of a star or other heavenly body, whose projection on the celestial sphere is represented by M in the figure, is its angular distance north or south of the celestial equator. It is regarded as positive if the body is north of the equator, negative if the body is south. The declination of the body M in Fig. 111 is measured by the arc KM of the hour circle of the body. Declination corresponds to latitude on the earth.

The **hour angle** of the body M is the angle at the pole between the celestial meridian (i.e., the hour circle of the observer) and the hour circle through the body. It is the angle ZPM in the figure, and may be measured by the arc QK of the celestial equator. It is usually measured from the celestial meridian, toward the west, from 0° to 360° or from 0 to 24 hours. Since the celestial sphere apparently rotates through 360° in 24 hours, 1 hour corresponds to $\frac{1}{24} \times 360° = 15°$, and we have the following relations between measures of time and angular measure:

$$1 \text{ hour} = 15 \text{ degrees} \qquad (1^h = 15°),$$
$$1 \text{ minute of time} = 15 \text{ minutes of arc } (1^m = 15'),$$
$$1 \text{ second of time} = 15 \text{ seconds of arc } (1^s = 15'').$$

The **altitude** of the body M is its distance above the horizon, and is measured by the arc HM.* The altitude is taken as positive if the body is above the horizon, negative if it is below.

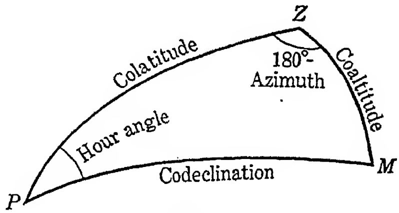


FIG. 112

The **azimuth** of the body is the angle at the zenith between the celestial meridian $PZQS$ and the great circle $ZMHZ'$ through the zenith and the body. It may be measured from north or from south. If, for example, it is measured from the south, the azimuth of M in Fig. 111 is the angle SZM.

A heavenly body may be located by its declination and its hour angle, or by its altitude and azimuth.

134. Astronomical triangle.

The spherical triangle PZM whose vertices are the celestial pole,† the zenith, and the projection of a heavenly body on the celestial sphere, is called the **astronomical triangle.**

A study of Fig. 111 shows that

$$ZM = \textbf{coaltitude}, \tag{1}$$
$$MP = \textbf{codeclination}, \tag{2}$$
$$PZ = \textbf{colatitude}, \tag{3}$$

where the prefix "co" obviously denotes "complement of." Moreover,

$$P = \textbf{hour angle}, \tag{4}$$
$$Z = 180° - \textbf{azimuth}. \tag{5}$$

The angle M is of no special interest.

* It can easily be shown that the altitude of the north celestial pole, at any place of observation, is the latitude of the place.

† The north pole if the observer is in the northern hemisphere, the south pole if he is in the southern hemisphere.

If any three of the other five parts are known, the remaining two can be found. Thus, if an observer knows his latitude, and measures the altitude and azimuth of the sun, he can find PZ, ZM, and Z. From these he can compute the hour angle P. This would give the **local apparent time** (shown on a sundial).

From the American Nautical Almanac or the American Air Almanac (these are published by the United States Naval Observatory) can be obtained the declination of each of many heavenly bodies (sun, moon, planets, and several hundred stars) for any hour of the day. If an observer knows the time and measures the altitude of the sun, he has, after finding the declination of the heavenly body M from the Almanac, the values of ZM, MP, and P, from which he can compute PZ and hence his latitude.

Example 1.

An observation taken in St. Louis (latitude 38° 38′ N) showed the altitude of the sun to be 30° 30′. Its declination was found to be 10° 20′ N. What was the time of day?

Solution. In the astronomical triangle we have

$$\begin{aligned} m &= \text{colat.} = 90° - 38°\,38' = 51°\,22', \\ p &= \text{coalt.} = 90° - 30°\,30' = 59°\,30', \\ z &= \text{codec.} = 90° - 10°\,20' = 79°\,40'. \end{aligned}$$

This is Case I. Since only one angle is required, we use formula (9) of section 118 (page 216).

$$s = \tfrac{1}{2}(m + p + z).$$

$$\tan \tfrac{1}{2}P = \sqrt{\frac{\sin(s - m)\sin(s - z)}{\sin s \sin(s - p)}}$$

$$\log \tan \tfrac{1}{2}P$$
$$= \tfrac{1}{2}[\log\sin(s-m)+\log\sin(s-z)+\operatorname{colog}\sin s+\operatorname{colog}\sin(s-p)].$$

$2s$	$190° 32'$
s	$95° 16'$
$s - m$	$43° 54'$
$s - p$	$35° 46'$
$s - z$	$15° 36'$
CHECK. s	$95° 16'$
$\log \sin(s - m)$	$9.84098 - 10$
$\log \sin(s - z)$	$9.42962 - 10$
$\operatorname{colog} \sin s$	0.00184
$\operatorname{colog} \sin(s - p)$	0.23323
$\log \tan^2 \frac{1}{2}P$	$9.50567 - 10$
$\log \tan \frac{1}{2}P$	$9.75284 - 10$
$\frac{1}{2}P$	$29° 30.7'$
P	$59° \ 1'$

Reducing the hour angle P to units of time (see section 133), we get $P = 59° 1' \div 15 = 3^h 56^m$. If the observation was taken in the afternoon, the time was 3:56 p.m. If the observation was taken in the morning, the time was $12^h - 3^h 56^m = 8^h 4^m$, or 8:04 a.m. In either case the time is local apparent time.

Example 2.

The declination of a star is 7° 54′ N, its hour angle is 48° 51′. Find its azimuth, it being given that the observer is in latitude 67° 49′ N.

SOLUTION. In the astronomical triangle we have

$$\begin{aligned} z &= \text{codec.} = 90° - 7° 54' = 82° 6', \\ P &= \text{hr. } \angle = 48° 51', \\ m &= \text{colat.} = 90° - 67° 49' = 22° 11'. \end{aligned}$$

This is Case III.

$$\tan \tfrac{1}{2}(Z + M) = \frac{\cos \frac{1}{2}(z - m)}{\cos \frac{1}{2}(z + m)} \cot \tfrac{1}{2}P,$$

$$\tan \tfrac{1}{2}(Z - M) = \frac{\sin \frac{1}{2}(z - m)}{\sin \frac{1}{2}(z + m)} \cot \tfrac{1}{2}P,$$

$$\begin{aligned} \log \tan \tfrac{1}{2}(Z + M) = {} & \log \cos \tfrac{1}{2}(z - m) \\ & + \operatorname{colog} \cos \tfrac{1}{2}(z + m) + \log \cot \tfrac{1}{2}P, \end{aligned}$$

$$\log \tan \tfrac{1}{2}(Z - M) = \log \sin \tfrac{1}{2}(z - m) + \operatorname{colog} \sin \tfrac{1}{2}(z + m) + \log \cot \tfrac{1}{2}P.$$

$z + m$	$104° 17'$
$z - m$	$59° 55'$
$\frac{1}{2}(z + m)$	$52° \ 8.5'$
$\frac{1}{2}(z - m)$	$29° 57.5'$
$\frac{1}{2}P$	$24° 25.5'$
$\log \cos \frac{1}{2}(z - m)$	$9.93772 - 10$
$\operatorname{colog} \cos \frac{1}{2}(z + m)$	0.21204
$\log \cot \frac{1}{2}P$	0.34280
$\log \sin \frac{1}{2}(z - m)$	$9.69842 - 10$
$\operatorname{colog} \sin \frac{1}{2}(z + m)$	0.10263
$\log \tan \frac{1}{2}(Z + M)$	0.49256
$\log \tan \frac{1}{2}(Z - M)$	0.14385
$\frac{1}{2}(Z + M)$	$72° 10.0'$
$\frac{1}{2}(Z - M)$	$54° 19.2'$
Z	$126° 29.2'$
M	$17° 50.8'$

$$\text{Azimuth} = 180° - Z = 53° 31'.$$

CHECK.
$$\frac{\sin Z}{\sin z} \quad \frac{\sin M}{\sin m} = x,$$

$$\log x = \log \sin Z - \log \sin z = \log \sin M - \log \sin m.$$

$\log \sin Z$	$9.90525 - 10$
$\log \sin z$	$9.99586 - 10$
$\log x$	$9.90939 - 10$
$\log \sin M$	$9.48639 - 10$
$\log \sin m$	$9.57700 - 10$
$\log x$	$9.90939 - 10$

Example 3.

An observer in the northern hemisphere finds the altitude of the sun to be 35° 23′ at 9:15 a.m., local apparent time. If the declination of the sun is 10° 48′ S, what is the latitude of the place of observation?

SOLUTION. In the astronomical triangle we have

$$z = MP = \text{codec.} = 90° + 10°\,48' = 100°\,48',$$
$$p = ZM = \text{coalt.} = 90° - 35°\,23' = 54°\,37',$$
$$P = \text{hr.} \angle = 12^h - 9^h\,15^m = 2^h\,45^m = 41°\,15'.$$

This is Case V.

$$\sin Z = \frac{\sin z \sin P}{\sin p}.$$

$$\log \sin Z = \log \sin z + \log \sin P + \text{colog} \sin p.$$

$\log \sin z$	$9.99224 - 10$
$\log \sin P$	$9.81911 - 10$
$\text{colog} \sin p$	0.08868
$\log \sin Z$	$9.90003 - 10$
	$52°\,36'$ * or $127°\,24'$

$$\tan \tfrac{1}{2}m = \frac{\sin \frac{1}{2}(Z + P)}{\sin \frac{1}{2}(Z - P)} \tan \tfrac{1}{2}(z - p),$$

$$\log \tan \tfrac{1}{2}m = \log \sin \tfrac{1}{2}(Z + P) + \text{colog} \sin \tfrac{1}{2}(Z - P) + \log \tan \tfrac{1}{2}(z - p).$$

$Z + P$	$168°\,39'$
$Z - P$	$86°\;\;9'$
$z - p$	$46°\,11'$
$\frac{1}{2}(Z + P)$	$84°\,19.5'$
$\frac{1}{2}(Z - P)$	$43°\;\;4.5'$
$\frac{1}{2}(z - p)$	$23°\;\;5.5'$
$\log \sin \frac{1}{2}(Z + P)$	$9.99786 - 10$
$\text{colog} \sin \frac{1}{2}(Z - P)$	0.16561
$\log \tan \frac{1}{2}(z - p)$	$9.62978 - 10$
$\log \tan \frac{1}{2}m$	$9.79325 - 10$
$\frac{1}{2}m$	$31°\,51'$
m	$63°\,42'$

Since m = colat., lat. $= 90° - 63°\,42' = 26°\,18'$ N.

EXERCISES XVII. B

1. An observation taken in New York (40° 43′ N) showed the altitude of the sun to be 52° 25′. Its declination was found

* Discarded, since Z and z must terminate in the same quadrant.

to be 12° 15′. What was the local apparent time of the observation if it was taken in the morning?

2. An afternoon observation at Montreal (45° 30′ N) determined the altitude of the sun to be 26° 30′. Given that the declination of the sun was 8° 0′ S, find the local apparent time of the observation.

3. Find the altitude and the azimuth of the sun at 3 p.m. in latitude 47° 38′ N, its declination being 7° 18′.

4. The declination of a star is 22° 1′, its hour angle is 15° 8′. The latitude of the place of observation is 51° 19′ N. Find the altitude and the azimuth of the star.

5. The declination of a star is −26° 19′, its altitude is 31° 5′, and its azimuth is S 18° 9′ W. Find the latitude of the observer.

6. The altitude of the sun is 50° 32′, its declination is 12° 38′, its azimuth S 12° 6′ W. Find the latitude and the local apparent time.

7. Find the local apparent time of sunset in Chicago (41° 50′ N) on a day when the declination of the sun is −7° 30′.

   SUGGESTION. At sunset the altitude of the sun is 0°.

   NOTE. In practice a correction must be made in problems of this type for the refraction of the rays of the sun by the atmosphere of the earth. Another correction must be made for the angular radius of the sun.

8. Find the length of the day (sunrise to sunset) in New Orleans (29° 57′ N) when the declination of the sun is −20°.

9. On the longest day of the year the declination of the sun is 23° 27′. Find the length of the longest day in latitude (a) 25°, (b) 45°, (c) 65°.

10. On the shortest day of the year the declination of the sun is −23° 27′. Find the length of the shortest day in latitude (a) 25°, (b) 45°, (c) 65°.

IMPORTANT FORMULAS

INDEX

ANSWERS

Important Formulas

$$\sin\theta\csc\theta = 1 \qquad \sin^2\theta + \cos^2\theta = 1$$
$$\cos\theta\sec\theta = 1 \qquad 1 + \tan^2\theta = \sec^2\theta$$
$$\tan\theta\cot\theta = 1 \qquad 1 + \cot^2\theta = \csc^2\theta$$
$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$
$$\sin(\theta + \phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$$
$$\sin(\theta - \phi) = \sin\theta\cos\phi - \cos\theta\sin\phi$$
$$\cos(\theta + \phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$$
$$\cos(\theta - \phi) = \cos\theta\cos\phi + \sin\theta\sin\phi$$
$$\tan(\theta + \phi) = \frac{\tan\theta + \tan\phi}{1 - \tan\theta\tan\phi}$$
$$\cot(\theta + \phi) = \frac{\cot\theta\cot\phi - 1}{\cot\phi + \cot\theta}$$
$$\tan(\theta - \phi) = \frac{\tan\theta - \tan\phi}{1 + \tan\theta\tan\phi}$$
$$\cot(\theta - \phi) = \frac{\cot\theta\cot\phi + 1}{\cot\phi - \cot\theta}$$
$$\sin 2\theta = 2\sin\theta\cos\theta \qquad \cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta} \qquad \cot 2\theta = \frac{\cot^2\theta - 1}{2\cot\theta}$$
$$\sin\tfrac{1}{2}\theta = \pm\sqrt{1 - \cos\theta} \qquad \cos\tfrac{1}{2}\theta = \pm\sqrt{\frac{1 + \cos\theta}{2}}$$
$$\tan\tfrac{1}{2}\theta = \pm\sqrt{\frac{1 - \cos}{1 + \cos\theta}} = \frac{1 - \cos}{\sin\theta} = \frac{\sin\theta}{1 + \cos\theta}$$
$$\sin\theta + \sin\phi = 2\sin\tfrac{1}{2}(\theta + \phi)\cos\tfrac{1}{2}(\theta - \phi)$$
$$\sin\theta - \sin\phi = 2\cos\tfrac{1}{2}(\theta + \phi)\sin\tfrac{1}{2}(\theta - \phi)$$

$$\cos\theta + \cos\phi = 2\cos\tfrac{1}{2}(\theta+\phi)\cos\tfrac{1}{2}(\theta-\phi)$$
$$\cos\theta - \cos\phi = -2\sin\tfrac{1}{2}(\theta+\phi)\sin\tfrac{1}{2}(\theta-\phi)$$

Plane triangles.

Law of sines: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Law of cosines: $a^2 = b^2 + c^2 - 2bc\cos A$ *

Law of tangents: $\dfrac{a-b}{a+b} = \dfrac{\tan\frac{1}{2}(A-B)}{\tan\frac{1}{2}(A+B)}$ *

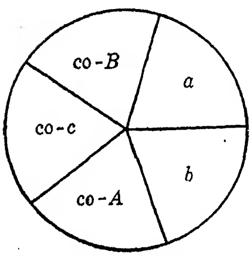
$\tan\frac{1}{2}A =$ a' $\qquad s = (a+b+c),$

$$= \sqrt{\frac{(s-a)(s-b)(s-c)}{s}}$$

Spherical triangles.

Napier's rules (for right triangles):

$$\sin(\text{mid. part}) = \tan(\text{adj. part})\tan(\text{adj. part})$$
$$= \cos(\text{opp. part})\cos(\text{opp. part})$$



Law of sines: $\dfrac{\sin a}{\sin A} = \dfrac{\sin b}{\sin B} = \dfrac{\sin c}{\sin C}$

Law of cosines for sides:

$\cos a = \cos b\cos c + \sin b\sin c\cos A$ *

Law of cosines for angles:

$\cos A = -\cos B\cos C + \sin B\sin C\cos a$ *

Napier's analogies:

$$\frac{\sin\frac{1}{2}(A-B)}{\sin\frac{1}{2}(A+B)} = \frac{\tan\frac{1}{2}(a-b)}{\tan\frac{1}{2}c} \text{ *}$$

$$\frac{\cos\frac{1}{2}(A-B)}{\cos\frac{1}{2}(A+B)} = \frac{\tan\frac{1}{2}(a+b)}{\tan\frac{1}{2}c} \text{ *}$$

* Two other formulas may be obtained by changing the letters.

$$\frac{\sin \frac{1}{2}(a-b)}{\sin \frac{1}{2}(a+b)} = \frac{\tan \frac{1}{2}(A-B)}{\cot \frac{1}{2}C} *$$

$$\frac{\cos \frac{1}{2}(a-b)}{\cos \frac{1}{2}(a+b)} = \frac{\tan \frac{1}{2}(A+B)}{\cot \frac{1}{2}C} *$$

$$\tan \tfrac{1}{2}A = \frac{\tan r}{\sin(s-a)},* \qquad s = \tfrac{1}{2}(a+b+c),$$

$$\tan r = \sqrt{\frac{\sin(s-a)\sin(s-b)\sin(s-c)}{\sin s}}$$

$$\tan \tfrac{1}{2}a = \tan R \cos(S-A),* \qquad S = \tfrac{1}{2}(A+B+C),$$

$$\tan R \quad \sqrt{\frac{-\cos S}{\cos(S-A)\cos(S-B)\cos(S-C)}}$$

* Two other formulas may be obtained by changing the letters.

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Answers to Odd-Numbered Exercises

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	sin A cos B	cos A sin B	tan A cot B	csc A sec B	sec A csc B	cot A tan B
		$\frac{3}{5}$	$\frac{4}{3}$	$\frac{5}{4}$		
3.	$\frac{2\sqrt{13}}{13}$	$\frac{3\sqrt{13}}{13}$	$\frac{2}{3}$	$\frac{\sqrt{13}}{2}$	$\frac{\sqrt{13}}{3}$	$\frac{3}{2}$
	$\frac{2}{3}$	$\frac{\sqrt{5}}{3}$	$\frac{2\sqrt{5}}{5}$	$\frac{3}{2}$	$\frac{3\sqrt{5}}{5}$	$\frac{\sqrt{5}}{2}$
7.	$\frac{8}{17}$	$\frac{15}{17}$	$\frac{8}{15}$	$\frac{17}{8}$	$\frac{17}{15}$	
9.	$\frac{7}{25}$	$\frac{24}{25}$	$\frac{7}{24}$	$\frac{25}{7}$	$\frac{25}{24}$	
11.	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
13.	$\frac{3\sqrt{10}}{10}$	$\frac{\sqrt{10}}{10}$	[illegible]	$\sqrt{10}$ / [illegible]	$\sqrt{10}$	$\frac{1}{3}$

15. $\frac{13}{15}$. **17.** $\frac{1}{48}$. **19.** (a) $\frac{3}{4}$, $\frac{\sqrt{7}}{4}$, $3\sqrt{7}$ / [illegible]; (b) $\sqrt{7}$ / [illegible], $\frac{3}{4}$, $\frac{\sqrt{7}}{3}$.

Exercises I. C, page 8

	sin A	cos A	tan A	csc A	sec A	cot A
	$\frac{3}{5}$		$\frac{3}{4}$	$\frac{5}{3}$	$\frac{5}{4}$	$\frac{4}{3}$
	$\frac{5\sqrt{26}}{26}$	$\frac{\sqrt{26}}{26}$	5	$\frac{\sqrt{26}}{5}$	$\sqrt{26}$	
5.	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$		1
7.		$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$		$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
9.	$\frac{2\sqrt{29}}{29}$	$\frac{5\sqrt{29}}{29}$		$\frac{\sqrt{29}}{2}$	$\frac{\sqrt{29}}{5}$	$\frac{5}{2}$

	$\sin A$	$\cos A$	$\tan A$	$\csc A$	$\sec A$	$\cot A$
11.	$\frac{2\sqrt{29}}{29}$	$\frac{5\sqrt{29}}{29}$	$\frac{2}{5}$	$\frac{\sqrt{29}}{2}$	$\frac{\sqrt{29}}{5}$	
13.	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$		$\frac{\sqrt{3}}{3}$
15.	$\frac{\sqrt{5}}{5}$	$\frac{2\sqrt{5}}{5}$		$\sqrt{5}$	$\frac{\sqrt{5}}{2}$	2
17.		$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$
19.	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$		2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
21.		$\frac{3\sqrt{5}}{7}$	$\frac{2\sqrt{5}}{15}$	$\frac{7}{2}$	$\frac{7\sqrt{5}}{15}$	$\frac{3\sqrt{5}}{2}$

23. $\cos A = \dfrac{m^2 - n^2}{m^2 + n^2}$, $\tan A = \dfrac{2mn}{m^2 - n^2}$, $\csc A = \dfrac{m^2 + n^2}{2mn}$, $\sec A = \dfrac{m^2 + n^2}{m^2 - n^2}$, $\cot A = \dfrac{m^2 - n^2}{2mn}$.

Exercises I. D, page 11

1. 0.8802. **3.** 0.2805. **5.** 0.7112. **7.** 0.0029. **9.** 343.77. **11.** 36° 40′. **13.** 17° 0′. **15.** 68° 30′. **17.** 8° 20′. **19.** 77° 10′. **21.** 24° 0′. **23.** 0.8420. No.

Exercises II. A, page 19

1. $B = 55°$, $a = 2.87$, $b = 4.10$.
3. $B = 53°$, $a = 39.94$, $c = 66.37$.
5. $A = 53° 30'$, $B = 36° 30'$, $c = 28.60$.
7. $A = 72° 30'$, $a = 293.1$, $c = 307.3$.
9. $A = 16° 40'$, $B = 73° 20'$, $c = 0.8937$. **11.** 37.3 ft., 38.6 ft.
13. 46°. **15.** 63.1 ft. **17.** 1418 ft. **19.** 120.6 ft.

Exercises II. B, page 23

1. 0.5185. **3.** 0.8887. **5.** 0.8200. **7.** 0.3528. **9.** 0.7001. **11.** 0.0026. **13.** 49.923. **15.** 0.4603. **17.** 21° 18′. **19.** 21° 19′. **21.** 19° 12′. **23.** 67° 46′. **25.** 0° 45′. **27.** 6° 5′. **29.** 11° 28′.
31. $A = 20°$, $B = 70°$, $b = 18.79$.
33. $B = 32° 48'$, $a = 0.0240$, $b = 0.0155$.
35. $A = 29° 49'$, $B = 60° 11'$, $b = 32.27$.
37. $B = 70° 16'$, $b = 63.56$, $c = 67.54$.
39. $B = 44° 58'$, $a = 8.230$, $c = 11.63$.
41. $A = 7° 22'$, $B = 82° 38'$, $b = 1.825$.

43. $B = 78° 59'$, $a = 19.42$, $b = 99.73$.
45. $A = 7° 4'$, $B = 82° 56'$, $b = 99.54$.
47. 161.4 ft., 32° 36′, 57° 24′. **49.** 80.87 ft. **51.** 130.9 ft.
53. 2.48 ft. **55.** 3.47 ft. **57.** 116.1 ft.

Exercises II. C, page 28

1. 14.2 knots, S 28° 12′ W. **3.** 24.2 ft./sec., 65° 34′.
5. (a) 53° 8′ with upstream direction; (b) 15 min.
7. 90° 58′. **9.** 86.04 lb.

Exercises II. D, page 30

1. 99° 30′, 9.83 in., 47.6 sq. in. **3.** 21° 58′, 79° 1′, 79° 1′.
5. 122° 6′. **7.** 8.42 in. **9.** 41° 25′, 198.4 sq. ft.
11. (a) 16.18 in., 15.39 in., 769.4 sq. in.; (b) 21.93 in., 20.61 in., 1391 sq. in.; (c) 21.60 in., 21.33 in., 1442 sq. in.
13. 15.35 ft., 12.42 ft.

Exercises II. E, page 34

1. $C = 70°$, $b = 29.5$, $c = 28.2$. **3.** $B = 74° 2'$, $C = 35° 58'$, $b = 8.2$.
5. $A = 95° 44'$, $B = 40° 27'$, $C = 43° 48'$.
7. $A = 50° 16'$, $B = 29° 44'$, $b = 52.9$.
9. 0.13 mi. = 686 ft. **11.** 127 ft. **13.** 105 ft. **15.** 409 ft.

Exercises III. A, page 39

1. 12.3, 29.9, 4.1, 1.40, 0.25, 0.22, 68, 63.2, 2.000, 2.000, 2.36, 2.34, 2.35, 2.35.
3. 0.002, 0.00005, 0.00001, 0.25, 0.02.
5. $1\underline{0}.\underline{0}2$, $1\underline{0}.2\underline{0}$, $0.2\underline{0}$, 0.02, $0.02\underline{0}$, $25\underset{?}{0}\underset{?}{0}\underset{?}{0}$ $25\underline{0}6$, $0.003\underline{00}$, $0.2\underline{0}5\underline{00}$, $2\underline{0}5\underset{?}{0}\underset{?}{0}$.
7. 18,000,000, 0.000,023,5, 848,200,000, 0.000,000,003,7.

Exercises III. B, page 43

1. 1490. **3.** 55.04. **5.** 231700. **7.** 18800. **9.** 1,242,800.
11. 2.93. **13.** 27.95. **15.** 147.2. **17.** 190500. **19.** 2.60.
21. 41.02. **23.** 4.241. **25.** 0.8272.

Exercises IV. A, page 48

1. 2. **3.** 3. **5.** −1. **7.** −1. **9.** −3. **11.** −1.
13. 1. **15.** 3. **17.** 0. **19.** 5. **21.** −2. **23.** 1.
25. 1. **27.** 3. **29.** −1. **31.** −2. **33.** 7. **35.** −1.

Exercises IV. B, page 50

1. 1.83251. 3. 2.55509. 5. 0.30103. 7. 3.69897.
9. 3.92572. 11. 8.33365 − 10. 13. 5.39794. 15. 0.89492.
17. 1.20276. 19. 0.47195. 21. 3.83154. 23. 4.73501.
25. 0.80023. 27. 6.94298 − 10. 29. 0.99992. 31. 4.99999.
33. 6.00004 − 10. 35. 2.91908.

Exercises IV. C, page 51

1. 5.0000. 3. 863.00. 5. 0.64980. 7. 0.000,000,578,80.
9. 0.069890. 11. 0.049074. 13. 0.001,576,4. 15. 0.066567.
17. 1,427,700. 19. 6.8305. 21. 88.202. 23. 10.002.

Exercises IV. D, page 56

1. 1489. 3. 1.16. 5. 15700. 7. 1217. 9. 0.2247.
11. 5.117. 13. 0.9564. 15. 92,024,000. 17. 0.62764. 19. 7.2292.
21. 38,122,000,000,000. 23. 299.83. 25. 0.97422. 27. 0.4544.
29. 47.002. 31. 1.146×10^{14}. 33. 2.1064. 35. 2.7314.
37. 2.9295. 39. −0.020629. 41. −21.544. 43. 19.594.

Exercises IV. E, page 59

In exercises 1–23, −10 is to be appended.

1. 9.68557. 3. 9.99067. 5. 10.50704. 7. 9.34276.
9. 9.81519. 11. 9.13078. 13. 10.23101. 15. 9.84933.
17. 9.71647. 19. 9.22613. 21. 9.92504. 23. 10.71142.
25. 20° 14′. 27. 63° 41′. 29. 57° 0.5′. 31. 11° 0.1′. 33. 57° 37.8′.
35. 38° 12.4′. 37. 39° 11.8′. 39. 81° 13.5′. 41. 49° 25.5′. 43. 88° 24.4′.
45. 87° 15.0′. 47. Impossible. 49. 2.855. 51. 97.035. 53. 0.18058.
55. 147.33. 57. 0.86142. 59. 1362.4. 61. 37° 52.9′.

Exercises V. A, page 63

1. $A = 39° 25'$, $B = 50° 35'$, $c = 1250$; 383100.
3. $A = 47° 53'$, $B = 42° 7'$, $b = 0.1846$; 0.01885.
5. $A = 51° 52'$, $B = 38° 8'$, $a = 6385$; 16,000,000.
7. $A = 31° 45'$, $b = 77.63$, $c = 91.29$; 1865.
9. $A = 66° 51'$, $a = 1765$, $c = 1920$; 666200.
11. $A = 26° 23.0'$, $B = 63° 37.0'$, $b = 5728.8$; 8,139,400.
13. $A = 33° 39.4'$, $B = 56° 20.6'$, $a = 574.16$; 247560.
15. $A = 63° 42.8'$, $b = 165.90$, $c = 374.61$; 27861.
17. $A = 37° 50.2'$, $a = 44.909$, $b = 57.820$; 1298.3.
19. (a) 101.05; (b) 7319.2. 21. 12.478 cm.

Exercises VI. A, page 70

	sin	cos	tan	csc	sec	cot
1.	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1	$\sqrt{2}$	$-\sqrt{2}$	-1
3.	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	-2	$-\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
5.	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$		$-\sqrt{2}$	$-\sqrt{2}$	
7.	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$	-2	$\frac{2\sqrt{3}}{3}$	$-\sqrt{3}$

9. $\frac{3}{2} + \frac{\sqrt{3}}{2}$. **11.** $-\frac{1}{2} + 5\sqrt{3}$. **13.** $-3 - \frac{2\sqrt{3}}{3}$ **15.** $-\frac{13}{4} + \sqrt{3}$.

17. $\frac{3}{2} - \sqrt{2}$. **19.** $2 + {}^{4\sqrt{3}}$ **21.** 4. **23.** 4. **25.** **27.** 0.

Exercises VI. B, page 78

1. (a) $\sin 20°$ or $\cos 70°$; (b) $-\cos 35°$ or $-\sin 55°$;
(c) $-\tan 80°$ or $-\cot 10'$ (d) $\csc 50°$ or $\sec 40°$;
(e) $-\sec 8°$ or $-\csc 82°$; (f) $-\cot 82°$ or $-\tan 8°$;
(g) $\sin 43°$ or $\cos 47°$; (h) $-\cos 84° 50'$ or $-\sin 5° 10'$;
(i) $-\tan 17° 56'$ or $-\cot 72° 4'$; (j) $-\cot 54° 42'$ or $-\tan 35° 18'$;
(k) $\sin 65° 39'$ or $\cos 24° 21'$; (l) $-\cos 87° 47.2'$ or $-\sin 2° 12.8'$.

3. (a) 0.57358; (b) −0.40674; (c) −3.7321; (d) 1.5617;
(e) 0.77715; (f) −0.97499; (g) −0.60626; (h) 0.97622;
(i) −0.29654; (j) 0.30486; (k) −0.36397; (l) 0.09277.

5. 0.

7. (a) 18° or 162°; (b) 60° 10′; (c) 70° 50′; (d) 30° 20′;
(e) 42° 10′ or 137° 50′; (f) 140° 30′.

Exercises VII. A, page 83

1. $C = 30°$, $b = 12.6$, $c = 6.4$. **3.** $B = 37° 10'$, $a = 3.5$, $c = 4.1$.
5. $A = 93° 40'$, $a = 324$, $c = 314$. **7.** 9.4, 6.7. **9.** 12.6, 5.34.
11. 92.2 ft. **13.** 110 ft.

Exercises VII. B, page 87

1. $B = 23° 41'$, $C = 116° 19'$, $c = 11.2$.
3. $A = 23° 48'$, $C = 120° 2'$, $c = 45.5$.
5. $B = 43° 37'$, $C = 63° 3'$, $c = 2.3$.
7. $A = 84° 12'$, $B = 80° 8'$, $b = 34.7$;
$A' = 95° 48'$, $B' = 68° 32'$, $b' = 32.7$.
9. 7.48 in., 8.03 in. **11.** 54.3 ft.

Exercises VII. C, page 90

1. $A = 51°$, $C = 69°$, $b = 5.6$. **3.** $B = 41°$, $C = 121°$, $a = 0.77$.
5. $A = 53° 25'$, $B = 31° 35'$, $c = 285$. **7.** 14.4 mi. **9.** 3.62 in., 7.20 in.
11. 175 yd.

Exercises VII. D, page 91

1. $A = 28° 57'$, $B = 46° 34'$, $C = 104° 29'$.
3. $A = 75° 26'$, $B = 56° 4'$, $C = 48° 30'$.
5. $A = 16° 16'$, $B = 73° 44'$, $C = 90° 0'$.
7. $A = 38° 56'$, $B = 34° 11'$, $C = 106° 54'$.
9. $35° 42'$ E or W of S. **11.** $57° 10'$, $122° 50'$, 23.5 in. **13.** 12.07.

Exercises VII. E, page 94

1. $A = 33° 9.9'$, $a = 435.71$, $c = 787.53$; 156030.
3. $B = 15° 57.0'$, $b = 5.4420$, $c = 17.865$; 36.400.
5. $B = 111° 11.3'$, $a = 102.19$, $b = 491.06$; 21190.
7. $B = 42° 12.8'$, $a = 514.73$, $c = 1025.0$; 177250.
9. $A = 42° 7.7'$, $a = 0.18940$, $c = 0.26964$; 0.013004.
11. 15.223 in., 18.439 in.

Exercises VII. F, page 95

1. $A = 57° 59.9'$, $C = 23° 36.6'$, $c = 29.526$; 913.08.
3. $A = 104° 32.3'$, $B = 40° 1.9'$, $a = 5888.4$; 6,678,200;
$A' = 4° 36.1'$, $B' = 139° 58.1'$, $a' = 488.04$; 553500.
5. $A = 63° 8.3'$, $B = 67° 32.8'$, $b = 89.534$; 2933.9;
$A' = 116° 51.7'$, $B' = 13° 49.4'$, $b' = 23.147$; 758.48.
7. $A = 103° 21.9'$, $C = 48° 48.8'$, $a = 0.67733$; 0.082812;
$A' = 20° 59.5'$, $C' = 131° 11.2'$, $a' = 0.24939$; 0.030491.
9. $A = 134° 37.3'$, $C = 25° 8.2'$, $a = 94.370$; 919.44;
$A' = 4° 53.7'$, $C' = 154° 51.8'$, $a' = 11.314$; 110.23.
11. No solution. **13.** 7423 ft. or 3344 ft.

Exercises VII. G, page 99

The answer for the third side may differ slightly from that given; it depends on the formula used.

1. $A = 57° 50'$, $B = 58° 32'$, $c = 300.9$; 36490.
3. $A = 38° 52.7'$, $B = 8° 49.0'$, $c = 43.017$; 120.36.
5. $A = 153° 17.5'$, $C = 14° 14.0'$, $b = 32.381$; 268.22.
7. $A = 23° 26.2'$, $C = 19° 2.6'$, $b = 819.00$; 64450.
9. $B = 46° 23.8'$, $C = 90°$, $a = 17120$; 153,880,000.
11. 2577 ft.

Exercises VII. H, page 103

1. $A = 44° 4.8', B = 101° 44.4', C = 34° 10.8'$; 6212.4.
3. $A = 30° 41.8', B = 99° 25.2', C = 49° 53.2'$; 74.745.
5. $A = 33° 32.6', B = 50° 40.8', C = 95° 46.6'$; 1,742,200,000.
7. $A = 53° 34.0', B = 26° 5.0', C = 100° 21.0'$; 483.07.
9. $A = 28° 11.8', B = 34° 4.8', C = 117° 43.2'$; 1.8836.
11. 41.51 ft.

Exercises VII. I, page 105

1. $C = 52° 15.9', b = 621.94, c = 516.16$; 132100.
3. $A = 65° 21.8', b = 1.6389, c = 4.7821$; 3.5621.
5. $A = 127° 9.4', B = 6° 24.4', C = 46° 26.2'$; 0.027977.
7. $A = 27° 28.0', B = 125° 55.4', c = 265.29$; 29345.
9. $A = 46° 26.3', B = 6° 24.4', b = 74260$; 279,762,000.
11. $B = 81° 12.2', a = 303.45, c = 271.32$; 40682.
13. $A = 46° 23.8', C = 29° 21.2', b = 9.8396$; 17.730.
15. $A = 26° 21.6', B = 106° 40.6', C = 46° 57.8'$; 788.70.
17. $C = 33° 43.0', a = 487.51, b = 689.63$; 93310.
19. $A = 99° 40.1', B = 28° 20.0', c = 182.37$; 9873.5.
21. 975.25 ft. 23. N 80° 2′ W, S 19° 6′ E. 25. 885.2 ft.
27. 31830 ft. 29. 927.0 ft., 742.6 ft., 35° 26.5′.
31. 751.5 ft. 33. 39° 41′. 35. 42.9 ft. 37. 19.806, 35.690, 44.504.
39. 57.67 rd., 96.11 rd., 134.56 rd. 49. 48° 26′.

Exercises VII. J, page 112

1. 15.18 lb., 44° 24′.
3. 30° with vertical and from front to back of windows.
5. 49° 28′. 7. 36.5 mi./hr., N 18° 21′ W.
9. 127° 10′, 90° 22′, 142° 27′.

Exercises VIII. A, page 117

	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
1.		$\frac{5}{13}$	$\frac{12}{5}$	$\frac{13}{12}$	$\frac{13}{5}$	$\frac{5}{12}$
3.	$-\frac{2\sqrt{13}}{13}$	$\frac{3\sqrt{13}}{13}$		$-\frac{\sqrt{13}}{2}$	$\frac{\sqrt{13}}{3}$	$-\frac{3}{2}$
5.	$\frac{\sqrt{21}}{5}$		$-\frac{\sqrt{21}}{2}$	$\frac{5\sqrt{21}}{21}$	$-\frac{5}{2}$	$-\frac{2\sqrt{21}}{21}$
7.	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	-1	$-\sqrt{2}$		-1
	$-\frac{7}{25}$	$-\frac{24}{25}$		$-\frac{25}{7}$	$-\frac{25}{24}$	$\frac{24}{7}$

11.		$\pm\frac{\sqrt{3}}{2}$	$\sqrt{3}$	2	$\pm\frac{2\sqrt{3}}{3}$	$\pm\sqrt{3}$
13.	$\pm\frac{2\sqrt{29}}{29}$	$\mp\frac{5\sqrt{29}}{29}$		$\pm\frac{\sqrt{29}}{2}$	$\mp\frac{\sqrt{29}}{5}$	$-\frac{5}{2}$
15.	$\pm\frac{2\sqrt{29}}{29}$	$\pm\frac{5\sqrt{29}}{29}$	$\frac{2}{5}$	$\pm\frac{\sqrt{29}}{2}$	$\pm\sqrt{29}$	
17.	$\pm\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\mp\sqrt{3}$	$\pm\frac{2\sqrt{3}}{3}$		$\mp\frac{\sqrt{3}}{3}$
19.	$\pm\frac{\sqrt{5}}{5}$	$\pm\frac{2\sqrt{5}}{5}$		$\pm\sqrt{5}$	$\pm\frac{\sqrt{5}}{2}$	2
21.	$\frac{1}{3}$	$\pm 2\sqrt{2}$	$\pm\sqrt{2}$		$\pm\frac{3\sqrt{2}}{4}$	$\pm 2\sqrt{2}$
23.	$\pm\frac{\sqrt{3}}{2}$	$\mp\frac{1}{2}$		$\pm\frac{2\sqrt{3}}{3}$	∓ 2	$-\frac{\sqrt{3}}{3}$
25.	$\pm\frac{2\sqrt{2}}{3}$		$\mp 2\sqrt{2}$	$\pm\frac{3\sqrt{2}}{4}$	-3	$\mp\frac{\sqrt{2}}{4}$
27.	$\pm\frac{10\sqrt{101}}{101}$	$\pm\frac{\sqrt{101}}{101}$	10	$\pm\frac{\sqrt{101}}{10}$	$\pm\sqrt{101}$	
29.	$\sqrt{6}$	$\pm\frac{\sqrt{3}}{3}$		$\pm\frac{\sqrt{6}}{2}$	$\pm\sqrt{3}$	$\frac{\sqrt{2}}{2}$

31. (a) $\pm\frac{33}{40}, \pm\frac{29}{120}$; (b) $\pm\frac{608}{425}, \pm\frac{208}{425}$; (c) $\frac{199}{85}, \frac{39}{85}$; (d) $\pm\frac{5}{9}$;
(e) $\pm\frac{527}{56}, \pm\frac{289}{56}$; (f) $\frac{147}{115}, \frac{3}{115}, \frac{21}{5}, \frac{3}{35}$;
(g) $\frac{4958}{425}, \frac{518}{85}, \frac{1742}{425}, \frac{182}{85}$;
(h) $(192m^2 \pm 416mn + 105n^2)/192, (192m^2 \pm 304mn - 105n^2)/192.$

Exercises VIII. B, page 120

41.

$\sin\theta =$	$\sin\theta$	$\pm\sqrt{1-\cos^2\theta}$	$\pm\dfrac{\tan\theta}{\sqrt{1+\tan^2\theta}}$	$\dfrac{1}{\csc\theta}$	$\pm\dfrac{\sqrt{\sec^2\theta-1}}{\sec\theta}$	$\pm\dfrac{1}{\sqrt{1+\cot^2\theta}}$
$\cos\theta =$	$\pm\sqrt{1-\sin^2\theta}$	$\cos\theta$	$\pm\dfrac{1}{\sqrt{1+\tan^2\theta}}$	$\pm\dfrac{\sqrt{\csc^2\theta-1}}{\csc\theta}$	$\dfrac{1}{\sec\theta}$	$\pm\dfrac{\cot\theta}{\sqrt{1+\cot^2\theta}}$
$\tan\theta =$	$\pm\dfrac{\sin\theta}{\sqrt{1-\sin^2\theta}}$	$\pm\dfrac{\sqrt{1-\cos^2\theta}}{\cos\theta}$	$\tan\theta$	$\pm\dfrac{1}{\sqrt{\csc^2\theta-1}}$	$\pm\sqrt{\sec^2\theta-1}$	$\dfrac{1}{\cot\theta}$
$\csc\theta =$	$\dfrac{1}{\sin\theta}$	$\pm\dfrac{1}{\sqrt{1-\cos^2\theta}}$	$\pm\dfrac{\sqrt{1+\tan^2\theta}}{\tan\theta}$	$\csc\theta$	$\pm\dfrac{\sec\theta}{\sqrt{\sec^2\theta-1}}$	$\pm\sqrt{1+\cot^2\theta}$
$\sec\theta =$	$\pm\dfrac{1}{\sqrt{1-\sin^2\theta}}$	$\dfrac{1}{\cos\theta}$	$\pm\sqrt{1+\tan^2\theta}$	$\pm\dfrac{\csc\theta}{\sqrt{\csc^2\theta-1}}$	$\sec\theta$	$\pm\dfrac{\sqrt{1+\cot^2\theta}}{\cot\theta}$
$\cot\theta =$	$\pm\dfrac{\sqrt{1-\sin^2\theta}}{\sin\theta}$	$\pm\dfrac{\cos\theta}{\sqrt{1-\cos^2\theta}}$	$\dfrac{1}{\tan\theta}$	$\pm\sqrt{\csc^2\theta-1}$	$\pm\dfrac{1}{\sqrt{\sec^2\theta-1}}$	$\cot\theta$

Exercises VIII. C, page 126

3. $\frac{1}{4}(\sqrt{6}-\sqrt{2})$, $\frac{1}{4}(\sqrt{6}+\sqrt{2})$, $2-\sqrt{3}$, $2+\sqrt{3}$. **9.** $\cos\theta$. **11.** 0.
19. (a) $-\frac{185}{697}$; (b) $\frac{672}{697}$; (c) $-\frac{185}{672}$; (d) $-\frac{672}{185}$; (e) $-\frac{455}{697}$; (f) $\frac{528}{697}$; (g) $-\frac{455}{528}$; (h) $-\frac{528}{455}$.
21. (a) $\pm\frac{171}{221}$; (b) $\pm\frac{140}{221}$; (c) $\frac{171}{140}$; (d) $\frac{140}{171}$; (e) $\pm\frac{21}{221}$; (f) $\pm\frac{220}{21}$; (g) $\frac{21}{220}$; (h) $\frac{220}{21}$.

Exercises VIII. D, page 130

3. $\dfrac{\sqrt{3}}{2}, -\dfrac{1}{2}, -\sqrt{3}, -\dfrac{\sqrt{3}}{3}$.
5. $\frac{1}{4}(\sqrt{6}-\sqrt{2})$, $\frac{1}{4}(\sqrt{6}+\sqrt{2})$, $2-\sqrt{3}$, $2+\sqrt{3}$.
7. (a) $\pm\frac{720}{1681}$; (b) $-\frac{1519}{1681}$; (c) $\pm\frac{720}{1519}$; (d) $\pm\frac{1519}{720}$; (e) $\pm\dfrac{5\sqrt{41}}{41}, \pm\dfrac{4\sqrt{41}}{41}$; (f) $\pm\dfrac{4\sqrt{41}}{41}, \pm\dfrac{5\sqrt{41}}{41}$; (g) $\dfrac{5}{4}, \dfrac{4}{5}$; (h) $\dfrac{4}{5}, \dfrac{5}{4}$.

Exercises VIII. E, page 132

1. $2\sin 30^\circ \cos 10^\circ = \cos 10^\circ$. **3.** $2\cos 50^\circ \cos 10^\circ$. **5.** $2\cos 40^\circ \cos 2^\circ$.
7. $2\sin 32\frac{1}{2}^\circ \cos 7\frac{1}{2}^\circ$. **9.** $2\sin 50^\circ \cos 18^\circ = 2\cos 40^\circ \sin 72^\circ$.
11. $2\sin 47^\circ \cos 3^\circ = 2\cos 43^\circ \sin 87^\circ$. **13.** $2\sin 2\theta \cos\theta$.
15. $2\sin\frac{3}{4}\theta \cos\frac{1}{4}\theta$. **17.** $2\cos\frac{7}{2}\theta \cos\frac{1}{2}\theta$.

Exercises VIII. F, page 133

23. (a) $\pm\frac{64}{1025}, \pm\frac{496}{1025}$; (b) $\pm\frac{1023}{1025}, \pm\frac{897}{1025}$; (c) $\frac{64}{1023}, \frac{496}{897}$; (d) $\frac{1023}{64}, \frac{897}{496}$; (e) $\pm\frac{496}{1025}, \pm\frac{64}{1025}$; (f) $\pm\frac{897}{1025}, \pm\frac{1023}{1025}$; (g) $\frac{496}{897}, \frac{64}{1023}$; (h) $\frac{897}{496}, \frac{1023}{64}$; (i) $\frac{336}{625}$; (j) $\frac{527}{625}$; (k) $\frac{336}{527}$; (l) $\dfrac{527}{336}$; (m) $\pm\dfrac{\sqrt{2}}{10}, \pm\dfrac{7\sqrt{2}}{10}$; (n) $\pm\dfrac{7\sqrt{2}}{10}, \pm\dfrac{\sqrt{2}}{10}$; (o) $\dfrac{1}{7}, -7$;
(p) $7, -\dfrac{1}{7}$; (q) $\pm\dfrac{9\sqrt{82}}{82}$; (r) $\pm\dfrac{\sqrt{82}}{82}$; (s) ± 9; (t) $\pm\dfrac{1}{9}$;
(u) $\pm\frac{512}{1025}, \pm\frac{62}{1025}$; (v) $\pm\frac{512}{1025}, \pm\frac{62}{1025}$; (w) $-\frac{16}{1025}$, $-$
(x) $\frac{1984}{1025}, \frac{16}{1025}$.
27. $\frac{1}{4}\sqrt{10-2\sqrt{5}}$, $\frac{1}{4}(1+\sqrt{5})$, $\sqrt{5-2\sqrt{5}}$, $\frac{1}{5}\sqrt{25+10\sqrt{5}}$.
29. $\frac{1}{16}(\sqrt{6}+\sqrt{2})(\sqrt{5}-1) - \frac{1}{8}(\sqrt{3}-1)\sqrt{5+\sqrt{5}}$,
$\frac{1}{8}(\sqrt{3}+1)\sqrt{5+\sqrt{5}} + \frac{1}{16}(\sqrt{6}-\sqrt{2})(\sqrt{5}-1)$.
31. 120 ft.

Exercises VIII. G, page 138

1. $\sqrt{2}\sin(\theta-45^\circ)$. **3.** $13\cos(\theta+\phi)$, $\phi = \operatorname{arccot}\frac{12}{5} = 22^\circ 37'$.
5. $2\cos(\theta-60^\circ)$. **7.** $\sqrt{2}\cos(\theta-45^\circ)$. **9.** $1.2997\cos(\theta+73^\circ 44')$.

Exercises IX. A, page 140

1. (a) $\frac{\pi}{18}$; (b) $\frac{7\pi}{36}$; (c) $\frac{4\pi}{15}$; (d) $\frac{7\pi}{18}$; (e) $\frac{5\pi}{6}$; (f) $\frac{14\pi}{9}$; (g) $\frac{\pi}{10}$;
(h) $\frac{20\pi}{9}$; (i) $\frac{7\pi}{120}$; (j) $\frac{11\pi}{80}$; (k) $\frac{641\pi}{240}$; (l) $\frac{13\pi}{135}$.

3. (a) 18°; (b) 15°; (c) 12°; (d) 10°; (e) 120°; (f) 135°; (g) 270°; (h) 150°; (i) 36°; (j) 72°; (k) 108°; (l) 144°; (m) 54°; (n) 84°; (o) 75°; (p) 140°.

5. (a) 28° 38′ 52.4″; (b) 38° 11′ 49.9″; (c) 16° 22′ 12.8″; (d) 162° 20′ 17″; (e) 183° 20′ 47.4″; (f) 70° 49′ 3.3″; (g) 7° 4′ 54.3″; (h) 22° 14′ 52.8″.

7. (a) $\frac{\pi}{3}$; (b) $\frac{5\pi}{6}$; (c) $\frac{\pi}{4}$; (d) $\frac{3\pi}{8}$.

9. (a) $\frac{\pi}{12}$; (b) $\frac{\pi}{720}$; (c) $\frac{5\pi}{18}$; (d) 6π; (e) $\frac{19\pi}{24}$.

11. (a) $\frac{\sqrt{3}}{2}$; (b) $-\frac{1}{2}$; (c) 1; (d) $-\sqrt{3}$; (e) $-\sqrt{2}$; (f) 2; (g) -1;
(h) 0.76604; (i) 0.15838; (j) −2.0765; (k) −0.28173;
(l) 0.97095; (m) 0.84147; (n) −0.66628; (o) 1.8856; (p) 2.1520;
(q) 0.01000; (r) 0.86232.

Exercises IX. B, page 144

1. 1.4. **3.** 3 ft. $6\frac{1}{2}$ in. **5.** 10 in. **7.** 1.9263 in. **9.** 2640.
11. (a) $60\pi^{(r)}$/sec.; (b) 240π ft./sec.

Exercises IX. C, page 146

1. 13.5 sq. in., 1.2305 sq. in. **3.** $1\frac{1}{5}^{(r)}$. **5.** 10.05 in.
7. 144 sq. in. **9.** (a) 15 sq. in.; (b) 4.687 cu. in. **11.** 103.0.

Exercises IX. D, page 150

Table IIIa of the Macmillan Logarithmic and Trigonometric Tables was used in obtaining some of these answers.

1. (a) 0.02132; (b) 0.02132; (c) 46.903.
3. (a) 8.19904 − 10; (b) 8.19910 − 10; (c) 1.80090.
5. 153.6. **7.** 2160 mi. **9.** 2.5×10^{13} mi. **11.** 238500 mi.
13. $A = 0° 45.2'$, $B = 89° 14.8'$, $c = 57.958$.
15. $A = 174° 15.4'$, $B = 3° 3.5'$, $C = 2° 41.1'$.
17. $A = 59° 25.0'$, $b = 0.13531$, $c = 0.072393$.

Exercises IX. E, page 152

1. 40. **3.** 2100 ft. **5.** 83 mils. **7.** 43 mils. **9.** 20.
11. $0° \, 33' \, 45''$, $2° \, 48' \, 45''$, $5° \, 37' \, 30''$.

Exercises X. A, page 163

15. $\frac{\pi}{4} + n\pi$.

23(1). 2π. **23(3).** 2π. **23(5).** 4π. **23(7).** 2π. **23(9).** $\frac{\pi}{2}$.
23(11). 4.

Exercises XI. A, page 173

3. $\frac{3\pi}{4}, 2n\pi \pm \frac{3\pi}{4}$. **5.** $\frac{\pi}{2}, 2n\pi \pm \frac{\pi}{2}$. **7.** $\frac{\pi}{4}, \frac{\pi}{4} + n\pi$.

9. $-\frac{\pi}{3}, -\frac{\pi}{3} + n\pi$.

11. $0.240, n\pi + (-1)^n \, 0.240$. **13.** $0.980, 0.980 + n\pi$.
15. $1.581, 2n\pi \pm 1.581$. **17.** $0.7297, n\pi + (-1)^n \, 0.7297$.
19. $1.1071, 1.1071 + n\pi$. **21.** $\frac{3}{4}$. **23.** $\frac{9}{13}$. **25.** $-\frac{8}{15}$. **27.** $\pm\frac{20}{29}$. **29.** $\pm\frac{3}{4}$.
31. $-\frac{1}{3}$. **33.** x. **35.** $\pm\frac{x}{\sqrt{1 - x^2}}$. **37.** $\pm\frac{\quad}{\sqrt{1 - x^2}}$ **39.** $\pm\frac{x}{\sqrt{1 + x^2}}$
41. $\pm\sqrt{1 + x^2}$. **45.** $-$ **47.** $1, -\frac{7}{9}$. **49.** $-\frac{1}{9}$. **51.** $\frac{435}{308}, -\frac{525}{92}$.
53. $\pm\frac{611}{1189}$ **55.** $\pm\frac{24}{25}$ $\pm\frac{2\sqrt{6}}{25}$ **57.** $\pm\frac{943}{1105}, \pm\frac{47}{1105}, \pm\frac{1073}{1105}, \pm\frac{817}{1105}$.
77. $n\pi + (-1)^n\theta$. **79.** $\theta + n\pi$.

Exercises XII. A, page 181

1. $n \cdot 180°$. **3.** $45° + n \cdot 180°$. **5.** $75° \, 58' + n \cdot 180°$.
7. $90° + n \cdot 180°, 210° + n \cdot 360°, 330° + n \cdot 360°$.
9. $90° + n \cdot 180°, 26° \, 34' + n \cdot 180°$.
11. $45° + n \cdot 180°, 161° \, 34' + n \cdot 180°$.
15. $60° + n \cdot 180°$. **17.** $11\frac{1}{4}° + n \cdot 22\frac{1}{2}°$.
19. $12° + n \cdot 36°$. **21.** $26° \, 34' + n \cdot 180°$.
23. $n \cdot 360°, 90° + n \cdot 360°$. **25.** $126° \, 13' + n \cdot 360°, 174° \, 25' + n \cdot 360°$.
27. $15° + n \cdot 360°, 285° + n \cdot 360°$. **29.** $n \cdot 180° \pm 45°, 90° + n \cdot 180°$.
31. $n \cdot 360°, 45° + n \cdot 90°$. **33.** $n \cdot 360° \pm 50° \, 36', n \cdot 360° \pm 129° \, 24'$.
35. $n \cdot 180°, 220° \, 39' + n \cdot 360°, 319° \, 21' + n \cdot 360°$.
37. $240° + n \cdot 360°, 300° + n \cdot 360°$.

$$\overline{x^2 + y^2}, \theta = \text{Arctan}\,\frac{y}{x} + 2n\pi,$$

$$r = -\sqrt{x^2 + y^2}, \theta \quad \pi + \text{Arctan}\,\frac{y}{x} + 2n\pi;$$

$x < 0, r = \qquad \pi + \text{Arctan}\,\frac{y}{x} + 2n\pi,$

$r = -\sqrt{x^2 + y^2}, \theta = \text{Arctan}\,\frac{y}{x} + 2n\pi;$

$x = 0, y > 0, r = \pm y, \theta = \pm\frac{\pi}{2} + 2n\pi,$

$y < 0, r = \pm y, \theta = \mp\frac{\pi}{2} + 2n\pi,$

$y = 0, r = 0, \theta$ meaningless.

41. $\theta = 45°\,50' + (-1)^m \cdot 30°\,20' + (m + 2k) \cdot 180°,$
$\phi = 45°\,50' - (-1)^m \cdot 30°\,20' + (m + 2l) \cdot 180°,$
where k, l, m are any integers.

43. $\theta = 50°\,46' + m \cdot 360°,\ \ \phi = 37°\,46' + n \cdot 360°;$
$\theta = 129°\,14' + m \cdot 360°, \phi = 217°\,46' + n \cdot 360°;$
$\theta = 230°\,46' + m \cdot 360°, \phi = 142°\,14' + n \cdot 360°;$
$\theta = 309°\,14' + m \cdot 360°, \phi = 322°\,14' + n \cdot 360°.$

47. 1.9346. **49.** 0.4797.* **51.** ± 0.8241. **53.** 2.8632.
55. 0, ± 0.9477. **57.** -3.1423.* **59.** Identity. **61.** $n \cdot 180°$.
63. Identity. **65.** Identity.

Exercises XIII. A, page 187

1. $8 + 6i$. **3.** $2 + 5i$. **5.** $6 + 5i$. **7.** $-1 + 7i$. **9.** $1 + 3i$. **11.** 14.
13. $5 - 2i$. **15.** $-5i$. **17.** $11 + 3i$.

Exercises XIII. B, page 189

1. $5\sqrt{2}$ cis 135°. **3.** 2 cis 30°. **5.** 5 cis 306° 52′. **7.** 6 cis 90°.
9. 17 cis 241° 56′. **11.** $\sqrt{13}$ cis 56° 19′. **13.** $\sqrt{26}$ cis 348° 41′.
15. $7\sqrt{2}$ cis 225°. **17.** 10 cis 306° 52′. **19.** $\sqrt{53}$ cis 164° 3′.
21. $\frac{\sqrt{13}}{6}$ cis 33° 41′. **23.** $\frac{5\sqrt{2}}{2} + \frac{5i\sqrt{2}}{2}$. **25.** $-\frac{3\sqrt{2}}{2} - \frac{3i\sqrt{2}}{2}$.
27. $10i$. **29.** $-4i$. **31.** $1 - i$. **33.** $8.1915 - 5.7358i$.
35. $-4.6984 - 1.7101i$. **37.** $7.6604 + 6.4279i$.

Exercises XIII. C, page 190

1. 15 cis 110°. **3.** $2\sqrt{2}$ cis 105°. **5.** 12 cis 110°. **7.** 3 cis 90° $= 3i$.
9. $\frac{3\sqrt{2}}{2}$ cis 195°.

Exercises XIII. D, page 193

1. 343 cis 54°. **3.** 32 cis 90° $32i$. **5.** 2500 cis 180° -2500.
7. cis 176°. **9.** cis 180° $= -1$.

* Other solutions exist.

11. 10^{-6} cis $300° = 0.000,000,5(1 - i\sqrt{3})$. **13.** 3 cis 40°, 3 cis 220°.
15. 3 cis 9°, 3 cis 129°, 3 cis 249°.
17. $\sqrt[3]{2}$ cis $20° = 1.1839 + 0.43092i$, $\sqrt[3]{2}$ cis $140°$
$= -0.96514 + 0.80986i$, $\sqrt[3]{2}$ cis $260° = -0.21878 - 1.2408i$.
19. cis $0° = 1$, cis $120° = -\frac{1}{2} + \frac{i\sqrt{3}}{2}$, cis $240° = -\frac{1}{2} - \frac{i\sqrt{3}}{2}$.
21. $\sqrt{2}$ cis $45° = 1 + i$, $\sqrt{2}$ cis $105° = -0.36603 + 1.3660i$,
$\sqrt{2}$ cis $165° = -1.3660 + 0.36603i$, $\sqrt{2}$ cis $225° = -1 - i$,
$\sqrt{2}$ cis $285° = 0.36603 - 1.3660i$, $\sqrt{2}$ cis $345° = 1.3660 - 0.36603i$.
23. $\sqrt{2}$ cis $45° = 1 + i$, $\sqrt{2}$ cis $117° = -0.64204 + 1.2601i$,
$\sqrt{2}$ cis $189° = -1.3968 - 0.22123i$, $\sqrt{2}$ cis $261°$
$= -0.22123 - 1.3968i$, $\sqrt{2}$ cis $333° = 1.2601 - 0.64204i$.
25. 1, $0.30902 \pm 0.95106i$, $-0.80902 \pm 0.58779i$. **27.** $\pm\frac{\sqrt{2}}{2}(1 \pm i)$.
29. $\pm(1.8478 + 0.76536i)$, $\pm(0.76536 - 1.8478i)$.
31. Same as Ex. 25, discarding $x = 1$.

Exercises XV. A, page 207

1. $B = 153°\,58.3'$, $a = 67°\,7.0'$, $b = 155°\,46.7'$.
3. $A = 105°\,52.3'$, $a = 117°\,13.7'$, $b = 33°\,32.7'$.
5. $a = 69°\,34.9'$, $b = 134°\,59.4'$, $c = 104°\,16.8'$.
7. $A = 81°\,43.0'$, $a = 70°\,16.2'$, $c = 107°\,58.2'$;
$A' = 98°\,17.0'$, $a' = 109°\,43.8'$, $c' = 72°\,1.8'$.
9. $A = 78°\,31.9'$, $b = 112°\,48.5'$, $c = 94°\,46.8'$.
11. $A = 127°\,23.3'$, $B = 109°\,52.2'$, $b = 115°\,19.6'$.
13. $A = 74°\,15.2'$, $B = 30°\,30.8'$, $a = 57°\,41.5'$.
15. No solution.
17. $B = 72°\,54.2'$, $b = 46°\,29.5'$, $c = 49°\,21.5'$;
$B' = 107°\,5.8'$, $b' = 133°\,30.5'$, $c' = 130°\,38.5'$.
19. $B = 20°\,49.8'$, $a = 44°\,44.0'$, $c = 46°\,40.1'$.
21. arctan $\sqrt{2} = 54°\,44'$.

Exercises XV. B, page 208

1. $A = 64°\,40.4'$, $B = 49°\,47.1'$, $C = 106°\,2.0'$.
3. $B = 111°\,25.9'$, $a = 117°\,4.3'$, $b = 108°\,59.2'$.
5. $B = 28°\,14.0'$, $C = 78°\,53.3'$, $b = 28°\,49.4'$;
$B' = 151°\,46.0'$, $C' = 101°\,6.7'$, $b' = 151°\,10.6'$.
7. $A = 118°\,32.6'$, $B = 33°\,20.4'$, $C = 66°\,28.3'$.
9. $A = 47°\,25.6'$, $C = 107°\,50.2'$, $a = 50°\,40.8'$;
$A' = 132°\,34.4'$, $C' = 72°\,9.8'$, $a' = 129°\,19.2'$.

Exercises XV. C, page 209

1. $B = 100° 14.4'$, $a = c = 71° 19.9'$.
3. $A = C = 103° 28.4'$, $b = 110° 37.6'$.
5. $B = C = 49° 1.3'$, $b = c = 78° 20.3'$;
   $B' = C' = 130° 58.7'$, $b' = c' = 101° 39.7'$.
7. $a = b = 94° 16.1'$, $c = 99° 48.2'$.
9. $B = 119° 35.4'$, $C = 62° 1.5'$, $b = 110° 32.6'$.
11. $A = B = C = 60° 15.2'$. 13. $A = B = C = 102° 7.8'$.
15. $a = b = c = 98° 30.5'$.

Exercises XVI. A, page 220

1. (a) Obtuse; (b) acute; (c) acute. 3. Obtuse. 5. a obtuse, c acute.
7. Acute: A; obtuse: $\frac{1}{2}(A + C)$, $\frac{1}{2}(B + C)$, B, C; 90°: $\frac{1}{2}(A + B)$.

Exercises XVI. B, page 223

1. $A = 128° 4.2'$, $B = 51° 34.2'$, $C = 73° 14.6'$.
3. $A = 65° 10.0'$, $B = 98° 50.6'$, $C = 125° 17.8'$.
5. $A = 77° 36.0'$, $B = 63° 17.0'$, $C = 107° 23.2'$.
7. $a = 47° 44.8'$, $b = 132° 40.6'$, $c = 103° 11.6'$.
9. No solution.
11. $A = 45° 25.0'$, $B = 33° 59.4'$, $C = 118° 42.0'$.
13. $a = 83° 5.8'$, $b = 102° 31.6'$, $c = 94° 26.2'$.
15. No solution.
17. $a = 126° 36.6'$, $b = 118° 13.4'$, $c = 83° 24.0'$.
19. $a = 46° 11.4'$, $b = 74° 15.4'$, $c = 86° 10.8'$.

Exercises XVI. C, page 227

1. $A = 55° 52.4'$, $B = 20° 10.0'$, $c = 66° 20.8'$.
3. $A = 144° 33.3'$, $B = 112° 46.5'$, $c = 136° 50.8'$.
5. $A = 121° 33.5'$, $B = 43° 13.5'$, $c = 62° 11.6'$.
7. $a = 95° 38.0'$, $b = 41° 52.2'$, $C = 110° 48.8'$.
9. $a = 123° 21.4'$, $c = 84° 15.4'$, $B = 129° 4.6'$.
11. $B = 95° 38.1'$, $C = 97° 26.5'$, $a = 64° 23.2'$.
13. $a = 89° 30.3'$, $c = 62° 32.1'$, $B = 1° 41.4'$.
15. $A = 96° 2.3'$, $B = 125° 43.7'$, $c = 100° 48.0'$.
17. $a = 47° 29.3'$, $b = 50° 6.3'$, $C = 129° 58.6'$.
19. $A = 142° 16.3'$, $B = 46° 7.1'$, $c = 89° 28.2'$.

Exercises XVI. D, page 232

1. $B = 22° 34.8'$, $C = 101° 16.0'$, $c = 50° 36.6'$.
3. $B = 59° 24.4'$, $C = 115° 39.8'$, $c = 97° 33.2'$;

$B' = 120° 35.6'$, $C' = 27° 0.2'$, $c' = 29° 57.4'$.

5. No solution.

7. $C = 101° 42.0'$, $b = 31° 24.7'$, $c = 147° 10.6'$; $C' = 36° 45.4'$, $b' = 148° 35.3'$, $c' = 19° 20.8'$.

9. No solution.

11. $B = 87° 34.5'$, $C = 53° 6.6'$, $c = 52° 27.2'$; $B' = 92° 25.5'$, $C' = 25° 26.2'$, $c' = 25° 12.0'$.

13. $B = 97° 21.4'$, $a = 59° 3.2'$, $b = 120° 9.4'$; $B' = 58° 55.4'$, $a' = 120° 56.8'$, $b' = 48° 19.2'$.

15. $B = 148° 6.3'$, $C = 130° 21.4'$, $c = 62° 9.0'$; $B' = 31° 53.7'$, $C' = 6° 17.6'$, $c' = 7° 18.4'$.

17. $C = 36° 38.8'$, $b = 51° 17.9'$, $c = 41° 4.6'$.

19. $C = 8° 17.6'$, $b = 125° 23.2'$, $c = 6° 51.2'$; $C' = 139° 39.0'$, $b' = 54° 36.8'$, $c' = 147° 36.8'$.

Exercises XVI. E, page 233

1. $A = 38° 27.5'$, $B = 92° 38.3'$, $c = 23° 59.0'$.

3. $a = 80° 5.2'$, $b = 70° 10.4'$, $c = 145° 5.0'$.

5. $A = 80° 14.8'$, $b = 145° 55.2'$, $c = 119° 22.6'$.

7. $B = 31° 53.7'$, $C = 6° 17.6'$, $c = 7° 18.4'$; $B' = 148° 6.3'$, $C' = 130° 21.4'$, $c' = 62° 9.0'$.

9. $A = 98° 56.0'$, $B = 66° 18.0'$, $c = 103° 30.6'$.

11. $a = 98° 44.8'$, $b = 83° 25.0'$, $c = 75° 23.2'$.

13. $a = 74° 36.4'$, $b = 112° 16.6'$, $c = 72° 33.4'$.

15. $C = 36° 38.8'$, $b = 51° 17.9'$, $c = 41° 4.6'$.

17. $A = 50° 30.2'$, $B = 135° 5.5'$, $a = 70° 20.4'$.

19. $A = 53° 30.4'$, $B = 51° 58.4'$, $C = 149° 13.4'$.

21. $B = 85° 41.2'$, $a = 47° 48.4'$, $c = 59° 39.2'$.

23. $A = 23° 17.8'$, $B = 146° 25.6'$, $C = 35° 53.4'$.

25. $C = 53° 30.4'$, $a = 88° 20.8'$, $b = 66° 46.0'$.

27. $C = 139° 39.0'$, $b = 54° 36.8'$, $c = 147° 36.8'$; $C' = 8° 17.6'$, $b' = 125° 23.2'$, $c' = 6° 51.2'$.

29. $C = 155° 51.0'$, $b = 125° 22.7'$, $c = 155° 48.0'$.

31. 21.67 in., 25.89 sq. in. **33.** 1.645 in.

Exercises XVII. A, page 238

Distances are given in nautical miles. To convert to statute miles, multiply by 1.1516. In Exercises 1–7 the first direction is the bearing of the second point from the first, the second direction is the bearing of the first point from the second.

1. 2229, N 78° 19′ W, N 69° 54′ E. **3.** 6797, S 63° 54′ E, N 55° 32′ W.

5. 5754, S 65° 29′ E, N 51° 19′ W. **7.** 7297, S 15° 34′ E, S 14° 0′ W.
9. 527 mi. **11.** (a) S 42° 54′ E; (b) S 44° 0′ E. **13.** 190.

Exercises XVII. B, page 245

1. 10:08 a.m. **3.** 34° 30′, S 58° 20′ W. **5.** 30° 13′ N. **7.** 5:33 p.m
9. (a) $13^h\ 33^m$; (b) $15^h\ 26^m$; (c) $21^h\ 8^m$.

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LOGARITHMIC AND TRIGONOMETRIC TABLES

REVISED EDITION

PREPARED UNDER THE DIRECTION OF

EARLE RAYMOND HEDRICK

ENTIRELY RE-SET IN A NEW TYPE FACE

NEW YORK

THE MACMILLAN COMPANY

Set up and electrotyped. Revised edition published August, 1920. Reprinted April, 1924; May, September, December, 1925; April, June, 1926; January, July, 1927; March, December, 1928; November, 1929; October, November, 1930; April, October, 1931; February, 1932; April, 1933; May, 1933; November, 1933; July, 1935; May, 1936; December, 1937; May, June, 1938; December, 1939; August, October, 1940; May, June, 1941; March, April, May, 1942; February, May, 1943; June, September, November, 1943.

PRINTED IN THE UNITED STATES OF AMERICA

PREFACE

The present edition of this book contains all of the tables in the previous editions. All have been reset in a new and very readable type.

Great care has been exercised to preserve and to increase the great degree of reliability that existed in the previous edition. For careful reading of the proofs, either in the first proofs made from type or in the proofs made from cast plates, I am indebted to my daughter Elisabeth and her husband, Mr. Richard L. Miller, to several of my own students, and to the following friends in other institutions, sometimes with the aid of their students: Professor C. H. Currier, Brown University; Professor H. T. Davis, University of Indiana; Professor H. B. Dwight, Massachusetts Institute of Technology; Professor W. B. Ford, University of Michigan; Professor A. M. Harding, University of Arkansas; Professor C. G. Jaeger, Pomona College; Professor L. S. Johnston, University of Detroit; Professors A. J. Kempner and C. A. Hutchinson, University of Colorado; Professor G. W. Mullins, Barnard College (Columbia University); Professor L. M. Passano, Massachusetts Institute of Technology; Professors H. L. Rietz, Roscoe Woods, and J. F. Reilly, University of Iowa; Professor E. E. Watson, Iowa State Teachers College at Cedar Falls; Dr. E. W. Wilson, Cambridge, Mass.; and Professor Kathryn Wyant, Athens College, Athens, Alabama. Each of these persons or groups has read the complete proof. With deep feeling, I may record also that the late Professor Louis Ingold of the University of Missouri read the proofs up to page 54, and had sent me the last of these pages within a week of his sudden death on January 25, 1935.

These careful readings render the possibility of printers' errors extremely remote. While the calculation of the probability that an undiscovered error exists is not simple, a strict account has been kept of each error found and of the total number not found by any one group of readers, so that a basis for a statistical calculation is known: the resulting probability that even one undiscovered printers' error exists is not more than one in many thousands.

I desire to express here my thanks to all those, particularly those mentioned above, who have assisted in the effort to make these tables so free from errors and therefore so reliable. I know of no comparable method for securing this quality in a set of tables.

I repeat also my acknowledgment made in the original edition to many previously existing tables, particularly those of Vega and those of Hoüel. During the proof-reading, those who have assisted have compared these tables with a great variety of existing tables, including several high-place tables, and the values have been recalculated and checked whenever a disagreement has been discovered.

Finally, I wish to mention the excellent cooperation of the editorial staff of the Macmillan Company under the able direction of Mr. F. T. Sutphen.

E. R. Hedrick

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EXPLANATION OF THE TABLES

TABLE I. FIVE-PLACE COMMON LOGARITHMS OF NUMBERS FROM 1 TO 10 000

1. Common Logarithms. The power to which 10 must be raised to produce any number n is called the common logarithm * of n. Thus $\log 10 = 1$, $\log 100 = 2$, $\log 1000 = 3$, etc.; $\log 1 = 0$, $\log 0.1 = -1$; $\log 0.01 = -2$, $\log 0.001 = -3$, etc. In general, if $10^l = n$, l is called the **common logarithm** of n, and is denoted by $\log n$.

2. Fundamental Principles. Logarithms constitute a great labor-saving device in arithmetical computations. The principles of their application are stated as follows:

I. *The logarithm of a product is equal to the sum of the logarithms of the factors:* $\log ab = \log a + \log b$. This follows from the fact that if $10^l = a$ and $10^L = b$, $10^{l+L} = a \cdot b$. In brief: *to multiply, add logarithms.*

II. *The logarithm of a fraction is equal to the difference obtained by subtracting the logarithm of the denominator from the logarithm of the numerator:* $\log (a/b) = \log a - \log b$. For, if $10^l = a$ and $10^L = b$, then $10^{l-L} = a \div b$. In brief: *to divide, subtract logarithms.*

III. *The logarithm of a power is equal to the logarithm of the base multiplied by the exponent of the power:* $\log a^b = b \log a$. This follows from the fact that if $10^l = a$, then $10^{lb} = a^b$.

IV. *The logarithm of a root of a number is found by dividing the logarithm of the number by the index of the root:* $\log \sqrt[b]{a} = (\log a)/b$. This follows from the fact that if $10^l = a$, then $10^{l/b} = a^{1/b} = \sqrt[b]{a}$.

Corollary of II. *The logarithm of the reciprocal of a number is the negative of the logarithm of the number:* $\log (1/a) = -\log a$, since $\log 1 = 0$.

3. Characteristic and Mantissa. Every real positive number has a real common logarithm. If a and b are any two real positive numbers such that $a < b$, then $\log a < \log b$. Neither zero nor any negative number has a real logarithm.

a	1	10	100	1000	10000	100000	1000000	10000000
$\log a$	0	1	2	3	4	5	6	7

Inspection of the preceding table shows that

the logarithm of every number between 1 and 10 is a proper fraction,

the logarithm of every number between 10 and 100 is 1 + a fraction,

the logarithm of every number between 100 and 1000 is 2 + a fraction;

* Common logarithms are exponents of the base 10; other systems of logarithms have bases different from 10; Napierian logarithms (see Table VII, p. 112) have a base denoted by e, an irrational number whose value is approximately 2.71828. When it is necessary to call attention to the base, the expression $\log_{10} n$ will mean common logarithm of n; $\log_e n$ will mean the Napierian logarithm, etc.; but in this book $\log n$ denotes $\log_{10} n$ unless otherwise explicitly stated.

and so on. It is evident that the logarithm of every number (not an exact power of 10) consists of a whole number + a fraction (usually written as a decimal). The whole number is called the **characteristic**; the decimal is called the **mantissa.** The characteristic of the logarithm of any number greater than 1 may be determined as follows:

RULE I. *The characteristic of any number greater than 1 is one less than the number of digits before the decimal point.*

The following table shows that

a	.0000001	.000001	.00001	.0001	.001	.01	.1	1
$\log a$	−7	−6	−5	−4	−3	−2	−1	0

the logarithm of every number between 0.1 and 1 is $-1 +$ a fraction,
the logarithm of every number between 0.01 and 0.1 is $-2 +$ a fraction,
the logarithm of every number between 0.001 and 0.01 is $-3 +$ a fraction;
and so on.

Thus the characteristic of every number between 0 and 1 is a negative whole number; there is a great practical advantage, however, in computing, to write these characteristics as follows: $-1 = 9 - 10$, $-2 = 8 - 10$, $-3 = 7 - 10$, etc. Thus, the logarithm of 0.562 is $-1 + 0.74974$, but this should be written $9.74974 - 10$; and similarly for all numbers less than 1.

RULE II. *The characteristic of a number less than 1 is found by subtracting from 9 the number of ciphers between the decimal point and the first significant digit, and writing -10 after the result.*

Thus, the characteristic of log 645 is 2 by Rule I; the characteristic of log 64.5 is 1 by (I); of log 6.45 is 0 by (I); of log 0.645 is $9 - 10$ by (II); of log 0.0645 is $8 - 10$ by (II).

To move the decimal point in a given number one place to the right is equivalent to adding one unit to its logarithm, because this is equivalent to multiplying the given number by 10. Likewise, to move the decimal point one place to the left is equivalent to subtracting one unit from the logarithm. Hence, moving the decimal point any number of places to the right or left does not change the mantissa but only the characteristic.*

Thus, 5345, 5.345, 534.5, 0.05345, 534500 all have the same mantissa.

4. Use of the Table. To use logarithms in computation we need a table arranged so as to enable us to find, with as little effort and time as possible, the logarithms of given numbers and, vice versa, to find numbers when their logarithms are known. Since the characteristics may be found by means of Rules I and II, p. viii, only mantissas are given. This is done in Table I. Most of the numbers in this table are irrational, and must be represented in the decimal system by approximations. A five-place table is one which gives the values correct to five places of decimals.

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* Another rule for finding the characteristic, based on this property, is often useful: if the decimal point were just after the first significant figure, the characteristic would be zero; start at this point and count the digits passed over to the left or right to the actual decimal point; the number obtained is the characteristic, except for sign; the sign is negative if the movement was to the left, positive if the movement was to the right.

PROBLEM 1. *To find the logarithm of a given number.* First, determine the characteristic, then look in the table for the mantissa.

To find the mantissa in the table when the given number (neglecting the decimal point) consists of four, or less, digits (exclusive of ciphers at the beginning or end), look in the column marked N for the first three digits and select the column headed by the fourth digit: the mantissa will be found at the intersection of this row and this column. Thus to find the logarithm of 72050, observe first (Rule I) that the characteristic is 4. To find the mantissa, fix attention on the digits 7205; find 720 in column N, and opposite it in column 5 is the desired mantissa, 0.85763; hence log 72050 = 4.85763. The mantissa of 0.07826 is found opposite 782 in column 6 and is 0.89354; hence log 0.07826 = 8.89354 − 10.

5. Interpolation. If there are more than four significant figures in the given number, its mantissa is not printed in the table; but it can be found approximately by assuming that the mantissa varies as the number varies in the small interval not tabulated; while this assumption is not strictly correct, it is sufficiently accurate for use with this table.

Thus, to find the logarithm of 72054 we observe that log 72050 = 4.85763 and that log 72060 = 4.85769. Hence a change of 10 in the number causes a change of 0.00006 in the mantissa; we assume therefore that a change of 4 in the number will cause, approximately, a change of $0.4 \times 0.00006 = 0.00002$ (dropping the sixth place) in the mantissa; and we write log 72054 = 4.85763 + 0.00002 = 4.85765.

The difference between two successive values printed in the table is called a **tabular difference** (0.00006, above). The proportional part of this difference to be added to one of the tabular values is called the **correction** (0.00002, above), and is found by multiplying the tabular difference by the appropriate fraction (0.4, above). These proportional parts are usually written *without the zeros*, and are printed at the right-hand side of each page, to be used when mental multiplications seem uncertain.

Example 1. Find the logarithm of 0.0012647. Opposite 126 in column 4 find 0.10175; the tabular difference is 34 (zeros dropped); 0.7×34 is given in the margin as 24; this correction added gives 0.10199 as the mantissa of 0.0012647; hence log 0.0012647 = 7.10199 − 10.

Example 2. Find the logarithm of 1.85643. Opposite 185 in column 6 find 0.26858; tabular difference 23; 0.43×23 is given in the margin as 10; this correction added gives 0.26868 as the mantissa of 1.85643; hence log 1.85643 = 0.26868.

6. Reverse Reading of the Table. PROBLEM 2. *To find the number when its logarithm is known.** First, fixing attention on the mantissa only, find from the table the number having this mantissa, then place the decimal point by means of the two following rules: †

RULE III. *If the characteristic of the logarithm is positive (in which case the mantissa is not followed by −10), begin at the left, count digits one more than the characteristic, and place the decimal point to the right of the last digit counted.*

* The number whose logarithm is k is often called the **antilogarithm** of k.

† Another convenient form of these rules is as follows: if the characteristic were zero, the decimal point would fall just after the first significant figure; move the decimal point one place to the right for each positive unit in the characteristic, one place to the left for each negative unit in the characteristic.

RULE IV. *If the characteristic is negative (in which case the mantissa will be preceded by a number n and followed by -10), prefix $9 - n$ ciphers, and place the decimal point to the left of these ciphers.*

Example 1. Given $\log x = 1.22737$, to find x.

Since the mantissa is 22737, we look for 22 in the first column and to the right and below for 737, which we find in column 8 opposite 168. The number is therefore 1688. Since the characteristic is $+1$, we begin at the left, count 2 places, and place the point; hence $x = 16.88$.

Example 2. Given $\log x = 2.24912$, to find x.

This mantissa is not found in the table; in such cases we interpolate as follows: select the mantissa in the table next less than the given mantissa, and write down the corresponding number; here, 1774; the tabular difference is 25; the actual difference (found by subtracting the mantissa of 1774 from the given mantissa) is 17; hence the proportionality factor is 17/25 = 0.68 or 0.7 (to the nearest tenth). Since moving the decimal point does not affect the mantissa, it follows that the digits in the required number are 17747 (to five places). The characteristic 2 directs to count 3 places from the left; hence $x = 177.47$.

RULE. *In general, when the given mantissa is not found in the table, write down four digits of the number corresponding to the mantissa in the table next less than the given mantissa, determine a fifth figure by dividing the actual difference by the tabular difference, and locate the decimal point by means of the characteristic.*

7. Cologarithms. We might add the logarithms of the factors in the numerator and from this sum subtract the logarithm of the denominator; but we can shorten the operation by *adding* the negative of the logarithm of the denominator instead of subtracting the logarithm itself. The negative of the logarithm of a number (when written in convenient form for computation) is called the **cologarithm** of the number. We may find the negative of any number by subtracting it from zero, and it is convenient in logarithmic computation to write zero in the form $10.00000 - 10$. Thus the negative of 2.17 is $7.83 - 10$; the negative of $1.1432 - 10$ is 8.8568. Remembering that the cologarithm of a number is its negative we have the following rule:

To find the cologarithm of a number begin at the left of its logarithm (including the characteristic) and subtract each digit from 9, except the last, which subtract from 10; if the logarithm has not -10 after the mantissa, write -10 after the result; if the logarithm has -10 after the mantissa, do not write -10 after the result.*

By this rule the cologarithm of a number can be read directly out of the table without taking the trouble to write down the logarithm. Attention must be given not to forget the characteristic. The use of the cologarithm is governed by the principle:

Adding the cologarithm is equivalent to subtracting the logarithm.

Ia. CONDENSED LOGARITHMS AND ANTILOGARITHMS

8. Method of Computing Logarithms. This table is a rearrangement of the condensed table given by Hoüel.† From it, the logarithm of any number whatever may be obtained to within 5 in the fifteenth place; or to any desired degree of accuracy less than this.

To illustrate the process, we shall compute $\log \pi$ to nine places. Taking $\pi = 3.14159\,26535\,8979$, we divide it by 3, the first significant digit, obtaining

* If the logarithm ends in one or more ciphers, the last significant digit is to be understood here.

† HOÜEL, *Recueil de Formules et de Tables numériques*, 3d ed., Paris, Gauthier-Villars, 1901.

$\pi/3 = 1.04719\ 755 \cdots$. We then divide this quotient by 1.04, etc., obtaining finally

$$\pi = 3(1.04)(1.006)(1.0009)(1.00001\ 52172\ 23).$$

We can obtain the logarithm of each of the first four factors from this table. The logarithm of the last factor can be obtained by multiplying its decimal part by $M = 0.43429\ 44819$; for the error made in writing

$$\log(1 + x) = Mx$$

is less than $Mx^2/2$. We find Mx either by using the fact that the last column in this table gives multiples of M, or (preferably) by Table VIII, page 115. Adding the five logarithms just mentioned, we find

$$\log \pi = 0.49714\ 98727\ 4,$$

which is surely correct to within 1 in the tenth place. The correct value is $0.49714\ 98726\ 9 \cdots$.

The process may be applied to any other number in an analogous manner. Such high-place logarithms are occasionally needed in statistical work and in the preparation of tables.

9. Method of Computing Antilogarithms. The condensed table of antilogarithms gives eleven significant figures (ten decimal places). From it, the antilogarithm of any number can be computed to within 5 in the tenth significant digit.

Thus, to compute the antilogarithm of .4342944819 to 8 significant figures, we may write

$$10^{0.43429\ 44819} = (10^{0.4})(10^{0.03})(10^{0.004})(10^{0.0002})(10^{0.00009})(10^{0.00000\ 44819}).$$

The first five factors may be obtained directly from the table. The last factor may be calculated from the formula $10^x = 1 + (1/M)x$. The error in this formula is less than 3 in the $(2k)$th decimal place if x is less than $(0.1)^k$, where $k > 1$.

However, a much more rapid process depends on the use of Tables I and XI with this table. Thus, by Table I, $10^{0.43429} = 2.718$, nearly. By Table XI, $\log\ 2.718 = 0.43424\ 94524 \cdots$. Hence $10^{0.43429\ 44819} = (2.718)(10^{0.00004\ 50295})$ $= (2.718)(10^{0.00004})(10^{0.00000\ 50295})$. Obtaining the second factor from this table, and the last factor from the formula $10^x = 1 + (1/M)x$, by Table VIII, we find $10^{0.43429\ 44819} = 2.71828\ 1828$; the correct value is $2.718281828459 \cdots$. This process requires only *two* long multiplications.

II. FIVE-PLACE TABLE OF THE ACTUAL VALUES OF THE TRIGONOMETRIC FUNCTIONS OF ANGLES

10. Direct Readings. This table gives the sines, cosines, tangents, and cotangents of the angles from 0° to 45°; and by a simple device, indicated by the printing, the values of these functions for angles from 45° to 90° may be read directly from the same table. For angles less than 45° read down the page, the degrees being found at the top and the minutes on the left; for angles greater than 45° read up the page, the degrees being found at the bottom and the minutes on the right.

To find a function of an angle (such as 15° 27′.6, for example) which does not reduce to an integral number of minutes, we employ the process of inter-

polation. To illustrate, let us find tan 15° 27′.6. In the table we find tan 15° 27′ = 0.27638 and tan 15° 28′ = 0.27670; we know that tan 15° 27′.6 lies between these two numbers. The process of interpolation depends on the assumption that between 15° 27′ and 15° 28′ the tangent of the angle varies directly as the angle; while this assumption is not strictly true, it gives an approximation sufficiently accurate for a five-place table. Thus we should assume that tan 15° 27′.5 is halfway between 0.27638 and 0.27670. We may state the problem as follows: An increase of 1′ in the angle increases the tangent 0.00032; assuming that the tangent varies as the angle, an increase of 0′.6 in the angle will increase the tangent by $0.6 \times 0.00032 = 0.00019$ (retaining only five places); hence

$$\tan 15^\circ 27'.6 = 0.27638 + 0.00019 = 0.27657.$$

The difference between two successive values in the table is called, as in Table I, the *tabular difference* (0.00032 above). The proportional part of the tabular difference which is used is called the *correction* (0.00019 above), and is found by multiplying the tabular difference by the appropriate fraction of the smallest unit given in the table.

Example 1. Find sin 63° 52′.8.

We find $\sin 63^\circ 52' = 0.89777$;

tabular difference = 0.00013 (subtracted mentally from the table),

correction $= 0.8 \times 0.00013 = 0.00010$ (to be added).

Hence $\sin 63^\circ 52'.8 = 0.89787$.

Example 2. Find cos 65° 24′.8.

$\cos 65^\circ 24' = 0.41628$;

tabular difference = 26; $0.8 \times 26 = 21$

(to be subtracted because the cosine decreases as the angle increases).

Hence $\cos 65^\circ 24'.8 = 0.41607$.

RULE. *To find a trigonometric function of an angle by interpolation: select the angle in the table which is next smaller than the given angle, and read its sine (cosine or tangent or cotangent as the case may be) and the tabular difference. Compute the correction as the proper proportional part of the tabular difference. In case of sines or tangents* ***add*** *the correction; in case of cosines or cotangents,* ***subtract*** *it.*

11. Reverse Readings. Interpolation is also used in finding the angle when one of its functions is given.

Example 1. Given $\sin x = 0.32845$, to find x.

Looking in the table we find the sine which is next less than the given sine to be .32832, and this belongs to 19° 10′. Subtract the value of the sine selected from the given sine to obtain the actual difference = 0.00013; note that the tabular difference = 0.00027. The actual difference divided by the tabular difference gives the correction = 13/27 = 0.5 as the decimal of a minute (to be added). Hence $x = 19^\circ 10'.5$.

Example 2. Given $\cos x = 0.28432$, to find x.

The cosine in the table next less than this is 0.28429 and belongs to 73° 29′; the tabular difference is 28; the actual difference is 3; correction = 3/28 = 0.1 (to be subtracted). Hence $x = 73^\circ 28'.9$.

RULE. *To find an angle when one of its trigonometric functions is given: select from the table the same named function which is next less than the given function, noting the corresponding angle and the tabular difference; compute the actual difference (between the selected value of the function and the given value) and divide*

it by the tabular difference; this gives the correction which is to be added if the given function is sine or tangent, and to be subtracted if the given function is cosine or cotangent.

III. FIVE-PLACE COMMON LOGARITHMS OF THE TRIGONOMETRIC FUNCTIONS

12. Use of the Table. If it is required to find the numerical value of $x = 27.85 \times \sin 51° 27'$, we may apply logarithms as follows:

$$\begin{aligned} \log 27.85 &= 1.44483. \\ \log \sin 51° 27' &= \underline{9.89324 - 10} \text{ (add)} \\ \log x &= 1.33807 \qquad x = 21.78 \end{aligned}$$

The only new idea here is the method of finding log sin 51° 27′, which means the logarithm of the sine of 51° 27′. The most obvious way is to find in Table II, sin 51° 27′ = 0.78206, and then to find in Table I, log 0.78206 = 9.89324 − 10, but this involves consulting two tables. To avoid the necessity of doing this, Table III gives the logarithms of the sines, cosines, tangents, and cotangents. The arrangement and the principles of interpolation are similar to those given on p. vii for Table I. The sines and cosines of all acute angles, the tangents of all acute angles less than 45° and the cotangents of all acute angles greater than 45° are proper fractions, and their logarithms end with −10, which is not printed in the table, but which should be written down whenever such a logarithm is used.

In the printed table, values are stated so that 10 should be subtracted in every case.

Example 1. Find log sin 68° 25′.4.

On the page having 68° at the bottom, and in the row having 25′ on the right find log sin 68° 25′ = 9.96843 − 10; the tabular difference is 5; 0.4 × 5 is given in the margin as 2; this is the correction to be added, giving log sin 68° 25′.4 = 9.96845 − 10.

(In case of sine and tangent *add* the correction. In case of cosine and cotangent, *subtract* the correction.)

Example 2. Given log cos x = 9.72581 − 10, to find x.

The logarithmic cosine next less than the given one is 9.72562 − 10 and belongs to 57° 53′; the actual difference is 19; the tabular difference is 20; hence the correction is 19/20 = 1.0 (to the nearest tenth); (subtract); hence x = 57° 52′.0.

In finding log ctn α for any angle α, note that log ctn α = −log tan α, since ctn α = 1/tan α. Hence *the tabular differences for* log ctn *are precisely the same as those for* log tan *throughout the table, but taken in reversed order.* Likewise, log sec α = −log cos α, log csc α = −log sin α; hence the values of log sec α and log csc α are omitted.

For angles near 0° or near 90°, the interpolations are not very accurate if the differences are large. For the calculation of sine or tangent near 0°, Table IIIa, page 45, gives the values of

$$S = \log \sin A - \log A' \qquad \text{and} \qquad T = \log \tan A - \log A',$$

where A is the given angle and A' is the number of minutes in A, for values of A between 0° and 3°. Then

$$\log \sin A = \log A' + S \qquad \text{and} \qquad \log \tan A = \log A' + T,$$

for small angles. Moreover, since we have $\cos A = \sin (90° - A)$ and $\text{ctn } A = \tan (90° - A)$,

$$\log \cos A = \log (90° - A)' + S \qquad \text{and} \qquad \log \operatorname{ctn} A = \log (90° - A)' + T,$$

when A is near 90°.

Another method practically equivalent to the preceding is to use the approximate relations

$$\log \sin A - \log \sin B = \log A' - \log B'$$

and

$$\log \tan A - \log \tan B = \log A' - \log B',$$

where A is the given angle and B is the nearest angle to A that is given in the table. If $A < 3°$ and $|A - B| < 1'$, these formulas give $\log \sin A$ and $\log \tan A$ to five decimal places.

IV–V. RADIAN MEASURE

13. Computations in Radian Measure. The reduction of degrees to radians is facilitated by Table IV—*Conversion of Degrees to Radians.* Since π radians $= 180°$, this table may be regarded as a table of multiples of $\pi/180$.

The values of $\sin x$, $\cos x$, $\tan x$, are stated for every angle x from 0.00 to 1.60 radians at intervals of 0.01 radian in Table V—*Trigonometric Functions in Radian Measure.* The values of any of these functions for larger values of x may be computed by first converting the value of the angle in radian measure to degree measure, by Table V*a*, and then finding the value of the function from Table II.

The reduction of radians to degrees can be performed directly by Table V; or, for greater accuracy, by the supplementary Table V*a*.

VI. POWERS—ROOTS—RECIPROCALS

14. Arrangement. This table is arranged so that the square, cube, square root, cube root, or reciprocal can be read directly to five decimal places for any number n of three significant figures. To attain this, not only n^2, n^3, $\sqrt{n}$, $\sqrt[3]{n}$, $1/n$, but also $\sqrt{10\,n}$, $\sqrt[3]{10\,n}$, $\sqrt[3]{100\,n}$ are printed on every page. All values have been carefully recomputed and checked.

Thus to find $\sqrt{1.17}$, read in $\sqrt{n}$ column the result: 1.08167. To find $\sqrt{11.7}$, read *in the same line*, in $\sqrt{10\,n}$ column the result: 3.42053. To find $\sqrt{117}$, read 10 times the entry in $\sqrt{n}$ column, since $\sqrt{117} = 10\sqrt{1.17}$.

Similarly, $\sqrt[3]{1.17} = 1.05373$ from $\sqrt[3]{n}$ column; $\sqrt[3]{11.7} = 2.27019$ from *the same line* in $\sqrt[3]{10\,n}$ column; $\sqrt[3]{117} = 4.89097$ from *the same line* in $\sqrt[3]{100\,n}$ column.

The effect of a change in the decimal point in n^2, n^3, and $1/n$ is only to shift the decimal point in the result, without altering the digits printed.

VII. NAPIERIAN OR NATURAL LOGARITHMS

15. The Base *e*.—Natural Logarithms. The number $e = 2.7182818 \cdots$ is called the **natural base** of logarithms. The logarithms of numbers to this base are given in Table VII at intervals of 0.01 from 0.01 to 10.09, and at unit intervals from 10 to 409. The fundamental relation $\log_e n = \log_e 10 \times \log_{10} n$ enables us to transfer from the base 10 to the base e, or conversely; where $\log_e 10 = 2.30258509 \cdots$.

VIII. MULTIPLES OF M AND OF $1/M$

16. Multiples of M and $1/M$. This table is convenient whenever a number is to be multiplied by M or by $1/M$. This occurs whenever it is desired to change from common logarithms to natural logarithms, or conversely, since $M = \log_{10} e$ and since we have

$$\log_{10} x = (\log_e x)(\log_{10} e) = M \log_e x \quad \text{and} \quad \log_e x = (1/M) \log_{10} x.$$

Other formulas that require these multiples are

$$\log_{10} e^x = x \log_{10} e = x\cdot M \quad \text{and} \quad \log_e (10^n\cdot x) = \log_e x + n(1/M);$$

and the approximate formulas (see §§ 8, 9, pp. x, xi)

$$\log_{10} (1 \pm x) = \pm x\cdot M \quad \text{and} \quad 10^{\pm x} = 1 \pm (1/M)x.$$

IX. VALUES AND LOGARITHMS OF HYPERBOLIC FUNCTIONS

17. Hyperbolic Functions. This table gives the values of e^x, e^{-x}, sinh x, cosh x, tanh x; and the logarithms of e^x, sinh x, cosh x, at varying intervals from $x = 0$ to $x = 10$. It is to be noted that $\log e^{-x} = - \log e^x$ and $\log \tanh x = \log \sinh x - \log \cosh x$. The table may be extended indefinitely by means of Table VIII, since $\log_{10} e^x = x\cdot M\cdot$ for this reason Table VIII may be regarded as a table of values of $\log_{10} e^x$.

X. VALUES AND LOGARITHMS OF HAVERSINES

18. Haversines. This table gives the values and the logarithms of the haversines of angles from 0° to 180° at intervals of 10′. The haversine, which means *half of the versed sine*, is

$$\text{hav } A = (\tfrac{1}{2}) \text{ vers } A = (\tfrac{1}{2})(1 - \cos A);$$

hence its values to five places may be computed from the table of cosines. It is used extensively in navigation, and it may be used to advantage in the solution of ordinary oblique triangles.

XI. FACTOR TABLE—LOGARITHMS OF PRIMES

19. Factors of Composite Numbers. Logarithms of Primes. The uses of this table are evident in questions involving factoring, and for finding high-place logarithms of numbers whose prime factors are less than 2018.

We shall illustrate the finding of logarithms of other numbers by finding $\log \pi$. Taking $\pi = 3.14159\,26536$, divide by 3 (the first digit), obtaining $1.04719\,75512 \cdots$. Divide this quotient by 1.047 (in general, by the nearest first four digits), obtaining $1.00018\,8683 \cdots$. By Table VIII, the approximate formula $\log (1 \pm x) = \pm x\cdot M$ gives

	log 1.00018 8683	= 0.00008 1944	(Table VIII)
	log 3	= 0.47712 12547	(Table XI)
log 1.047 =	log 3 + log 0.349	= 0.01994 66817	(Table XI)
	log π	= 0.49714 9880	

while the true value of $\log \pi$ is 0.49714 98726 9, so that the error is less than 1 in the eighth place. In general, this process will give the logarithm of *any* number to within 6 in the eighth decimal place, and the *probable error* is less than 1.5 in the eighth place. For still greater accuracy, see Table I*a* and § 10.

XII. INTEREST TABLES

20. Interest Tables. Tables XII *a*, *b*, *c*, *d* give compound interest and annuity data for various per cents up to fifty years. Aside from the obvious uses, formulas involving these data will be found in works on statistics, accounting, and the mathematics of business.

Table XII*e* gives the logarithms of $(1 + r)$ to fifteen places, for all ordinary values of r from ½% to 10%. For other values of r, $\log (1 + r)$ may be computed from Table I*a* (see § 8). The final result in interest calculations may be obtained to nine significant figures by the antilogarithms of Table I*a* (see § 9).

Table XII*f* is the American Experience Mortality Table.

XIV. FOUR-PLACE TABLES

21. Four-place Tables. These are duplicates of the preceding five-place tables, reduced to four places, and with larger intervals between the tabulations. The value of such four-place tables consists in the greater speed with which they can be used, in case the degree of accuracy they afford is sufficient for the purpose in hand.

XIV*a*. Logarithms of Numbers. The only special feature of this table is that *the proportional parts are printed for every tenth in every row;* hence the logarithm of any number of *four* significant figures can be read directly.

XIV*b*. Antilogarithms. This table will be found to facilitate approximate calculations to a marked degree. The proportional parts are stated in the right-hand margin for each row separately. This arrangement, with the corresponding one in Table XIV*a*, makes the tables *effectively* four-place each way.

XIV*c*. Values and Logarithms of Trigonometric Functions. In this table, the values of $\sin \alpha$, $\cos \alpha$, $\tan \alpha$, $\operatorname{ctn} \alpha$, and their common logarithms, are stated for each 10-minute interval in α. The characteristics of the logarithms are omitted, since they can be supplied readily from the value.

Greek Alphabet

Letters		Names	Letters		Names	Letters		Names	Letters		Names
Α	α	Alpha	Η	η	Eta	Ν	ν	Nu	Τ	τ	Tau
Β	β	Beta	Θ	θ	Theta	Ξ	ξ	Xi	Υ	υ	Upsilon
Γ	γ	Gamma	Ι		Iota	Ο	ο	Omicron	Φ	φ	Phi
Δ	δ	Delta	Κ	κ	Kappa	Π	π	Pi	Χ	χ	Chi
Ε	ε	Epsilon	Λ	λ	Lambda	Ρ	ρ	Rho	Ψ	ψ	Psi
Ζ	ζ	Zeta	Μ	μ	Mu	Σ	σ ς	Sigma	Ω	ω	Omega

LOGARITHMIC AND TRIGONOMETRIC TABLES

TABLE I

COMMON LOGARITHMS OF NUMBERS

FROM

1 TO 10 000

TO

FIVE DECIMAL PLACES

1 — 100

N	Log	N	Log	N	Log	N	Log	N	Log
0	———	**20**	1.30 103	**40**	1.60 206	**60**	1.77 815	**80**	1.90 309
1	0.00 000	21	1.32 222	41	1.61 278	61	1.78 533	81	1.90 849
2	0.30 103	22	1.34 242	42	1.62 325	62	1.79 239	82	1.91 381
3	0.47 712	23	1.36 173	43	1.63 347	63	1.79 934	83	1.91 908
4	0.60 206	24	1.38 021	44	1.64 345	64	1.80 618	84	1.92 428
5	0.69 897	25	1.39 794	45	1.65 321	65	1.81 291	85	1.92 942
6	0.77 815	26	1.41 497	46	1.66 276	66	1.81 954	86	1.93 450
7	0.84 510	27	1.43 136	47	1.67 210	67	1.82 607	87	1.93 952
8	0.90 309	28	1.44 716	48	1.68 124	68	1.83 251	88	1.94 448
9	0.95 424	29	1.46 240	49	1.69 020	69	1.83 885	89	1.94 939
10	1.00 000	**30**	1.47 712	**50**	1.69 897	**70**	1.84 510	**90**	1.95 424
11	1.04 139	31	1.49 136	51	1.70 757	71	1.85 126	91	1.95 904
12	1.07 918	32	1.50 515	52	1.71 600	72	1.85 733	92	1.96 379
13	1.11 394	33	1.51 851	53	1.72 428	73	1.86 332	93	1.96 848
14	1.14 613	34	1.53 148	54	1.73 239	74	1.86 923	94	1.97 313
15	1.17 609	35	1.54 407	55	1.74 036	75	1.87 506	95	1.97 772
16	1.20 412	36	1.55 630	56	1.74 819	76	1.88 081	96	1.98 227
17	1.23 045	37	1.56 820	57	1.75 587	77	1.88 649	97	1.98 677
18	1.25 527	38	1.57 978	58	1.76 343	78	1.89 209	98	1.99 123
19	1.27 875	39	1.59 106	59	1.77 085	79	1.89 763	99	1.99 564
N	Log	N	Log	N	Log	N	Log	N	Log

N.	0	1	2	3	4	5	6	7	8	9
100	.00 000	043	087	130	173	217	260	303	346	389
101	432	475	518	561	604	647	689	732	775	817
102	860	903	945	988	*030	*072	*115	*157	*199	*242
103	01 284	326	368	410	452	494	536	578	620	662
104	703	745	787	828	870	912	953	995	*036	*078
105	02 119	160	202	243	284	325	366	407	449	490
106	531	572	612	653	694	735	776	816	857	898
107	938	979	*019	*060	*100	*141	*181	*222	*262	*302
108	03 342	383	423	463	503	543	583	623	663	703
109	743	782	822	862	902	941	981	*021	*060	*100
110	04 139	179	218	258	297	336	376	415	454	493
111	532	571	610	650	689	727	766	805	844	883
112	922	961	999	*038	*077	*115	*154	*192	*231	*269
113	05 308	346	385	423	461	500	538	576	614	652
114	690	729	767	805	843	881	918	956	994	*032
115	06 070	108	145	183	221	258	296	333	371	408
116	446	483	521	558	595	633	670	707	744	781
117	819	856	893	930	967	*004	*041	*078	*115	*151
118	07 188	225	262	298	335	372	408	445	482	518
119	555	591	628	664	700	737	773	809	846	882
120	918	954	990	*027	*063	*099	*135	*171	*207	*243
121	08 279	314	350	386	422	458	493	529	565	600
122	636	672	707	743	778	814	849	884	920	955
123	991	*026	*061	*096	*132	*167	*202	*237	*272	*307
124	09 342	377	412	447	482	517	552	587	621	656
125	691	726	760	795	830	864	899	934	968	*003
126	10 037	072	106	140	175	209	243	278	312	346
127	380	415	449	483	517	551	585	619	653	687
128	721	755	789	823	857	890	924	958	992	*025
129	11 059	093	126	160	193	227	261	294	327	361
130	394	428	461	494	528	561	594	628	661	694
131	727	760	793	826	860	893	926	959	992	*024
132	12 057	090	123	156	189	222	254	287	320	352
133	385	418	450	483	516	548	581	613	646	678
134	710	743	775	808	840	872	905	937	969	*001
135	13 033	066	098	130	162	194	226	258	290	322
136	354	386	418	450	481	513	545	577	609	640
137	672	704	735	767	799	830	862	893	925	956
138	988	*019	*051	*082	*114	*145	*176	*208	*239	*270
139	14 301	333	364	395	426	457	489	520	551	582
140	613	644	675	706	737	768	799	829	860	891
141	922	953	983	*014	*045	*076	*106	*137	*168	*198
142	15 229	259	290	320	351	381	412	442	473	503
143	534	564	594	625	655	685	715	746	776	806
144	836	866	897	927	957	987	*017	*047	*077	*107
145	16 137	167	197	227	256	286	316	346	376	406
146	435	465	495	524	554	584	613	643	673	702
147	732	761	791	820	850	879	909	938	967	997
148	17 026	056	085	114	143	173	202	231	260	289
149	319	348	377	406	435	464	493	522	551	580
150	609	638	667	696	725	754	782	811	840	869
N.	0	1	2	3	4	5	6	7	8	9

Prop. Pts.

	44	43	42
1	4.4	4.3	4.2
2	8.8	8.6	8.4
3	13.2	12.9	12.6
4	17.6	17.2	16.8
5	22.0	21.5	21.0
6	26.4	25.8	25.2
7	30.8	30.1	29.4
8	35.2	34.4	33.6
9	39.6	38.7	37.8

	41	40	39
1	4.1	4.0	3.9
2	8.2	8.0	7.8
3	12.3	12.0	11.7
4	16.4	16.0	15.6
5	20.5	20.0	19.5
6	24.6	24.0	23.4
7	28.7	28.0	27.3
8	32.8	32.0	31.2
9	36.9	36.0	35.1

	38	37	36
1	3.8	3.7	3.6
2	7.6	7.4	7.2
3	11.4	11.1	10.8
4	15.2	14.8	14.4
5	19.0	18.5	18.0
6	22.8	22.2	21.6
7	26.6	25.9	25.2
8	30.4	29.6	28.8
9	34.2	33.3	32.4

	35	34	33
1	3.5	3.4	3.3
2	7.0	6.8	6.6
3	10.5	10.2	9.9
4	14.0	13.6	13.2
5	17.5	17.0	16.5
6	21.0	20.4	19.8
7	24.5	23.8	23.1
8	28.0	27.2	26.4
9	31.5	30.6	29.7

	32	31	30
1	3.2	3.1	3.0
2	6.4	6.2	6.0
3	9.6	9.3	9.0
4	12.8	12.4	12.0
5	16.0	15.5	15.0
6	19.2	18.6	18.0
7	22.4	21.7	21.0
8	25.6	24.8	24.0
9	28.8	27.9	27.0

N.	0	1	2	3	4	5	6	7	8	9
150	17 609	638	667	696	725	754	782	811	840	869
151	898	926	955	984	*013	*041	*070	*099	*127	*156
152	18 184	213	241	270	298	327	355	384	412	441
153	469	498	526	554	583	611	639	667	696	724
154	752	780	808	837	865	893	921	949	977	*005
155	19 033	061	089	117	145	173	201	229	257	285
156	312	340	368	396	424	451	479	507	535	562
157	590	618	645	673	700	728	756	783	811	838
158	866	893	921	948	976	*003	*030	*058	*085	*112
159	20 140	167	194	222	249	276	303	330	358	385
160	412	439	466	493	520	548	575	602	629	656
161	683	710	737	763	790	817	844	871	898	925
162	952	978	*005	*032	*059	*085	*112	*139	*165	*192
163	21 219	245	272	299	325	352	378	405	431	458
164	484	511	537	564	590	617	643	669	696	722
165	748	775	801	827	854	880	906	932	958	985
166	22 011	037	063	089	115	141	167	194	220	246
167	272	298	324	350	376	401	427	453	479	505
168	531	557	583	608	634	660	686	712	737	763
169	789	814	840	866	891	917	943	968	994	*019
170	23 045	070	096	121	147	172	198	223	249	274
171	300	325	350	376	401	426	452	477	502	528
172	553	578	603	629	654	679	704	729	754	779
173	805	830	855	880	905	930	955	980	*005	*030
174	24 055	080	105	130	155	180	204	229	254	279
175	304	329	353	378	403	428	452	477	502	527
176	551	576	601	625	650	674	699	724	748	773
177	797	822	846	871	895	920	944	969	993	*018
178	25 042	066	091	115	139	164	188	212	237	261
179	285	310	334	358	382	406	431	455	479	503
180	527	551	575	600	624	648	672	696	720	744
181	768	792	816	840	864	888	912	935	959	983
182	26 007	031	055	079	102	126	150	174	198	221
183	245	269	293	316	340	364	387	411	435	458
184	482	505	529	553	576	600	623	647	670	694
185	717	741	764	788	811	834	858	881	905	928
186	951	975	998	*021	*045	*068	*091	*114	*138	*161
187	27 184	207	231	254	277	300	323	346	370	393
188	416	439	462	485	508	531	554	577	600	623
189	646	669	692	715	738	761	784	807	830	852
190	875	898	921	944	967	989	*012	*035	*058	*081
191	28 103	126	149	171	194	217	240	262	285	307
192	330	353	375	398	421	443	466	488	511	533
193	556	578	601	623	646	668	691	713	735	758
194	780	803	825	847	870	892	914	937	959	981
195	29 003	026	048	070	092	115	137	159	181	203
196	226	248	270	292	314	336	358	380	403	425
197	447	469	491	513	535	557	579	601	623	645
198	667	688	710	732	754	776	798	820	842	863
199	885	907	929	951	973	994	*016	*038	*060	*081
200	30 103	125	146	168	190	211	233	255	276	298
N.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

Prop. Pts.

	29	28	27	26	25	24	23	22	21
1	2.9	2.8	2.7	2.6	2.5	2.4	2.3	2.2	2.1
2	5.8	5.6	5.4	5.2	5.0	4.8	4.6	4.4	4.2
3	8.7	8.4	8.1	7.8	7.5	7.2	6.9	6.6	6.3
4	11.6	11.2	10.8	10.4	10.0	9.6	9.2	8.8	8.4
5	14.5	14.0	13.5	13.0	12.5	12.0	11.5	11.0	10.5
6	17.4	16.8	16.2	15.6	15.0	14.4	13.8	13.2	12.6
7	20.3	19.6	18.9	18.2	17.5	16.8	16.1	15.4	14.7
8	23.2	22.4	21.6	20.8	20.0	19.2	18.4	17.6	16.8
9	26.1	25.2	24.3	23.4	22.5	21.6	20.7	19.8	18.9

Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9
200	30 103	125	146	168	190	211	233	255	276	298
201	320	341	363	384	406	428	449	471	492	514
202	535	557	578	600	621	643	664	685	707	728
203	750	771	792	814	835	856	878	899	920	942
204	963	984	*006	*027	*048	*069	*091	*112	*133	*154
205	31 175	197	218	239	260	281	302	323	345	366
206	387	408	429	450	471	492	513	534	555	576
207	597	618	639	660	681	702	723	744	765	785
208	806	827	848	869	890	911	931	952	973	994
209	32 015	035	056	077	098	118	139	160	181	201
210	222	243	263	284	305	325	346	366	387	408
211	428	449	469	490	510	531	552	572	593	613
212	634	654	675	695	715	736	756	777	797	818
213	838	858	879	899	919	940	960	980	*001	*021
214	33 041	062	082	102	122	143	163	183	203	224
215	244	264	284	304	325	345	365	385	405	425
216	445	465	486	506	526	546	566	586	606	626
217	646	666	686	706	726	746	766	786	806	826
218	846	866	885	905	925	945	965	985	*005	*025
219	34 044	064	084	104	124	143	163	183	203	223
220	242	262	282	301	321	341	361	380	400	420
221	439	459	479	498	518	537	557	577	596	616
222	635	655	674	694	713	733	753	772	792	811
223	830	850	869	889	908	928	947	967	986	*005
224	35 025	044	064	083	102	122	141	160	180	199
225	218	238	257	276	295	315	334	353	372	392
226	411	430	449	468	488	507	526	545	564	583
227	603	622	641	660	679	698	717	736	755	774
228	793	813	832	851	870	889	908	927	946	965
229	984	*003	*021	*040	*059	*078	*097	*116	*135	*154
230	36 173	192	211	229	248	267	286	305	324	342
231	361	380	399	418	436	455	474	493	511	530
232	549	568	586	605	624	642	661	680	698	717
233	736	754	773	791	810	829	847	866	884	903
234	922	940	959	977	996	*014	*033	*051	*070	*088
235	37 107	125	144	162	181	199	218	236	254	273
236	291	310	328	346	365	383	401	420	438	457
237	475	493	511	530	548	566	585	603	621	639
238	658	676	694	712	731	749	767	785	803	822
239	840	858	876	894	912	931	949	967	985	*003
240	38 021	039	057	075	093	112	130	148	166	184
241	202	220	238	256	274	292	310	328	346	364
242	382	399	417	435	453	471	489	507	525	543
243	561	578	596	614	632	650	668	686	703	721
244	739	757	775	792	810	828	846	863	881	899
245	917	934	952	970	987	*005	*023	*041	*058	*076
246	39 094	111	129	146	164	182	199	217	235	252
247	270	287	305	322	340	358	375	393	410	428
248	445	463	480	498	515	533	550	568	585	602
249	620	637	655	672	690	707	724	742	759	777
250	794	811	829	846	863	881	898	915	933	950
N.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

Prop. Pts.

log 2
=.30102 99957

	22	21
1	2.2	2.1
2	4.4	4.2
3	6.6	6.3
4	8.8	8.4
5	11.0	10.5
6	13.2	12.6
7	15.4	14.7
8	17.6	16.8
9	19.8	18.9

	20	19
1	2.0	1.9
2	4.0	3.8
3	6.0	5.7
4	8.0	7.6
5	10.0	9.5
6	12.0	11.4
7	14.0	13.3
8	16.0	15.2
9	18.0	17.1

	18	17
1	1.8	1.7
2	3.6	3.4
3	5.4	5.1
4	7.2	6.8
5	9.0	8.5
6	10.8	10.2
7	12.6	11.9
8	14.4	13.6
9	16.2	15.3

Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9
250	39 794	811	829	846	863	881	898	915	933	950
251	967	985	*002	*019	*037	*054	*071	*088	*106	*123
252	40 140	157	175	192	209	226	243	261	278	295
253	312	329	346	364	381	398	415	432	449	466
254	483	500	518	535	552	569	586	603	620	637
255	654	671	688	705	722	739	756	773	790	807
256	824	841	858	875	892	909	926	943	960	976
257	993	*010	*027	*044	*061	*078	*095	*111	*128	*145
258	41 162	179	196	212	229	246	263	280	296	313
259	330	347	363	380	397	414	430	447	464	481
260	497	514	531	547	564	581	597	614	631	647
261	664	681	697	714	731	747	764	780	797	814
262	830	847	863	880	896	913	929	946	963	979
263	996	*012	*029	*045	*062	*078	*095	*111	*127	*144
264	42 160	177	193	210	226	243	259	275	292	308
265	325	341	357	374	390	406	423	439	455	472
266	488	504	521	537	553	570	586	602	619	635
267	651	667	684	700	716	732	749	765	781	797
268	813	830	846	862	878	894	911	927	943	959
269	975	991	*008	*024	*040	*056	*072	*088	*104	*120
270	43 136	152	169	185	201	217	233	249	265	281
271	297	313	329	345	361	377	393	409	425	441
272	457	473	489	505	521	537	553	569	584	600
273	616	632	648	664	680	696	712	727	743	759
274	775	791	807	823	838	854	870	886	902	917
275	933	949	965	981	996	*012	*028	*044	*059	*075
276	44 091	107	122	138	154	170	185	201	217	232
277	248	264	279	295	311	326	342	358	373	389
278	404	420	436	451	467	483	498	514	529	545
279	560	576	592	607	623	638	654	669	685	700
280	716	731	747	762	778	793	809	824	840	855
281	871	886	902	917	932	948	963	979	994	*010
282	45 025	040	056	071	086	102	117	133	148	163
283	179	194	209	225	240	255	271	286	301	317
284	332	347	362	378	393	408	423	439	454	469
285	484	500	515	530	545	561	576	591	606	621
286	637	652	667	682	697	712	728	743	758	773
287	788	803	818	834	849	864	879	894	909	924
288	939	954	969	984	*000	*015	*030	*045	*060	*075
289	46 090	105	120	135	150	165	180	195	210	225
290	240	255	270	285	300	315	330	345	359	374
291	389	404	419	434	449	464	479	494	509	523
292	538	553	568	583	598	613	627	642	657	672
293	687	702	716	731	746	761	776	790	805	820
294	835	850	864	879	894	909	923	938	953	967
295	982	997	*012	*026	*041	*056	*070	*085	*100	*114
296	47 129	144	159	173	188	202	217	232	246	261
297	276	290	305	319	334	349	363	378	392	407
298	422	436	451	465	480	494	509	524	538	553
299	567	582	596	611	625	640	654	669	683	698
300	712	727	741	756	770	784	799	813	828	842
N.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

Prop. Pts.

	18	17
1	1.8	1.7
2	3.6	3.4
3	5.4	5.1
4	7.2	6.8
5	9.0	8.5
6	10.8	10.2
7	12.6	11.9
8	14.4	13.6
9	16.2	15.3

M
$=\log_{10} e$
$=\log_{10} 2.718\cdots$
$=.43429\ 44819$

	16	15
1	1.6	1.5
2	3.2	3.0
3	4.8	4.5
4	6.4	6.0
5	8.0	7.5
6	9.6	9.0
7	11.2	10.5
8	12.8	12.0
9	14.4	13.5

	14
1	1.4
2	2.8
3	4.2
4	5.6
5	7.0
6	8.4
7	9.8
8	11.2
9	12.6

Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9
300	47 712	727	741	756	770	784	799	813	828	842
301	857	871	885	900	914	929	943	958	972	986
302	48 001	015	029	044	058	073	087	101	116	130
303	144	159	173	187	202	216	230	244	259	273
304	287	302	316	330	344	359	373	387	401	416
305	430	444	458	473	487	501	515	530	544	558
306	572	586	601	615	629	643	657	671	686	700
307	714	728	742	756	770	785	799	813	827	841
308	855	869	883	897	911	926	940	954	968	982
309	996	*010	*024	*038	*052	*066	*080	*094	*108	*122
310	49 136	150	164	178	192	206	220	234	248	262
311	276	290	304	318	332	346	360	374	388	402
312	415	429	443	457	471	485	499	513	527	541
313	554	568	582	596	610	624	638	651	665	679
314	693	707	721	734	748	762	776	790	803	817
315	831	845	859	872	886	900	914	927	941	955
316	969	982	996	*010	*024	*037	*051	*065	*079	*092
317	50 106	120	133	147	161	174	188	202	215	229
318	243	256	270	284	297	311	325	338	352	365
319	379	393	406	420	433	447	461	474	488	501
320	515	529	542	556	569	583	596	610	623	637
321	651	664	678	691	705	718	732	745	759	772
322	786	799	813	826	840	853	866	880	893	907
323	920	934	947	961	974	987	*001	*014	*028	*041
324	51 055	068	081	095	108	121	135	148	162	175
325	188	202	215	228	242	255	268	282	295	308
326	322	335	348	362	375	388	402	415	428	441
327	455	468	481	495	508	521	534	548	561	574
328	587	601	614	627	640	654	667	680	693	706
329	720	733	746	759	772	786	799	812	825	838
330	851	865	878	891	904	917	930	943	957	970
331	983	996	*009	*022	*035	*048	*061	*075	*088	*101
332	52 114	127	140	153	166	179	192	205	218	231
333	244	257	270	284	297	310	323	336	349	362
334	375	388	401	414	427	440	453	466	479	492
335	504	517	530	543	556	569	582	595	608	621
336	634	647	660	673	686	699	711	724	737	750
337	763	776	789	802	815	827	840	853	866	879
338	892	905	917	930	943	956	969	982	994	*007
339	53 020	033	046	058	071	084	097	110	122	135
340	148	161	173	186	199	212	224	237	250	263
341	275	288	301	314	326	339	352	364	377	390
342	403	415	428	441	453	466	479	491	504	517
343	529	542	555	567	580	593	605	618	631	643
344	656	668	681	694	706	719	732	744	757	769
345	782	794	807	820	832	845	857	870	882	895
346	908	920	933	945	958	970	983	995	*008	*020
347	54 033	045	058	070	083	095	108	120	133	145
348	158	170	183	195	208	220	233	245	258	270
349	283	295	307	320	332	345	357	370	382	394
350	407	419	432	444	456	469	481	494	506	518
N.	0	1	2	3	4	5	6	7	8	9

Prop. Pts.

log 3 = .47712 12547

log π = .49714 98727

	15	14
1	1.5	1.4
2	3.0	2.8
3	4.5	4.2
4	6.0	5.6
5	7.5	7.0
6	9.0	8.4
7	10.5	9.8
8	12.0	11.2
9	13.5	12.6

	13	12
1	1.3	1.2
2	2.6	2.4
3	3.9	3.6
4	5.2	4.8
5	6.5	6.0
6	7.8	7.2
7	9.1	8.4
8	10.4	9.6
9	11.7	10.8

Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9
350	54 407	419	432	444	456	469	481	494	506	518
351	531	543	555	568	580	593	605	617	630	642
352	654	667	679	691	704	716	728	741	753	765
353	777	790	802	814	827	839	851	864	876	888
354	900	913	925	937	949	962	974	986	998	*011
355	55 023	035	047	060	072	084	096	108	121	133
356	145	157	169	182	194	206	218	230	242	255
357	267	279	291	303	315	328	340	352	364	376
358	388	400	413	425	437	449	461	473	485	497
359	509	522	534	546	558	570	582	594	606	618
360	630	642	654	666	678	691	703	715	727	739
361	751	763	775	787	799	811	823	835	847	859
362	871	883	895	907	919	931	943	955	967	979
363	991	*003	*015	*027	*038	*050	*062	*074	*086	*098
364	56 110	122	134	146	158	170	182	194	205	217
365	229	241	253	265	277	289	301	312	324	336
366	348	360	372	384	396	407	419	431	443	455
367	467	478	490	502	514	526	538	549	561	573
368	585	597	608	620	632	644	656	667	679	691
369	703	714	726	738	750	761	773	785	797	808
370	820	832	844	855	867	879	891	902	914	926
371	937	949	961	972	984	996	*008	*019	*031	*043
372	57 054	066	078	089	101	113	124	136	148	159
373	171	183	194	206	217	229	241	252	264	276
374	287	299	310	322	334	345	357	368	380	392
375	403	415	426	438	449	461	473	484	496	507
376	519	530	542	553	565	576	588	600	611	623
377	634	646	657	669	680	692	703	715	726	738
378	749	761	772	784	795	807	818	830	841	852
379	864	875	887	898	910	921	933	944	955	967
380	978	990	*001	*013	*024	*035	*047	*058	*070	*081
381	58 092	104	115	127	138	149	161	172	184	195
382	206	218	229	240	252	263	274	286	297	309
383	320	331	343	354	365	377	388	399	410	422
384	433	444	456	467	478	490	501	512	524	535
385	546	557	569	580	591	602	614	625	636	647
386	659	670	681	692	704	715	726	737	749	760
387	771	782	794	805	816	827	838	850	861	872
388	883	894	906	917	928	939	950	961	973	984
389	995	*006	*017	*028	*040	*051	*062	*073	*084	*095
390	59 106	118	129	140	151	162	173	184	195	207
391	218	229	240	251	262	273	284	295	306	318
392	329	340	351	362	373	384	395	406	417	428
393	439	450	461	472	483	494	506	517	528	539
394	550	561	572	583	594	605	616	627	638	649
395	660	671	682	693	704	715	726	737	748	759
396	770	780	791	802	813	824	835	846	857	868
397	879	890	901	912	923	934	945	956	966	977
398	988	999	*010	*021	*032	*043	*054	*065	*076	*086
399	60 097	108	119	130	141	152	163	173	184	195
400	206	217	228	239	249	260	271	282	293	304
N.	0	1	2	3	4	5	6	7	8	9

Prop. Pts.

	13	12
1	1.3	1.2
2	2.6	2.4
3	3.9	3.6
4	5.2	4.8
5	6.5	6.0
6	7.8	7.2
7	9.1	8.4
8	10.4	9.6
9	11.7	10.8

	11	10
1	1.1	1.0
2	2.2	2.0
3	3.3	3.0
4	4.4	4.0
5	5.5	5.0
6	6.6	6.0
7	7.7	7.0
8	8.8	8.0
9	9.9	9.0

Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9
400	60 206	217	228	239	249	260	271	282	293	304
401	314	325	336	347	358	369	379	390	401	412
402	423	433	444	455	466	477	487	498	509	520
403	531	541	552	563	574	584	595	606	617	627
404	638	649	660	670	681	692	703	713	724	735
405	746	756	767	778	788	799	810	821	831	842
406	853	863	874	885	895	906	917	927	938	949
407	959	970	981	991	*002	*013	*023	*034	*045	*055
408	61 066	077	087	098	109	119	130	140	151	162
409	172	183	194	204	215	225	236	247	257	268
410	278	289	300	310	321	331	342	352	363	374
411	384	395	405	416	426	437	448	458	469	479
412	490	500	511	521	532	542	553	563	574	584
413	595	606	616	627	637	648	658	669	679	690
414	700	711	721	731	742	752	763	773	784	794
415	805	815	826	836	847	857	868	878	888	899
416	909	920	930	941	951	962	972	982	993	*003
417	62 014	024	034	045	055	066	076	086	097	107
418	118	128	138	149	159	170	180	190	201	211
419	221	232	242	252	263	273	284	294	304	315
420	325	335	346	356	366	377	387	397	408	418
421	428	439	449	459	469	480	490	500	511	521
422	531	542	552	562	572	583	593	603	613	624
423	634	644	655	665	675	685	696	706	716	726
424	737	747	757	767	778	788	798	808	818	829
425	839	849	859	870	880	890	900	910	921	931
426	941	951	961	972	982	992	*002	*012	*022	*033
427	63 043	053	063	073	083	094	104	114	124	134
428	144	155	165	175	185	195	205	215	225	236
429	246	256	266	276	286	296	306	317	327	337
430	347	357	367	377	387	397	407	417	428	438
431	448	458	468	478	488	498	508	518	528	538
432	548	558	568	579	589	599	609	619	629	639
433	649	659	669	679	689	699	709	719	729	739
434	749	759	769	779	789	799	809	819	829	839
435	849	859	869	879	889	899	909	919	929	939
436	949	959	969	979	988	998	*008	*018	*028	*038
437	64 048	058	068	078	088	098	108	118	128	137
438	147	157	167	177	187	197	207	217	227	237
439	246	256	266	276	286	296	306	316	326	335
440	345	355	365	375	385	395	404	414	424	434
441	444	454	464	473	483	493	503	513	523	532
442	542	552	562	572	582	591	601	611	621	631
443	640	650	660	670	680	689	699	709	719	729
444	738	748	758	768	777	787	797	807	816	826
445	836	846	856	865	875	885	895	904	914	924
446	933	943	953	963	972	982	992	*002	*011	*021
447	65 031	040	050	060	070	079	089	099	108	118
448	128	137	147	157	167	176	186	196	205	215
449	225	234	244	254	263	273	283	292	302	312
450	321	331	341	350	360	369	379	389	398	408
N.	0	1	2	3	4	5	6	7	8	9

Prop. Pts.

	11	10
1	1.1	1.0
2	2.2	2.0
3	3.3	3.0
4	4.4	4.0
5	5.5	5.0
6	6.6	6.0
7	7.7	7.0
8	8.8	8.0
9	9.9	9.0

$\log M$
$= \log [\log e]$
$= 9.63778\,431$
-10

	9
1	0.9
2	1.8
3	2.7
4	3.6
5	4.5
6	5.4
7	6.3
8	7.2
9	8.1

Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9
450	65 321	331	341	350	360	369	379	389	398	408
451	418	427	437	447	456	466	475	485	495	504
452	514	523	533	543	552	562	571	581	591	600
453	610	619	629	639	648	658	667	677	686	696
454	706	715	725	734	744	753	763	772	782	792
455	801	811	820	830	839	849	858	868	877	887
456	896	906	916	925	935	944	954	963	973	982
457	992	*001	*011	*020	*030	*039	*049	*058	*068	*077
458	66 087	096	106	115	124	134	143	153	162	172
459	181	191	200	210	219	229	238	247	257	266
460	276	285	295	304	314	323	332	342	351	361
461	370	380	389	398	408	417	427	436	445	455
462	464	474	483	492	502	511	521	530	539	549
463	558	567	577	586	596	605	614	624	633	642
464	652	661	671	680	689	699	708	717	727	736
465	745	755	764	773	783	792	801	811	820	829
466	839	848	857	867	876	885	894	904	913	922
467	932	941	950	960	969	978	987	997	*006	*015
468	67 025	034	043	052	062	071	080	089	099	108
469	117	127	136	145	154	164	173	182	191	201
470	210	219	228	237	247	256	265	274	284	293
471	302	311	321	330	339	348	357	367	376	385
472	394	403	413	422	431	440	449	459	468	477
473	486	495	504	514	523	532	541	550	560	569
474	578	587	596	605	614	624	633	642	651	660
475	669	679	688	697	706	715	724	733	742	752
476	761	770	779	788	797	806	815	825	834	843
477	852	861	870	879	888	897	906	916	925	934
478	943	952	961	970	979	988	997	*006	*015	*024
479	68 034	043	052	061	070	079	088	097	106	115
480	124	133	142	151	160	169	178	187	196	205
481	215	224	233	242	251	260	269	278	287	296
482	305	314	323	332	341	350	359	368	377	386
483	395	404	413	422	431	440	449	458	467	476
484	485	494	502	511	520	529	538	547	556	565
485	574	583	592	601	610	619	628	637	646	655
486	664	673	681	690	699	708	717	726	735	744
487	753	762	771	780	789	797	806	815	824	833
488	842	851	860	869	878	886	895	904	913	922
489	931	940	949	958	966	975	984	993	*002	*011
490	69 020	028	037	046	055	064	073	082	090	099
491	108	117	126	135	144	152	161	170	179	188
492	197	205	214	223	232	241	249	258	267	276
493	285	294	302	311	320	329	338	346	355	364
494	373	381	390	399	408	417	425	434	443	452
495	461	469	478	487	496	504	513	522	531	539
496	548	557	566	574	583	592	601	609	618	627
497	636	644	653	662	671	679	688	697	705	714
498	723	732	740	749	758	767	775	784	793	801
499	810	819	827	836	845	854	862	871	880	888
500	897	906	914	923	932	940	949	958	966	975
N.	0	1	2	3	4	5	6	7	8	9

Prop. Pts.

	10	9
1	1.0	0.9
2	2.0	1.8
3	3.0	2.7
4	4.0	3.6
5	5.0	4.5
6	6.0	5.4
7	7.0	6.3
8	8.0	7.2
9	9.0	8.1

	8
1	0.8
2	1.6
3	2.4
4	3.2
5	4.0
6	4.8
7	5.6
8	6.4
9	7.2

Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9
500	69 897	906	914	923	932	940	949	958	966	975
501	984	992	*001	*010	*018	*027	*036	*044	*053	*062
502	70 070	079	088	096	105	114	122	131	140	148
503	157	165	174	183	191	200	209	217	226	234
504	243	252	260	269	278	286	295	303	312	321
505	329	338	346	355	364	372	381	389	398	406
506	415	424	432	441	449	458	467	475	484	492
507	501	509	518	526	535	544	552	561	569	578
508	586	595	603	612	621	629	638	646	655	663
509	672	680	689	697	706	714	723	731	740	749
510	757	766	774	783	791	800	808	817	825	834
511	842	851	859	868	876	885	893	902	910	919
512	927	935	944	952	961	969	978	986	995	*003
513	71 012	020	029	037	046	054	063	071	079	088
514	096	105	113	122	130	139	147	155	164	172
515	181	189	198	206	214	223	231	240	248	257
516	265	273	282	290	299	307	315	324	332	341
517	349	357	366	374	383	391	399	408	416	425
518	433	441	450	458	466	475	483	492	500	508
519	517	525	533	542	550	559	567	575	584	592
520	600	609	617	625	634	642	650	659	667	675
521	684	692	700	709	717	725	734	742	750	759
522	767	775	784	792	800	809	817	825	834	842
523	850	858	867	875	883	892	900	908	917	925
524	933	941	950	958	966	975	983	991	999	*008
525	72 016	024	032	041	049	057	066	074	082	090
526	099	107	115	123	132	140	148	156	165	173
527	181	189	198	206	214	222	230	239	247	255
528	263	272	280	288	296	304	313	321	329	337
529	346	354	362	370	378	387	395	403	411	419
530	428	436	444	452	460	469	477	485	493	501
531	509	518	526	534	542	550	558	567	575	583
532	591	599	607	616	624	632	640	648	656	665
533	673	681	689	697	705	713	722	730	738	746
534	754	762	770	779	787	795	803	811	819	827
535	835	843	852	860	868	876	884	892	900	908
536	916	925	933	941	949	957	965	973	981	989
537	997	*006	*014	*022	*030	*038	*046	*054	*062	*070
538	73 078	086	094	102	111	119	127	135	143	151
539	159	167	175	183	191	199	207	215	223	231
540	239	247	255	263	272	280	288	296	304	312
541	320	328	336	344	352	360	368	376	384	392
542	400	408	416	424	432	440	448	456	464	472
543	480	488	496	504	512	520	528	536	544	552
544	560	568	576	584	592	600	608	616	624	632
545	640	648	656	664	672	679	687	695	703	711
546	719	727	735	743	751	759	767	775	783	791
547	799	807	815	823	830	838	846	854	862	870
548	878	886	894	902	910	918	926	933	941	949
549	957	965	973	981	989	997	*005	*013	*020	*028
550	74 036	044	052	060	068	076	084	092	099	107

Prop. Pts.

log 5
=.69897 00043

	9	8
1	0.9	0.8
2	1.8	1.6
3	2.7	2.4
4	3.6	3.2
5	4.5	4.0
6	5.4	4.8
7	6.3	5.6
8	7.2	6.4
9	8.1	7.2

	7
1	0.7
2	1.4
3	2.1
4	2.8
5	3.5
6	4.2
7	4.9
8	5.6
9	6.3

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
550	74 036	044	052	060	068	076	084	092	099	107	
551	115	123	131	139	147	155	162	170	178	186	
552	194	202	210	218	225	233	241	249	257	265	
553	273	280	288	296	304	312	320	327	335	343	
554	351	359	367	374	382	390	398	406	414	421	
555	429	437	445	453	461	468	476	484	492	500	
556	507	515	523	531	539	547	554	562	570	578	
557	586	593	601	609	617	624	632	640	648	656	
558	663	671	679	687	695	702	710	718	726	733	
559	741	749	757	764	772	780	788	796	803	811	
560	819	827	834	842	850	858	865	873	881	889	
561	896	904	912	920	927	935	943	950	958	966	
562	974	981	989	997	*005	*012	*020	*028	*035	*043	
563	75 051	059	066	074	082	089	097	105	113	120	
564	128	136	143	151	159	166	174	182	189	197	
565	205	213	220	228	236	243	251	259	266	274	
566	282	289	297	305	312	320	328	335	343	351	
567	358	366	374	381	389	397	404	412	420	427	
568	435	442	450	458	465	473	481	488	496	504	
569	511	519	526	534	542	549	557	565	572	580	
570	587	595	603	610	618	626	633	641	648	656	
571	664	671	679	686	694	702	709	717	724	732	
572	740	747	755	762	770	778	785	793	800	808	
573	815	823	831	838	846	853	861	868	876	884	
574	891	899	906	914	921	929	937	944	952	959	
575	967	974	982	989	997	*005	*012	*020	*027	*035	
576	76 042	050	057	065	072	080	087	095	103	110	
577	118	125	133	140	148	155	163	170	178	185	
578	193	200	208	215	223	230	238	245	253	260	
579	268	275	283	290	298	305	313	320	328	335	
580	343	350	358	365	373	380	388	395	403	410	
581	418	425	433	440	448	455	462	470	477	485	
582	492	500	507	515	522	530	537	545	552	559	
583	567	574	582	589	597	604	612	619	626	634	
584	641	649	656	664	671	678	686	693	701	708	
585	716	723	730	738	745	753	760	768	775	782	
586	790	797	805	812	819	827	834	842	849	856	
587	864	871	879	886	893	901	908	916	923	930	
588	938	945	953	960	967	975	982	989	997	*004	
589	77 012	019	026	034	041	048	056	063	070	078	
590	085	093	100	107	115	122	129	137	144	151	
591	159	166	173	181	188	195	203	210	217	225	
592	232	240	247	254	262	269	276	283	291	298	
593	305	313	320	327	335	342	349	357	364	371	
594	379	386	393	401	408	415	422	430	437	444	
595	452	459	466	474	481	488	495	503	510	517	
596	525	532	539	546	554	561	568	576	583	590	
597	597	605	612	619	627	634	641	648	656	663	
598	670	677	685	692	699	706	714	721	728	735	
599	743	750	757	764	772	779	786	793	801	808	
600	815	822	830	837	844	851	859	866	873	880	
N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.

	8	7
1	0.8	0.7
2	1.6	1.4
3	2.4	2.1
4	3.2	2.8
5	4.0	3.5
6	4.8	4.2
7	5.6	4.9
8	6.4	5.6
9	7.2	6.3

N.	0	1	2	3	4	5	6	7	8	9
600	77 815	822	830	837	844	851	859	866	873	880
601	887	895	902	909	916	924	931	938	945	952
602	960	967	974	981	988	996	*003	*010	*017	*025
603	78 032	039	046	053	061	068	075	082	089	097
604	104	111	118	125	132	140	147	154	161	168
605	176	183	190	197	204	211	219	226	233	240
606	247	254	262	269	276	283	290	297	305	312
607	319	326	333	340	347	355	362	369	376	383
608	390	398	405	412	419	426	433	440	447	455
609	462	469	476	483	490	497	504	512	519	526
610	533	540	547	554	561	569	576	583	590	597
611	604	611	618	625	633	640	647	654	661	668
612	675	682	689	696	704	711	718	725	732	739
613	746	753	760	767	774	781	789	796	803	810
614	817	824	831	838	845	852	859	866	873	880
615	888	895	902	909	916	923	930	937	944	951
616	958	965	972	979	986	993	*000	*007	*014	*021
617	79 029	036	043	050	057	064	071	078	085	092
618	099	106	113	120	127	134	141	148	155	162
619	169	176	183	190	197	204	211	218	225	232
620	239	246	253	260	267	274	281	288	295	302
621	309	316	323	330	337	344	351	358	365	372
622	379	386	393	400	407	414	421	428	435	442
623	449	456	463	470	477	484	491	498	505	511
624	518	525	532	539	546	553	560	567	574	581
625	588	595	602	609	616	623	630	637	644	650
626	657	664	671	678	685	692	699	706	713	720
627	727	734	741	748	754	761	768	775	782	789
628	796	803	810	817	824	831	837	844	851	858
629	865	872	879	886	893	900	906	913	920	927
630	934	941	948	955	962	969	975	982	989	996
631	80 003	010	017	024	030	037	044	051	058	065
632	072	079	085	092	099	106	113	120	127	134
633	140	147	154	161	168	175	182	188	195	202
634	209	216	223	229	236	243	250	257	264	271
635	277	284	291	298	305	312	318	325	332	339
636	346	353	359	366	373	380	387	393	400	407
637	414	421	428	434	441	448	455	462	468	475
638	482	489	496	502	509	516	523	530	536	543
639	550	557	564	570	577	584	591	598	604	611
640	618	625	632	638	645	652	659	665	672	679
641	686	693	699	706	713	720	726	733	740	747
642	754	760	767	774	781	787	794	801	808	814
643	821	828	835	841	848	855	862	868	875	882
644	889	895	902	909	916	922	929	936	943	949
645	956	963	969	976	983	990	996	*003	*010	*017
646	81 023	030	037	043	050	057	064	070	077	084
647	090	097	104	111	117	124	131	137	144	151
648	158	164	171	178	184	191	198	204	211	218
649	224	231	238	245	251	258	265	271	278	285
650	291	298	305	311	318	325	331	338	345	351
N.	0	1	2	3	4	5	6	7	8	9

Prop. Pts.

	8	7
1	0.8	0.7
2	1.6	1.4
3	2.4	2.1
4	3.2	2.8
5	4.0	3.5
6	4.8	4.2
7	5.6	4.9
8	6.4	5.6
9	7.2	6.3

	6
1	0.6
2	1.2
3	1.8
4	2.4
5	3.0
6	3.6
7	4.2
8	4.8
9	5.4

Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9
650	81 291	298	305	311	318	325	331	338	345	351
651	358	365	371	378	385	391	398	405	411	418
652	425	431	438	445	451	458	465	471	478	485
653	491	498	505	511	518	525	531	538	544	551
654	558	564	571	578	584	591	598	604	611	617
655	624	631	637	644	651	657	664	671	677	684
656	690	697	704	710	717	723	730	737	743	750
657	757	763	770	776	783	790	796	803	809	816
658	823	829	836	842	849	856	862	869	875	882
659	889	895	902	908	915	921	928	935	941	948
660	954	961	968	974	981	987	994	*000	*007	*014
661	82 020	027	033	040	046	053	060	066	073	079
662	086	092	099	105	112	119	125	132	138	145
663	151	158	164	171	178	184	191	197	204	210
664	217	223	230	236	243	249	256	263	269	276
665	282	289	295	302	308	315	321	328	334	341
666	347	354	360	367	373	380	387	393	400	406
667	413	419	426	432	439	445	452	458	465	471
668	478	484	491	497	504	510	517	523	530	536
669	543	549	556	562	569	575	582	588	595	601
670	607	614	620	627	633	640	646	653	659	666
671	672	679	685	692	698	705	711	718	724	730
672	737	743	750	756	763	769	776	782	789	795
673	802	808	814	821	827	834	840	847	853	860
674	866	872	879	885	892	898	905	911	918	924
675	930	937	943	950	956	963	969	975	982	988
676	995	*001	*008	*014	*020	*027	*033	*040	*046	*052
677	83 059	065	072	078	085	091	097	104	110	117
678	123	129	136	142	149	155	161	168	174	181
679	187	193	200	206	213	219	225	232	238	245
680	251	257	264	270	276	283	289	296	302	308
681	315	321	327	334	340	347	353	359	366	372
682	378	385	391	398	404	410	417	423	429	436
683	442	448	455	461	467	474	480	487	493	499
684	506	512	518	525	531	537	544	550	556	563
685	569	575	582	588	594	601	607	613	620	626
686	632	639	645	651	658	664	670	677	683	689
687	696	702	708	715	721	727	734	740	746	753
688	759	765	771	778	784	790	797	803	809	816
689	822	828	835	841	847	853	860	866	872	879
690	885	891	897	904	910	916	923	929	935	942
691	948	954	960	967	973	979	985	992	998	*004
692	84 011	017	023	029	036	042	048	055	061	067
693	073	080	086	092	098	105	111	117	123	130
694	136	142	148	155	161	167	173	180	186	192
695	198	205	211	217	223	230	236	242	248	255
696	261	267	273	280	286	292	298	305	311	317
697	323	330	336	342	348	354	361	367	373	379
698	386	392	398	404	410	417	423	429	435	442
699	448	454	460	466	473	479	485	491	497	504
700	510	516	522	528	535	541	547	553	559	566
N.	0	1	2	3	4	5	6	7	8	9

Prop. Pts.

	7	6
1	0.7	0.6
2	1.4	1.2
3	2.1	1.8
4	2.8	2.4
5	3.5	3.0
6	4.2	3.6
7	4.9	4.2
8	5.6	4.8
9	6.3	5.4

Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9
700	84 510	516	522	528	535	541	547	553	559	566
701	572	578	584	590	597	603	609	615	621	628
702	634	640	646	652	658	665	671	677	683	689
703	696	702	708	714	720	726	733	739	745	751
704	757	763	770	776	782	788	794	800	807	813
705	819	825	831	837	844	850	856	862	868	874
706	880	887	893	899	905	911	917	924	930	936
707	942	948	954	960	967	973	979	985	991	997
708	85 003	009	016	022	028	034	040	046	052	058
709	065	071	077	083	089	095	101	107	114	120
710	126	132	138	144	150	156	163	169	175	181
711	187	193	199	205	211	217	224	230	236	242
712	248	254	260	266	272	278	285	291	297	303
713	309	315	321	327	333	339	345	352	358	364
714	370	376	382	388	394	400	406	412	418	425
715	431	437	443	449	455	461	467	473	479	485
716	491	497	503	509	516	522	528	534	540	546
717	552	558	564	570	576	582	588	594	600	606
718	612	618	625	631	637	643	649	655	661	667
719	673	679	685	691	697	703	709	715	721	727
720	733	739	745	751	757	763	769	775	781	788
721	794	800	806	812	818	824	830	836	842	848
722	854	860	866	872	878	884	890	896	902	908
723	914	920	926	932	938	944	950	956	962	968
724	974	980	986	992	998	*004	*010	*016	*022	*028
725	86 034	040	046	052	058	064	070	076	082	088
726	094	100	106	112	118	124	130	136	141	147
727	153	159	165	171	177	183	189	195	201	207
728	213	219	225	231	237	243	249	255	261	267
729	273	279	285	291	297	303	308	314	320	326
730	332	338	344	350	356	362	368	374	380	386
731	392	398	404	410	415	421	427	433	439	445
732	451	457	463	469	475	481	487	493	499	504
733	510	516	522	528	534	540	546	552	558	564
734	570	576	581	587	593	599	605	611	617	623
735	629	635	641	646	652	658	664	670	676	682
736	688	694	700	705	711	717	723	729	735	741
737	747	753	759	764	770	776	782	788	794	800
738	806	812	817	823	829	835	841	847	853	859
739	864	870	876	882	888	894	900	906	911	917
740	923	929	935	941	947	953	958	964	970	976
741	982	988	994	999	*005	*011	*017	*023	*029	*035
742	87 040	046	052	058	064	070	075	081	087	093
743	099	105	111	116	122	128	134	140	146	151
744	157	163	169	175	181	186	192	198	204	210
745	216	221	227	233	239	245	251	256	202	268
746	274	280	286	291	297	303	309	315	320	326
747	332	338	344	349	355	361	367	373	379	384
748	390	396	402	408	413	419	425	431	437	442
749	448	454	460	466	471	477	483	489	495	500
750	506	512	518	523	529	535	541	547	552	558
N.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

Prop. Pts.

log 7 =.84509 80400

	7	6
1	0.7	0.6
2	1.4	1.2
3	2.1	1.8
4	2.8	2.4
5	3.5	3.0
6	4.2	3.6
7	4.9	4.2
8	5.6	4.8
9	6.3	5.4

	5
1	0.5
2	1.0
3	1.5
4	2.0
5	2.5
6	3.0
7	3.5
8	4.0
9	4.5

Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9
750	87 506	512	518	523	529	535	541	547	552	558
751	564	570	576	581	587	593	599	604	610	616
752	622	628	633	639	645	651	656	662	668	674
753	679	685	691	697	703	708	714	720	726	731
754	737	743	749	754	760	766	772	777	783	789
755	795	800	806	812	818	823	829	835	841	846
756	852	858	864	869	875	881	887	892	898	904
757	910	915	921	927	933	938	944	950	955	961
758	967	973	978	984	990	996	*001	*007	*013	*018
759	88 024	030	036	041	047	053	058	064	070	076
760	081	087	093	098	104	110	116	121	127	133
761	138	144	150	156	161	167	173	178	184	190
762	195	201	207	213	218	224	230	235	241	247
763	252	258	264	270	275	281	287	292	298	304
764	309	315	321	326	332	338	343	349	355	360
765	366	372	377	383	389	395	400	406	412	417
766	423	429	434	440	446	451	457	463	468	474
767	480	485	491	497	502	508	513	519	525	530
768	536	542	547	553	559	564	570	576	581	587
769	593	598	604	610	615	621	627	632	638	643
770	649	655	660	666	672	677	683	689	694	700
771	705	711	717	722	728	734	739	745	750	756
772	762	767	773	779	784	790	795	801	807	812
773	818	824	829	835	840	846	852	857	863	868
774	874	880	885	891	897	902	908	913	919	925
775	930	936	941	947	953	958	964	969	975	981
776	986	992	997	*003	*009	*014	*020	*025	*031	*037
777	89 042	048	053	059	064	070	076	081	087	092
778	098	104	109	115	120	126	131	137	143	148
779	154	159	165	170	176	182	187	193	198	204
780	209	215	221	226	232	237	243	248	254	260
781	265	271	276	282	287	293	298	304	310	315
782	321	326	332	337	343	348	354	360	365	371
783	376	382	387	393	398	404	409	415	421	426
784	432	437	443	448	454	459	465	470	476	481
785	487	492	498	504	509	515	520	526	531	537
786	542	548	553	559	564	570	575	581	586	592
787	597	603	609	614	620	625	631	636	642	647
788	653	658	664	669	675	680	686	691	697	702
789	708	713	719	724	730	735	741	746	752	757
790	763	768	774	779	785	790	796	801	807	812
791	818	823	829	834	840	845	851	856	862	867
792	873	878	883	889	894	900	905	911	916	922
793	927	933	938	944	949	955	960	966	971	977
794	982	988	993	998	*004	*009	*015	*020	*026	*031
795	90 037	042	048	053	059	064	069	075	080	086
796	091	097	102	108	113	119	124	129	135	140
797	146	151	157	162	168	173	179	184	189	195
798	200	206	211	217	222	227	233	238	244	249
799	255	260	266	271	276	282	287	293	298	304
800	309	314	320	325	331	336	342	347	352	358
N.	0	1	2	3	4	5	6	7	8	9

Prop. Pts.

	6	5
1	0.6	0.5
2	1.2	1.0
3	1.8	1.5
4	2.4	2.0
5	3.0	2.5
6	3.6	3.0
7	4.2	3.5
8	4.8	4.0
9	5.4	4.5

Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9
800	90 309	314	320	325	331	336	342	347	352	358
801	363	369	374	380	385	390	396	401	407	412
802	417	423	428	434	439	445	450	455	461	466
803	472	477	482	488	493	499	504	509	515	520
804	526	531	536	542	547	553	558	563	569	574
805	580	585	590	596	601	607	612	617	623	628
806	634	639	644	650	655	660	666	671	677	682
807	687	693	698	703	709	714	720	725	730	736
808	741	747	752	757	763	768	773	779	784	789
809	795	800	806	811	816	822	827	832	838	843
810	849	854	859	865	870	875	881	886	891	897
811	902	907	913	918	924	929	934	940	945	950
812	956	961	966	972	977	982	988	993	998	*004
813	91 009	014	020	025	030	036	041	046	052	057
814	062	068	073	078	084	089	094	100	105	110
815	116	121	126	132	137	142	148	153	158	164
816	169	174	180	185	190	196	201	206	212	217
817	222	228	233	238	243	249	254	259	265	270
818	275	281	286	291	297	302	307	312	318	323
819	328	334	339	344	350	355	360	365	371	376
820	381	387	392	397	403	408	413	418	424	429
821	434	440	445	450	455	461	466	471	477	482
822	487	492	498	503	508	514	519	524	529	535
823	540	545	551	556	561	566	572	577	582	587
824	593	598	603	609	614	619	624	630	635	640
825	645	651	656	661	666	672	677	682	687	693
826	698	703	709	714	719	724	730	735	740	745
827	751	756	761	766	772	777	782	787	793	798
828	803	808	814	819	824	829	834	840	845	850
829	855	861	866	871	876	882	887	892	897	903
830	908	913	918	924	929	934	939	944	950	955
831	960	965	971	976	981	986	991	997	*002	*007
832	92 012	018	023	028	033	038	044	049	054	059
833	065	070	075	080	085	091	096	101	106	111
834	117	122	127	132	137	143	148	153	158	163
835	169	174	179	184	189	195	200	205	210	215
836	221	226	231	236	241	247	252	257	262	267
837	273	278	283	288	293	298	304	309	314	319
838	324	330	335	340	345	350	355	361	366	371
839	376	381	387	392	397	402	407	412	418	423
840	428	433	438	443	449	454	459	464	469	474
841	480	485	490	495	500	505	511	516	521	526
842	531	536	542	547	552	557	562	567	572	578
843	583	588	593	598	603	609	614	619	624	629
844	634	639	645	650	655	660	665	670	675	681
845	686	691	696	701	706	711	716	722	727	732
846	737	742	747	752	758	763	768	773	778	783
847	788	793	799	804	809	814	819	824	829	834
848	840	845	850	855	860	865	870	875	881	886
849	891	896	901	906	911	916	921	927	932	937
850	942	947	952	957	962	967	973	978	983	988
N.	0	1	2	3	4	5	6	7	8	9

Prop. Pts.

	6	5
1	0.6	0.5
2	1.2	1.0
3	1.8	1.5
4	2.4	2.0
5	3.0	2.5
6	3.6	3.0
7	4.2	3.5
8	4.8	4.0
9	5.4	4.5

Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9
850	92 942	947	952	957	962	967	973	978	983	988
851	993	998	*003	*008	*013	*018	*024	*029	*034	*039
852	93 044	049	054	059	064	069	075	080	085	090
853	095	100	105	110	115	120	125	131	136	141
854	146	151	156	161	166	171	176	181	186	192
855	197	202	207	212	217	222	227	232	237	242
856	247	252	258	263	268	273	278	283	288	293
857	298	303	308	313	318	323	328	334	339	344
858	349	354	359	364	369	374	379	384	389	394
859	399	404	409	414	420	425	430	435	440	445
860	450	455	460	465	470	475	480	485	490	495
861	500	505	510	515	520	526	531	536	541	546
862	551	556	561	566	571	576	581	586	591	596
863	601	606	611	616	621	626	631	636	641	646
864	651	656	661	666	671	676	682	687	692	697
865	702	707	712	717	722	727	732	737	742	747
866	752	757	762	767	772	777	782	787	792	797
867	802	807	812	817	822	827	832	837	842	847
868	852	857	862	867	872	877	882	887	892	897
869	902	907	912	917	922	927	932	937	942	947
870	952	957	962	967	972	977	982	987	992	997
871	94 002	007	012	017	022	027	032	037	042	047
872	052	057	062	067	072	077	082	086	091	096
873	101	106	111	116	121	126	131	136	141	146
874	151	156	161	166	171	176	181	186	191	196
875	201	206	211	216	221	226	231	236	240	245
876	250	255	260	265	270	275	280	285	290	295
877	300	305	310	315	320	325	330	335	340	345
878	349	354	359	364	369	374	379	384	389	394
879	399	404	409	414	419	424	429	433	438	443
880	448	453	458	463	468	473	478	483	488	493
881	498	503	507	512	517	522	527	532	537	542
882	547	552	557	562	567	571	576	581	586	591
883	596	601	606	611	616	621	626	630	635	640
884	645	650	655	660	665	670	675	680	685	689
885	694	699	704	709	714	719	724	729	734	738
886	743	748	753	758	763	768	773	778	783	787
887	792	797	802	807	812	817	822	827	832	836
888	841	846	851	856	861	866	871	876	880	885
889	890	895	900	905	910	915	919	924	929	934
890	939	944	949	954	959	963	968	973	978	983
891	988	993	998	*002	*007	*012	*017	*022	*027	*032
892	95 036	041	046	051	056	061	066	071	075	080
893	085	090	095	100	105	109	114	119	124	129
894	134	139	143	148	153	158	163	168	173	177
895	182	187	192	197	202	207	211	216	221	226
896	231	236	240	245	250	255	260	265	270	274
897	279	284	289	294	299	303	308	313	318	323
898	328	332	337	342	347	352	357	361	366	371
899	376	381	386	390	395	400	405	410	415	419
900	424	429	434	439	444	448	453	458	463	468
N.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

Prop. Pts.

	6	5
1	0.6	0.5
2	1.2	1.0
3	1.8	1.5
4	2.4	2.0
5	3.0	2.5
6	3.6	3.0
7	4.2	3.5
8	4.8	4.0
9	5.4	4.5

	4
1	0.4
2	0.8
3	1.2
4	1.6
5	2.0
6	2.4
7	2.8
8	3.2
9	3.6

Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9
900	95 424	429	434	439	444	448	453	458	463	468
901	472	477	482	487	492	497	501	506	511	516
902	521	525	530	535	540	545	550	554	559	564
903	569	574	578	583	588	593	598	602	607	612
904	617	622	626	631	636	641	646	650	655	660
905	665	670	674	679	684	689	694	698	703	708
906	713	718	722	727	732	737	742	746	751	756
907	761	766	770	775	780	785	789	794	799	804
908	809	813	818	823	828	832	837	842	847	852
909	856	861	866	871	875	880	885	890	895	899
910	904	909	914	918	923	928	933	938	942	947
911	952	957	961	966	971	976	980	985	990	995
912	999	*004	*009	*014	*019	*023	*028	*033	*038	*042
913	96 047	052	057	061	066	071	076	080	085	090
914	095	099	104	109	114	118	123	128	133	137
915	142	147	152	156	161	166	171	175	180	185
916	190	194	199	204	209	213	218	223	227	232
917	237	242	246	251	256	261	265	270	275	280
918	284	289	294	298	303	308	313	317	322	327
919	332	336	341	346	350	355	360	365	369	374
920	379	384	388	393	398	402	407	412	417	421
921	426	431	435	440	445	450	454	459	464	468
922	473	478	483	487	492	497	501	506	511	515
923	520	525	530	534	539	544	548	553	558	562
924	567	572	577	581	586	591	595	600	605	609
925	614	619	624	628	633	638	642	647	652	656
926	661	666	670	675	680	685	689	694	699	703
927	708	713	717	722	727	731	736	741	745	750
928	755	759	764	769	774	778	783	788	792	797
929	802	806	811	816	820	825	830	834	839	844
930	848	853	858	862	867	872	876	881	886	890
931	895	900	904	909	914	918	923	928	932	937
932	942	946	951	956	960	965	970	974	979	984
933	988	993	997	*002	*007	*011	*016	*021	*025	*030
934	97 035	039	044	049	053	058	063	067	072	077
935	081	086	090	095	100	104	109	114	118	123
936	128	132	137	142	146	151	155	160	165	169
937	174	179	183	188	192	197	202	206	211	216
938	220	225	230	234	239	243	248	253	257	262
939	267	271	276	280	285	290	294	299	304	308
940	313	317	322	327	331	336	340	345	350	354
941	359	364	368	373	377	382	387	391	396	400
942	405	410	414	419	424	428	433	437	442	447
943	451	456	460	465	470	474	479	483	488	493
944	497	502	506	511	516	520	525	529	534	539
945	543	548	552	557	562	566	571	575	580	585
946	589	594	598	603	607	612	617	621	626	630
947	635	640	644	649	653	658	663	667	672	676
948	681	685	690	695	699	704	708	713	717	722
949	727	731	736	740	745	749	754	759	763	768
950	772	777	782	786	791	795	800	804	809	813
N.	0	1	2	3	4	5	6	7	8	9

Prop. Pts.

	5	4
1	0.5	0.4
2	1.0	0.8
3	1.5	1.2
4	2.0	1.6
5	2.5	2.0
6	3.0	2.4
7	3.5	2.8
8	4.0	3.2
9	4.5	3.6

Prop. Pts.

950 — Logarithms of Numbers — 1000

N.	0	1	2	3	4	5	6	7	8	9
950	97 772	777	782	786	791	795	800	804	809	813
951	818	823	827	832	836	841	845	850	855	859
952	864	868	873	877	882	886	891	896	900	905
953	909	914	918	923	928	932	937	941	946	950
954	955	959	964	968	973	978	982	987	991	996
955	98 000	005	009	014	019	023	028	032	037	041
956	046	050	055	059	064	068	073	078	082	087
957	091	096	100	105	109	114	118	123	127	132
958	137	141	146	150	155	159	164	168	173	177
959	182	186	191	195	200	204	209	214	218	223
960	227	232	236	241	245	250	254	259	263	268
961	272	277	281	286	290	295	299	304	308	313
962	318	322	327	331	336	340	345	349	354	358
963	363	367	372	376	381	385	390	394	399	403
964	408	412	417	421	426	430	435	439	444	448
965	453	457	462	466	471	475	480	484	489	493
966	498	502	507	511	516	520	525	529	534	538
967	543	547	552	556	561	565	570	574	579	583
968	588	592	597	601	605	610	614	619	623	628
969	632	637	641	646	650	655	659	664	668	673
970	677	682	686	691	695	700	704	709	713	717
971	722	726	731	735	740	744	749	753	758	762
972	767	771	776	780	784	789	793	798	802	807
973	811	816	820	825	829	834	838	843	847	851
974	856	860	865	869	874	878	883	887	892	896
975	900	905	909	914	918	923	927	932	936	941
976	945	949	954	958	963	967	972	976	981	985
977	989	994	998	*003	*007	*012	*016	*021	*025	*029
978	99 034	038	043	047	052	056	061	065	069	074
979	078	083	087	092	096	100	105	109	114	118
980	123	127	131	136	140	145	149	154	158	162
981	167	171	176	180	185	189	193	198	202	207
982	211	216	220	224	229	233	238	242	247	251
983	255	260	264	269	273	277	282	286	291	295
984	300	304	308	313	317	322	326	330	335	339
985	344	348	352	357	361	366	370	374	379	383
986	388	392	396	401	405	410	414	419	423	427
987	432	436	441	445	449	454	458	463	467	471
988	476	480	484	489	493	498	502	506	511	515
989	520	524	528	533	537	542	546	550	555	559
990	564	568	572	577	581	585	590	594	599	603
991	607	612	616	621	625	629	634	638	642	647
992	651	656	660	664	669	673	677	682	686	691
993	695	699	704	708	712	717	721	726	730	734
994	739	743	747	752	756	760	765	769	774	778
995	782	787	791	795	800	804	808	813	817	822
996	826	830	835	839	843	848	852	856	861	865
997	870	874	878	883	887	891	896	900	904	909
998	913	917	922	926	930	935	939	944	948	952
999	957	961	965	970	974	978	983	987	991	996
1000	00 000	004	009	013	017	022	026	030	035	039
N.	0	1	2	3	4	5	6	7	8	9

Prop. Pts.

	5	4
1	0.5	0.4
2	1.0	0.8
3	1.5	1.2
4	2.0	1.6
5	2.5	2.0
6	3.0	2.4
7	3.5	2.8
8	4.0	3.2
9	4.5	3.6

Prop. Pts.

CONDENSED LOGARITHMS TO FIFTEEN DECIMAL PLACES

[The first digits of n are given in the first row at the top; the last digit of n i the left-hand column. The first column of logarithms are those of 1, 2, 3, ···, (The remaining columns give log $(1 + x)$, where $x = (0.1)^k$ times 1, 2, ···, 9.]

Last Digit	First Digit of n →	1.	1.0	1.00
	Log n	First Digits of log n →	.0	.00
1	00000 00000 00000	04139 26851 58225	0432 13737 82643	043 40774 79319
2	30102 99956 63981	07918 12460 47625	0860 01717 61918	086 77215 31227
3	47712 12547 19662	11394 33523 06837	1283 72247 05172	130 09330 20418
4	60205 99913 27962	14612 80356 78238	1703 33392 98780	173 37128 09001
5	69897 00043 36019	17609 12590 55681	2118 92990 69938	216 60617 56508
6	77815 12503 83644	20411 99826 55925	2530 58652 64770	259 79807 19909
7	84509 80400 14257	23044 89213 78274	2938 37776 85210	302 94705 53618
8	90308 99869 91944	25527 25051 03306	3342 37554 86950	346 05321 09506
9	95424 25094 39325	27875 36009 52829	3742 64979 40624	389 11662 36911

(continuation)

	1.000	1.0000	1.00000	1.000000	1.0000000	1.00000000
	.000	.0000	.00000	.000000	.0000000	.00000000
1	04 34272 76863	0 43429 23104	04342 94265	0434 29446	043 42945	04 34294
2	08 68502 11649	0 86858 02780	08685 88095	0868 58888	086 85890	08 68589
3	13 02688 05227	1 30286 39028	13028 81491	1302 88325	130 28834	13 02883
4	17 36830 58465	1 73714 31850	17371 74453	1737 17758	173 71779	17 37178
5	21 70929 72230	2 17141 81245	21714 66981	2171 47187	217 14724	21 71472
6	26 04985 47390	2 60568 87215	26057 59074	2605 76611	260 57668	26 05767
7	30 38997 84812	3 03995 49761	30400 50733	3040 06031	304 00613	30 40061
8	34 72966 85364	3 47421 68884	34743 41958	3474 35447	347 43557	34 74356
9	39 06892 49910	3 90847 44584	39086 32748	3908 64858	390 86502	39 08650

[For $x < .00000001$, log $(1 + x) = x \cdot M$, to within 3 in the 17th place, where $M = 0.43429448 \cdots$. Hence the last column gives multiples of M except for the decimal place. All the columns that would follow have the same significant digits displaced each time one place.]

CONDENSED ANTILOGARITHMS TO TEN DECIMAL PLACES

[The first digits of n are given in the first row at the top; $n = (0.1)^k x$; $x = 1, 2,$ 3, ···, 9 are given in the left-hand column. The first digits in 10^n are given in the second row at the top.]

x	$n = 0.1x$	$0.01x$	$0.001x$	$0.0001x$	$(0.1)^5x$	$(0.1)^6x$	$(0.1)^7x$
	10^n	1.	1.0	1.00	1.000	1.0000	1.00000
1	1.25892 54118	02329 29923	0230 52381	023 02850	02 30261	0 23026	02303
2	1.58489 31925	04712 85481	0461 57903	046 06231	04 60528	0 46052	04605
3	1.99526 23150	07151 93052	0693 16689	069 10142	06 90799	0 69078	06908
4	2.51188 64315	09647 81961	0925 28861	092 14583	09 21076	0 92104	09210
5	3.16227 76602	12201 84543	1157 94543	115 19555	11 51359	1 15130	11513
6	3.98107 17055	14815 36215	1391 13857	138 25058	13 81646	1 38156	13816
7	5.01187 23363	17489 75549	1624 86929	161 31092	16 11939	1 61182	16118
8	6.30957 34448	20226 44346	1859 13881	184 37657	18 42238	1 84209	18421
9	7.94328 23472	23026 87708	2093 94837	207 44753	20 72541	2 07235	20723

[For $n < 0.000001$, $10^n = 1 + n \cdot (1/M)$ to within 3 in the 12th decimal place, where $(1/M) = 2.302585 \cdots$. Hence the last column gives multiples of $(1/M)$ except for the decimal place. All the columns that would follow contain the same significant digits displaced one place for each new column.]

TABLE II

ACTUAL VALUES

OF THE

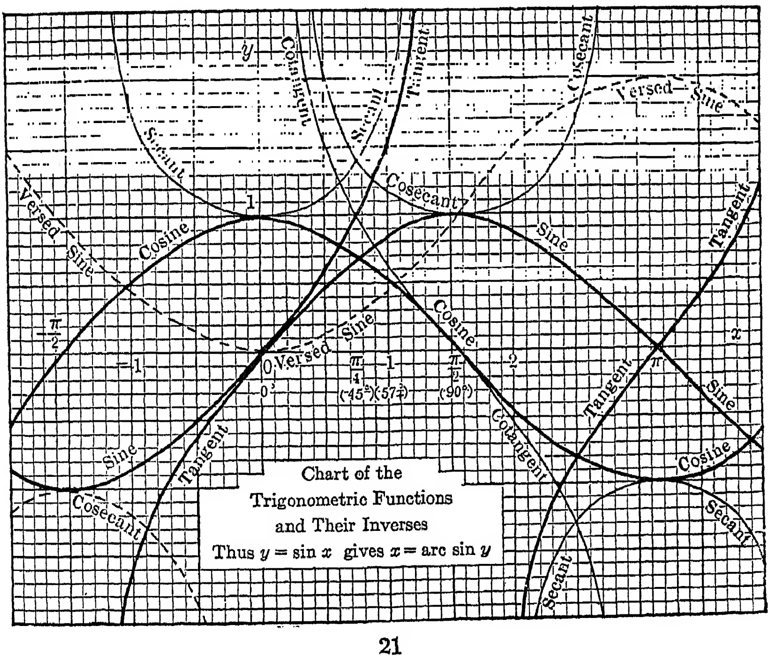
TRIGONOMETRIC FUNCTIONS

FROM

0° TO 90° AT INTERVALS OF ONE MINUTE

TO

FIVE DECIMAL PLACES



′	Sin	Tan	Ctn	Cos	
0	.00000	.00000	——	1.0000	60
1	029	029	3437.7	000	59
2	058	058	1718.9	000	58
3	087	087	1145.9	000	57
4	116	116	859.44	000	56
5	.00145	.00145	687.55	1.0000	55
6	175	175	572.96	000	54
7	204	204	491.11	000	53
8	233	233	429.72	000	52
9	262	262	381.97	000	51
10	.00291	.00291	343.77	1.0000	50
11	320	320	312.52	.99999	49
12	349	349	286.48	999	48
13	378	378	264.44	999	47
14	407	407	245.55	999	46
15	.00436	.00436	229.18	.99999	45
16	465	465	214.86	999	44
17	495	495	202.22	999	43
18	524	524	190.98	999	42
19	553	553	180.93	998	41
20	.00582	.00582	171.89	.99998	40
21	611	611	163.70	998	39
22	640	640	156.26	998	38
23	669	669	149.47	998	37
24	698	698	143.24	998	36
25	.00727	.00727	137.51	.99997	35
26	756	756	132.22	997	34
27	785	785	127.32	997	33
28	814	815	122.77	997	32
29	844	844	118.54	996	31
30	.00873	.00873	114.59	.99996	30
31	902	902	110.89	996	29
32	931	931	107.43	996	28
33	960	960	104.17	995	27
34	.00989	.00989	101.11	995	26
35	.01018	.01018	98.218	.99995	25
36	047	047	95.489	995	24
37	076	076	92.908	994	23
38	105	105	90.463	994	22
39	134	135	88.144	994	21
40	.01164	.01164	85.940	.99993	20
41	193	193	83.844	993	19
42	222	222	81.847	993	18
43	251	251	79.943	992	17
44	280	280	78.126	992	16
45	.01309	.01309	76.390	.99991	15
46	338	338	74.729	991	14
47	367	367	73.139	991	13
48	396	396	71.615	990	12
49	425	425	70.153	990	11
50	.01454	.01455	68.750	.99989	10
51	483	484	67.402	989	9
52	513	513	66.105	989	8
53	542	542	64.858	988	7
54	571	571	63.657	988	6
55	.01600	.01600	62.499	.99987	5
56	629	629	61.383	987	4
57	658	658	60.306	986	3
58	687	687	59.266	986	2
59	716	716	58.261	985	1
60	.01745	.01746	57.290	.99985	0
	Cos	Ctn	Tan	Sin	′

89°

′	Sin	Tan	Ctn	Cos	
0	.01745	.01746	57.290	.99985	60
1	774	775	56.351	984	59
2	803	804	55.442	984	58
3	832	833	54.561	983	57
4	862	862	53.709	983	56
5	.01891	.01891	52.882	.99982	55
6	920	920	52.081	982	54
7	949	949	51.303	981	53
8	.01978	.01978	50.549	980	52
9	.02007	.02007	49.816	980	51
10	.02036	.02036	49.104	.99979	50
11	065	066	48.412	979	49
12	094	095	47.740	978	48
13	123	124	47.085	977	47
14	152	153	46.449	977	46
15	.02181	.02182	45.829	.99976	45
16	211	211	45.226	976	44
17	240	240	44.639	975	43
18	269	269	44.066	974	42
19	298	298	43.508	974	41
20	.02327	.02328	42.964	.99973	40
21	356	357	42.433	972	39
22	385	386	41.916	972	38
23	414	415	41.411	971	37
24	443	444	40.917	970	36
25	.02472	.02473	40.436	.99969	35
26	501	502	39.965	969	34
27	530	531	39.506	968	33
28	560	560	39.057	967	32
29	589	589	38.618	966	31
30	.02618	.02619	38.188	.99966	30
31	647	648	37.769	965	29
32	676	677	37.358	964	28
33	705	706	36.956	963	27
34	734	735	36.563	963	26
35	.02763	.02764	36.178	.99962	25
36	792	793	35.801	961	24
37	821	822	35.431	960	23
38	850	851	35.070	959	22
39	879	881	34.715	959	21
40	.02908	.02910	34.368	.99958	20
41	938	939	34.027	957	19
42	967	968	33.694	956	18
43	.02996	.02997	33.366	955	17
44	.03025	.03026	33.045	954	16
45	.03054	.03055	32.730	.99953	15
46	083	084	32.421	952	14
47	112	114	32.118	952	13
48	141	143	31.821	951	12
49	170	172	31.528	950	11
50	.03199	.03201	31.242	.99949	10
51	228	230	30.960	948	9
52	257	259	30.683	947	8
53	286	288	30.412	946	7
54	316	317	30.145	945	6
55	.03345	.03346	29.882	.99944	5
56	374	376	29.624	943	4
57	403	405	29.371	942	3
58	432	434	29.122	941	2
59	461	463	28.877	940	1
60	.03490	.03492	28.636	.99939	0
	Cos	Ctn	Tan	Sin	′

88°

′	Sin	Tan	Ctn	Cos	
0	.03490	.03492	28.636	.99939	60
1	519	521	.399	938	59
2	548	550	28.166	937	58
3	577	579	27.937	936	57
4	606	609	.712	935	56
5	.03635	.03638	27.490	.99934	55
6	664	667	.271	933	54
7	693	696	27.057	932	53
8	723	725	26.845	931	52
9	752	754	.637	930	51
10	.03781	.03783	26.432	.99929	50
11	810	812	.230	927	49
12	839	842	26.031	926	48
13	868	871	25.835	925	47
14	897	900	.642	924	46
15	.03926	.03929	25.452	.99923	45
16	955	958	.264	922	44
17	.03984	.03987	25.080	921	43
18	.04013	.04016	24.898	919	42
19	042	046	.719	918	41
20	.04071	.04075	24.542	.99917	40
21	100	104	.368	916	39
22	129	133	.196	915	38
23	159	162	24.026	913	37
24	188	191	23.859	912	36
25	.04217	.04220	23.695	.99911	35
26	246	250	.532	910	34
27	275	279	.372	909	33
28	304	308	.214	907	32
29	333	337	23.058	906	31
30	.04362	.04366	22.904	.99905	30
31	391	395	.752	904	29
32	420	424	.602	902	28
33	449	454	.454	901	27
34	478	483	.308	900	26
35	.04507	.04512	22.164	.99898	25
36	536	541	22.022	897	24
37	565	570	21.881	896	23
38	594	599	.743	894	22
39	623	628	.606	893	21
40	.04653	.04658	21.470	.99892	20
41	682	687	.337	890	19
42	711	716	.205	889	18
43	740	745	21.075	888	17
44	769	774	20.946	886	16
45	.04798	.04803	20.819	.99885	15
46	827	833	.693	883	14
47	856	862	.569	882	13
48	885	891	.446	881	12
49	914	920	.325	879	11
50	.04943	.04949	20.206	.99878	10
51	.04972	.04978	20.087	876	9
52	.05001	.05007	19.970	875	8
53	030	037	.855	873	7
54	059	066	.740	872	6
55	.05088	.05095	19.627	.99870	5
56	117	124	.516	869	4
57	146	153	.405	867	3
58	175	182	.296	866	2
59	205	212	.188	864	1
60	.05234	.05241	19.081	.99863	0
	Cos	Ctn	Tan	Sin	′

87°

′	Sin	Tan	Ctn	Cos	
0	.05234	.05241	19.081	.99863	60
1	263	270	18.976	861	59
2	292	299	.871	860	58
3	321	328	.768	858	57
4	350	357	.666	857	56
5	.05379	.05387	18.564	.99855	55
6	408	416	.464	854	54
7	437	445	.366	852	53
8	466	474	.268	851	52
9	495	503	.171	849	51
10	.05524	.05533	18.075	.99847	50
11	553	562	17.980	846	49
12	582	591	.886	844	48
13	611	620	.793	842	47
14	640	649	.702	841	46
15	.05669	.05678	17.611	.99839	45
16	698	708	.521	838	44
17	727	737	.431	836	43
18	756	766	.343	834	42
19	785	795	.256	833	41
20	.05814	.05824	17.169	.99831	40
21	844	854	17.084	829	39
22	873	883	16.999	827	38
23	902	912	.915	826	37
24	931	941	.832	824	36
25	.05960	.05970	16.750	.99822	35
26	.05989	.05999	.668	821	34
27	.06018	.06029	.587	819	33
28	047	058	.507	817	32
29	076	087	.428	815	31
30	.06105	.06116	16.350	.99813	30
31	134	145	.272	812	29
32	163	175	.195	810	28
33	192	204	.119	808	27
34	221	233	16.043	806	26
35	.06250	.06262	15.969	.99804	25
36	279	291	.895	803	24
37	308	321	.821	801	23
38	337	350	.748	799	22
39	366	379	.676	797	21
40	.06395	.06408	15.605	.99795	20
41	424	438	.534	793	19
42	453	467	.464	792	18
43	482	496	.394	790	17
44	511	525	.325	788	16
45	.06540	.06554	15.257	.99786	15
46	569	584	.189	784	14
47	598	613	.122	782	13
48	627	642	15.056	780	12
49	656	671	14.990	778	11
50	.06685	.06700	14.924	.99776	10
51	714	730	.860	774	9
52	743	759	.795	772	8
53	773	788	.732	770	7
54	802	817	.669	768	6
55	.06831	.06847	14.606	.99766	5
56	860	876	.544	764	4
57	889	905	.482	762	3
58	918	934	.421	760	2
59	947	963	.361	758	1
60	.06976	.06993	14.301	.99756	0
	Cos	Ctn	Tan	Sin	′

86°

′	Sin	Tan	Ctn	Cos	
0	.06976	.06993	14.301	.99756	60
1	.07005	.07022	.241	754	59
2	034	051	.182	752	58
3	063	080	.124	750	57
4	092	110	.065	748	56
5	.07121	.07139	14.008	.99746	55
6	150	168	13.951	744	54
7	179	197	.894	742	53
8	208	227	.838	740	52
9	237	256	.782	738	51
10	.07266	.07285	13.727	.99736	50
11	295	314	.672	734	49
12	324	344	.617	731	48
13	353	373	.563	729	47
14	382	402	.510	727	46
15	.07411	.07431	13.457	.99725	45
16	440	461	.404	723	44
17	469	490	.352	721	43
18	498	519	.300	719	42
19	527	548	.248	716	41
20	.07556	.07578	13.197	.99714	40
21	585	607	.146	712	39
22	614	636	.096	710	38
23	643	665	13.046	708	37
24	672	695	12.996	705	36
25	.07701	.07724	12.947	.99703	35
26	730	753	.898	701	34
27	759	782	.850	699	33
28	788	812	.801	696	32
29	817	841	.754	694	31
30	.07846	.07870	12.706	.99692	30
31	875	899	.659	689	29
32	904	929	.612	687	28
33	933	958	.566	685	27
34	962	.07987	.520	683	26
35	.07991	.08017	12.474	.99680	25
36	.08020	046	.429	678	24
37	049	075	.384	676	23
38	078	104	.339	673	22
39	107	134	.295	671	21
40	.08136	.08163	12.251	.99668	20
41	165	192	.207	666	19
42	194	221	.163	664	18
43	223	251	.120	661	17
44	252	280	.077	659	16
45	.08281	.08309	12.035	.99657	15
46	310	339	11.992	654	14
47	339	368	.950	652	13
48	368	397	.909	649	12
49	397	427	.867	647	11
50	.08426	.08456	11.826	.99644	10
51	455	485	.785	642	9
52	484	514	.745	639	8
53	513	544	.705	637	7
54	542	573	.664	635	6
55	.08571	.08602	11.625	.99632	5
56	600	632	.585	630	4
57	629	661	.546	627	3
58	658	690	.507	625	2
59	687	720	.468	622	1
60	.08716	.08749	11.430	.99619	0
	Cos	Ctn	Tan	Sin	′

85°

′	Sin	Tan	Ctn	Cos	
0	.08716	.08749	11.430	.99619	60
1	745	778	.392	617	59
2	774	807	.354	614	58
3	803	837	.316	612	57
4	831	866	.279	609	56
5	.08860	.08895	11.242	.99607	55
6	889	925	.205	604	54
7	918	954	.168	602	53
8	947	.08983	.132	599	52
9	.08976	.09013	.095	596	51
10	.09005	.09042	11.059	.99594	50
11	034	071	11.024	591	49
12	063	101	10.988	588	48
13	092	130	.953	586	47
14	121	159	.918	583	46
15	.09150	.09189	10.883	.99580	45
16	179	218	.848	578	44
17	208	247	.814	575	43
18	237	277	.780	572	42
19	266	306	.746	570	41
20	.09295	.09335	10.712	.99567	40
21	324	365	.678	564	39
22	353	394	.645	562	38
23	382	423	.612	559	37
24	411	453	.579	556	36
25	.09440	.09482	10.546	.99553	35
26	469	511	.514	551	34
27	498	541	.481	548	33
28	527	570	.449	545	32
29	556	600	.417	542	31
30	.09585	.09629	10.385	.99540	30
31	614	658	.354	537	29
32	642	688	.322	534	28
33	671	717	.291	531	27
34	700	746	.260	528	26
35	.09729	.09776	10.229	.99526	25
36	758	805	.199	523	24
37	787	834	.168	520	23
38	816	864	.138	517	22
39	845	893	.108	514	21
40	.09874	.09923	10.078	.99511	20
41	903	952	.048	508	19
42	932	.09981	10.019	506	18
43	961	.10011	9.9893	503	17
44	.09990	040	.9601	500	16
45	.10019	.10069	9.9310	.99497	15
46	048	099	.9021	494	14
47	077	128	.8734	491	13
48	106	158	.8448	488	12
49	135	187	.8164	485	11
50	.10164	.10216	9.7882	.99482	10
51	192	246	.7601	479	9
52	221	275	.7322	476	8
53	250	305	.7044	473	7
54	279	334	.6768	470	6
55	.10308	.10363	9.6493	.99467	5
56	337	393	.6220	464	4
57	366	422	.5949	461	3
58	395	452	.5679	458	2
59	424	481	.5411	455	1
60	.10453	.10510	9.5144	.99452	0
	Cos	Ctn	Tan	Sin	′

84°

′	Sin	Tan	Ctn	Cos	
0	.10453	.10510	9.5144	.99452	60
1	482	540	.4878	449	59
2	511	569	.4614	446	58
3	540	599	.4352	443	57
4	569	628	.4090	440	56
5	.10597	.10657	9.3831	.99437	55
6	626	687	.3572	434	54
7	655	716	.3315	431	53
8	684	746	.3060	428	52
9	713	775	.2806	424	51
10	.10742	.10805	9.2553	.99421	50
11	771	834	.2302	418	49
12	800	863	.2052	415	48
13	829	893	.1803	412	47
14	858	922	.1555	409	46
15	.10887	.10952	9.1309	.99406	45
16	916	.10981	.1065	402	44
17	945	.11011	.0821	399	43
18	.10973	040	.0579	396	42
19	.11002	070	.0338	393	41
20	.11031	.11099	9.0098	.99390	40
21	060	128	8.9860	386	39
22	089	158	.9623	383	38
23	118	187	.9387	380	37
24	147	217	.9152	377	36
25	.11176	.11246	8.8919	.99374	35
26	205	276	.8686	370	34
27	234	305	.8455	367	33
28	263	335	.8225	364	32
29	291	364	.7996	360	31
30	.11320	.11394	8.7769	.99357	30
31	349	423	.7542	354	29
32	378	452	.7317	351	28
33	407	482	.7093	347	27
34	436	511	.6870	344	26
35	.11465	.11541	8.6648	.99341	25
36	494	570	.6427	337	24
37	523	600	.6208	334	23
38	552	629	.5989	331	22
39	580	659	.5772	327	21
40	.11609	.11688	8.5555	.99324	20
41	638	718	.5340	320	19
42	667	747	.5126	317	18
43	696	777	.4913	314	17
44	725	806	.4701	310	16
45	.11754	.11836	8.4490	.99307	15
46	783	865	.4280	303	14
47	812	895	.4071	300	13
48	840	924	.3863	297	12
49	869	954	.3656	293	11
50	.11898	.11983	8.3450	.99290	10
51	927	.12013	.3245	286	9
52	956	042	.3041	283	8
53	.11985	072	.2838	279	7
54	.12014	101	.2636	276	6
55	.12043	.12131	8.2434	.99272	5
56	071	160	.2234	269	4
57	100	190	.2035	265	3
58	129	219	.1837	262	2
59	158	249	.1640	258	1
60	.12187	.12278	8.1443	.99255	0
	Cos	Ctn	Tan	Sin	′

83°

′	Sin	Tan	Ctn	Cos	
0	.12187	.12278	8.1443	.99255	60
1	216	308	.1248	251	59
2	245	338	.1054	248	58
3	274	367	.0860	244	57
4	302	397	.0667	240	56
5	.12331	.12426	8.0476	.99237	55
6	360	456	.0285	233	54
7	389	485	8.0095	230	53
8	418	515	7.9906	226	52
9	447	544	.9718	222	51
10	.12476	.12574	7.9530	.99219	50
11	504	603	.9344	215	49
12	533	633	.9158	211	48
13	562	662	.8973	208	47
14	591	692	.8789	204	46
15	.12620	.12722	7.8606	.99200	45
16	649	751	.8424	197	44
17	678	781	.8243	193	43
18	706	810	.8062	189	42
19	735	840	.7882	186	41
20	.12764	.12869	7.7704	.99182	40
21	793	899	.7525	178	39
22	822	929	.7348	175	38
23	851	958	.7171	171	37
24	880	.12988	.6996	167	36
25	.12908	.13017	7.6821	.99163	35
26	937	047	.6647	160	34
27	966	076	.6473	156	33
28	.12995	106	.6301	152	32
29	.13024	136	.6129	148	31
30	.13053	.13165	7.5958	.99144	30
31	081	195	.5787	141	29
32	110	224	.5618	137	28
33	139	254	.5449	133	27
34	168	284	.5281	129	26
35	.13197	.13313	7.5113	.99125	25
36	226	343	.4947	122	24
37	254	372	.4781	118	23
38	283	402	.4615	114	22
39	312	432	.4451	110	21
40	.13341	.13461	7.4287	.99106	20
41	370	491	.4124	102	19
42	399	521	.3962	098	18
43	427	550	.3800	094	17
44	456	580	.3639	091	16
45	.13485	.13609	7.3479	.99087	15
46	514	639	.3319	083	14
47	543	669	.3160	079	13
48	572	698	.3002	075	12
49	600	728	.2844	071	11
50	.13629	.13758	7.2687	.99067	10
51	658	787	.2531	063	9
52	687	817	.2375	059	8
53	716	846	.2220	055	7
54	744	876	.2066	051	6
55	.13773	.13906	7.1912	.99047	5
56	802	935	.1759	043	4
57	831	965	.1607	039	3
58	860	.13995	.1455	035	2
59	889	.14024	.1304	031	1
60	.13917	.14054	7.1154	.99027	0
	Cos	Ctn	Tan	Sin	′

82°

′	Sin	Tan	Ctn	Cos	
0	.13917	.14054	7.1154	.99027	60
1	946	084	.1004	023	59
2	.13975	113	.0855	019	58
3	.14004	143	.0706	015	57
4	033	173	.0558	011	56
5	.14061	.14202	7.0410	.99006	55
6	090	232	.0264	.99002	54
7	119	262	7.0117	.98998	53
8	148	291	6.9972	994	52
9	177	321	.9827	990	51
10	.14205	.14351	6.9682	.98986	50
11	234	381	.9538	982	49
12	263	410	.9395	978	48
13	292	440	.9252	973	47
14	320	470	.9110	969	46
15	.14349	.14499	6.8969	.98965	45
16	378	529	.8828	961	44
17	407	559	.8687	957	43
18	436	588	.8548	953	42
19	464	618	.8408	948	41
20	.14493	.14648	6.8269	.98944	40
21	522	678	.8131	940	39
22	551	707	.7994	936	38
23	580	737	.7856	931	37
24	608	767	.7720	927	36
25	.14637	.14796	6.7584	.98923	35
26	666	826	.7448	919	34
27	695	856	.7313	914	33
28	723	886	.7179	910	32
29	752	915	.7045	906	31
30	.14781	.14945	6.6912	.98902	30
31	810	.14975	.6779	897	29
32	838	.15005	.6646	893	28
33	867	034	.6514	889	27
34	896	064	.6383	884	26
35	.14925	.15094	6.6252	.98880	25
36	954	124	.6122	876	24
37	.14982	153	.5992	871	23
38	.15011	183	.5863	867	22
39	040	213	.5734	863	21
40	.15069	.15243	6.5606	.98858	20
41	097	272	.5478	854	19
42	126	302	.5350	849	18
43	155	332	.5223	845	17
44	184	362	.5097	841	16
45	.15212	.15391	6.4971	.98836	15
46	241	421	.4846	832	14
47	270	451	.4721	827	13
48	299	481	.4596	823	12
49	327	511	.4472	818	11
50	.15356	.15540	6.4348	.98814	10
51	385	570	.4225	809	9
52	414	600	.4103	805	8
53	442	630	.3980	800	7
54	471	660	.3859	796	6
55	.15500	.15689	6.3737	.98791	5
56	529	719	.3617	787	4
57	557	749	.3496	782	3
58	586	779	.3376	778	2
59	615	809	.3257	773	1
60	.15643	.15838	6.3138	.98769	0
	Cos	Ctn	Tan	Sin	′

81°

′	Sin	Tan	Ctn	Cos	
0	.15643	.15838	6.3138	.98769	60
1	672	868	.3019	764	59
2	701	898	.2901	760	58
3	730	928	.2783	755	57
4	758	958	.2666	751	56
5	.15787	.15988	6.2549	.98746	55
6	816	.16017	.2432	741	54
7	845	047	.2316	737	53
8	873	077	.2200	732	52
9	902	107	.2085	728	51
10	.15931	.16137	6.1970	.98723	50
11	959	167	.1856	718	49
12	.15988	196	.1742	714	48
13	.16017	226	.1628	709	47
14	046	256	.1515	704	46
15	.16074	.16286	6.1402	.98700	45
16	103	316	.1290	695	44
17	132	346	.1178	690	43
18	160	376	.1066	686	42
19	189	405	.0955	681	41
20	.16218	.16435	6.0844	.98676	40
21	246	465	.0734	671	39
22	275	495	.0624	667	38
23	304	525	.0514	662	37
24	333	555	.0405	657	36
25	.16361	.16585	6.0296	.98652	35
26	390	615	.0188	648	34
27	419	645	6.0080	643	33
28	447	674	5.9972	638	32
29	476	704	.9865	633	31
30	.16505	.16734	5.9758	.98629	30
31	533	764	.9651	624	29
32	562	794	.9545	619	28
33	591	824	.9439	614	27
34	620	854	.9333	609	26
35	.16648	.16884	5.9228	.98604	25
36	677	914	.9124	600	24
37	706	944	.9019	595	23
38	734	.16974	.8915	590	22
39	763	.17004	.8811	585	21
40	.16792	.17033	5.8708	.98580	20
41	820	063	.8605	575	19
42	849	093	.8502	570	18
43	878	123	.8400	565	17
44	906	153	.8298	561	16
45	.16935	.17183	5.8197	.98556	15
46	964	213	.8095	551	14
47	.16992	243	.7994	546	13
48	.17021	273	.7894	541	12
49	050	303	.7794	536	11
50	.17078	.17333	5.7694	.98531	10
51	107	363	.7594	526	9
52	136	393	.7495	521	8
53	164	423	.7396	516	7
54	193	453	.7297	511	6
55	.17222	.17483	5.7199	.98506	5
56	250	513	.7101	501	4
57	279	543	.7004	496	3
58	308	573	.6906	491	2
59	336	603	.6809	486	1
60	.17365	.17633	5.6713	.98481	0
	Cos	Ctn	Tan	Sin	′

80°

′	Sin	Tan	Ctn	Cos	
0	.17365	.17633	5.6713	.98481	**60**
1	393	663	.6617	476	59
2	422	693	.6521	471	58
3	451	723	.6425	466	57
4	479	753	.6329	461	56
5	.17508	.17783	5.6234	.98455	**55**
6	537	813	.6140	450	54
7	565	843	.6045	445	53
8	594	873	.5951	440	52
9	623	903	.5857	435	51
10	.17651	.17933	5.5764	.98430	**50**
11	680	963	.5671	425	49
12	708	.17993	.5578	420	48
13	737	.18023	.5485	414	47
14	766	053	.5393	409	46
15	.17794	.18083	5.5301	.98404	**45**
16	823	113	.5209	399	44
17	852	143	.5118	394	43
18	880	173	.5026	389	42
19	909	203	.4936	383	41
20	.17937	.18233	5.4845	.98378	**40**
21	966	263	.4755	373	39
22	.17995	293	.4665	368	38
23	.18023	323	.4575	362	37
24	052	353	.4486	357	36
25	.18081	.18384	5.4397	.98352	**35**
26	109	414	.4308	347	34
27	138	444	.4219	341	33
28	166	474	.4131	336	32
29	195	504	.4043	331	31
30	.18224	.18534	5.3955	.98325	**30**
31	252	564	.3868	320	29
32	281	594	.3781	315	28
33	309	624	.3694	310	27
34	338	654	.3607	304	26
35	.18367	.18684	5.3521	.98299	**25**
36	395	714	.3435	294	24
37	424	745	.3349	288	23
38	452	775	.3263	283	22
39	481	805	.3178	277	21
40	.18509	.18835	5.3093	.98272	**20**
41	538	865	.3008	267	19
42	567	895	.2924	261	18
43	595	925	.2839	256	17
44	624	955	.2755	250	16
45	.18652	.18986	5.2672	.98245	**15**
46	681	.19016	.2588	240	14
47	710	046	.2505	234	13
48	738	076	.2422	229	12
49	767	106	.2339	223	11
50	.18795	.19136	5.2257	.98218	**10**
51	824	166	.2174	212	9
52	852	197	.2092	207	8
53	881	227	.2011	201	7
54	910	257	.1929	196	6
55	.18938	.19287	5.1848	.98190	**5**
56	967	317	.1767	185	4
57	.18995	347	.1686	179	3
58	.19024	378	.1606	174	2
59	052	408	.1526	168	1
60	.19081	.19438	5.1446	.98163	**0**
	Cos	**Ctn**	**Tan**	**Sin**	′

79°

′	Sin	Tan	Ctn	Cos	
0	.19081	.19438	5.1446	.98163	**60**
1	109	468	.1366	157	59
2	138	498	.1286	152	58
3	167	529	.1207	146	57
4	195	559	.1128	140	56
5	.19224	.19589	5.1049	.98135	**55**
6	252	619	.0970	129	54
7	281	649	.0892	124	53
8	309	680	.0814	118	52
9	338	710	.0736	112	51
10	.19366	.19740	5.0658	.98107	**50**
11	395	770	.0581	101	49
12	423	801	.0504	096	48
13	452	831	.0427	090	47
14	481	861	.0350	084	46
15	.19509	.19891	5.0273	.98079	**45**
16	538	921	.0197	073	44
17	566	952	.0121	067	43
18	595	.19982	5.0045	061	42
19	623	.20012	4.9969	056	41
20	.19652	.20042	4.9894	.98050	**40**
21	680	073	.9819	044	39
22	709	103	.9744	039	38
23	737	133	.9669	033	37
24	766	164	.9594	027	36
25	.19794	.20194	4.9520	.98021	**35**
26	823	224	.9446	016	34
27	851	254	.9372	010	33
28	880	285	.9298	.98004	32
29	908	315	.9225	.97998	31
30	.19937	.20345	4.9152	.97992	**30**
31	965	376	.9078	987	29
32	.19994	406	.9006	981	28
33	.20022	436	.8933	975	27
34	051	466	.8860	969	26
35	.20079	.20497	4.8788	.97963	**25**
36	108	527	.8716	958	24
37	136	557	.8644	952	23
38	165	588	.8573	946	22
39	193	618	.8501	940	21
40	.20222	.20648	4.8430	.97934	**20**
41	250	679	.8359	928	19
42	279	709	.8288	922	18
43	307	739	.8218	916	17
44	336	770	.8147	910	16
45	.20364	.20800	4.8077	.97905	**15**
46	393	830	.8007	899	14
47	421	861	.7937	893	13
48	450	891	.7867	887	12
49	478	921	.7798	881	11
50	.20507	.20952	4.7729	.97875	**10**
51	535	.20982	.7659	869	9
52	563	.21013	.7591	863	8
53	592	043	.7522	857	7
54	620	073	.7453	851	6
55	.20649	.21104	4.7385	.97845	**5**
56	677	134	.7317	839	4
57	706	164	.7249	833	3
58	734	195	.7181	827	2
59	763	225	.7114	821	1
60	.20791	.21256	4.7046	.97815	**0**
	Cos	**Ctn**	**Tan**	**Sin**	′

78°

′	Sin	Tan	Ctn	Cos	
0	.20791	.21256	4.7046	.97815	60
1	820	286	.6979	809	59
2	848	316	.6912	803	58
3	877	347	.6845	797	57
4	905	377	.6779	791	56
5	.20933	.21408	4.6712	.97784	55
6	962	438	.6646	778	54
7	.20990	469	.6580	772	53
8	.21019	499	.6514	766	52
9	047	529	.6448	760	51
10	.21076	.21560	4.6382	.97754	50
11	104	590	.6317	748	49
12	132	621	.6252	742	48
13	161	651	.6187	735	47
14	189	682	.6122	729	46
15	.21218	.21712	4.6057	.97723	45
16	246	743	.5993	717	44
17	275	773	.5928	711	43
18	303	804	.5864	705	42
19	331	834	.5800	698	41
20	.21360	.21864	4.5736	.97692	40
21	388	895	.5673	686	39
22	417	925	.5609	680	38
23	445	956	.5546	673	37
24	474	.21986	.5483	667	36
25	.21502	.22017	4.5420	.97661	35
26	530	047	.5357	655	34
27	559	078	.5294	648	33
28	587	108	.5232	642	32
29	616	139	.5169	636	31
30	.21644	.22169	4.5107	.97630	30
31	672	200	.5045	623	29
32	701	231	.4983	617	28
33	729	261	.4922	611	27
34	758	292	.4860	604	26
35	.21786	.22322	4.4799	.97598	25
36	814	353	.4737	592	24
37	843	383	.4676	585	23
38	871	414	.4615	579	22
39	899	444	.4555	573	21
40	.21928	.22475	4.4494	.97566	20
41	956	505	.4434	560	19
42	.21985	536	.4373	553	18
43	.22013	567	.4313	547	17
44	041	597	.4253	541	16
45	.22070	.22628	4.4194	.97534	15
46	098	658	.4134	528	14
47	126	689	.4075	521	13
48	155	719	.4015	515	12
49	183	750	.3956	508	11
50	.22212	.22781	4.3897	.97502	10
51	240	811	.3838	496	9
52	268	842	.3779	489	8
53	297	872	.3721	483	7
54	325	903	.3662	476	6
55	.22353	.22934	4.3604	.97470	5
56	382	964	.3546	463	4
57	410	.22995	.3488	457	3
58	438	.23026	.3430	450	2
59	467	056	.3372	444	1
60	.22495	.23087	4.3315	.97437	0
	Cos	Ctn	Tan	Sin	′

77°

′	Sin	Tan	Ctn	Cos	
0	.22495	.23087	4.3315	.97437	60
1	523	117	.3257	430	59
2	552	148	.3200	424	58
3	580	179	.3143	417	57
4	608	209	.3086	411	56
5	.22637	.23240	4.3029	.97404	55
6	665	271	.2972	398	54
7	693	301	.2916	391	53
8	722	332	.2859	384	52
9	750	363	.2803	378	51
10	.22778	.23393	4.2747	.97371	50
11	807	424	.2691	365	49
12	835	455	.2635	358	48
13	863	485	.2580	351	47
14	892	516	.2524	345	46
15	.22920	.23547	4.2468	.97338	45
16	948	578	.2413	331	44
17	.22977	608	.2358	325	43
18	.23005	639	.2303	318	42
19	033	670	.2248	311	41
20	.23062	.23700	4.2193	.97304	40
21	090	731	.2139	298	39
22	118	762	.2084	291	38
23	146	793	.2030	284	37
24	175	823	.1976	278	36
25	.23203	.23854	4.1922	.97271	35
26	231	885	.1868	264	34
27	260	916	.1814	257	33
28	288	946	.1760	251	32
29	316	.23977	.1706	244	31
30	.23345	.24008	4.1653	.97237	30
31	373	039	.1600	230	29
32	401	069	.1547	223	28
33	429	100	.1493	217	27
34	458	131	.1441	210	26
35	.23486	.24162	4.1388	.97203	25
36	514	193	.1335	196	24
37	542	223	.1282	189	23
38	571	254	.1230	182	22
39	599	285	.1178	176	21
40	.23627	.24316	4.1126	.97169	20
41	656	347	.1074	162	19
42	684	377	.1022	155	18
43	712	408	.0970	148	17
44	740	439	.0918	141	16
45	.23769	.24470	4.0867	.97134	15
46	797	501	.0815	127	14
47	825	532	.0764	120	13
48	853	562	.0713	113	12
49	882	593	.0662	106	11
50	.23910	.24624	4.0611	.97100	10
51	938	655	.0560	093	9
52	966	686	.0509	086	8
53	.23995	717	.0459	079	7
54	.24023	747	.0408	072	6
55	.24051	.24778	4.0358	.97065	5
56	079	809	.0308	058	4
57	108	840	.0257	051	3
58	136	871	.0207	044	2
59	164	902	.0158	037	1
60	.24192	.24933	4.0108	.97030	0
	Cos	Ctn	Tan	Sin	′

76°

′	Sin	Tan	Ctn	Cos	
0	.24192	.24933	4.0108	.97030	60
1	220	964	.0058	023	59
2	249	.24995	4.0009	015	58
3	277	.25026	3.9959	008	57
4	305	056	.9910	.97001	56
5	.24333	.25087	3.9861	.96994	55
6	362	118	.9812	987	54
7	390	149	.9763	980	53
8	418	180	.9714	973	52
9	446	211	.9665	966	51
10	.24474	.25242	3.9617	.96959	50
11	503	273	.9568	952	49
12	531	304	.9520	945	48
13	559	335	.9471	937	47
14	587	366	.9423	930	46
15	.24615	.25397	3.9375	.96923	45
16	644	428	.9327	916	44
17	672	459	.9279	909	43
18	700	490	.9232	902	42
19	728	521	.9184	894	41
20	.24756	.25552	3.9136	.96887	40
21	784	583	.9089	880	39
22	813	614	.9042	873	38
23	841	645	.8995	866	37
24	869	676	.8947	858	36
25	.24897	.25707	3.8900	.96851	35
26	925	738	.8854	844	34
27	954	769	.8807	837	33
28	.24982	800	.8760	829	32
29	.25010	831	.8714	822	31
30	.25038	.25862	3.8667	.96815	30
31	066	893	.8621	807	29
32	094	924	.8575	800	28
33	122	955	.8528	793	27
34	151	.25986	.8482	786	26
35	.25179	.26017	3.8436	.96778	25
36	207	048	.8391	771	24
37	235	079	.8345	764	23
38	263	110	.8299	756	22
39	291	141	.8254	749	21
40	.25320	.26172	3.8208	.96742	20
41	348	203	.8163	734	19
42	376	235	.8118	727	18
43	404	266	.8073	719	17
44	432	297	.8028	712	16
45	.25460	.26328	3.7983	.96705	15
46	488	359	.7938	697	14
47	516	390	.7893	690	13
48	545	421	.7848	682	12
49	573	452	.7804	675	11
50	.25601	.26483	3.7760	.96667	10
51	629	515	.7715	660	9
52	657	546	.7671	653	8
53	685	577	.7627	645	7
54	713	608	.7583	638	6
55	.25741	.26639	3.7539	.96630	5
56	769	670	.7495	623	4
57	798	701	.7451	615	3
58	826	733	.7408	608	2
59	854	764	.7364	600	1
60	.25882	.26795	3.7321	.96593	0
	Cos	Ctn	Tan	Sin	′

75°

′	Sin	Tan	Ctn	Cos	
0	.25882	.26795	3.7321	.96593	60
1	910	826	.7277	585	59
2	938	857	.7234	578	58
3	966	888	.7191	570	57
4	.25994	920	.7148	562	56
5	.26022	.26951	3.7105	.96555	55
6	050	.26982	.7062	547	54
7	079	.27013	.7019	540	53
8	107	044	.6976	532	52
9	135	076	.6933	524	51
10	.26163	.27107	3.6891	.96517	50
11	191	138	.6848	509	49
12	219	169	.6806	502	48
13	247	201	.6764	494	47
14	275	232	.6722	486	46
15	.26303	.27263	3.6680	.96479	45
16	331	294	.6638	471	44
17	359	326	.6596	463	43
18	387	357	.6554	456	42
19	415	388	.6512	448	41
20	.26443	.27419	3.6470	.96440	40
21	471	451	.6429	433	39
22	500	482	.6387	425	38
23	528	513	.6346	417	37
24	556	545	.6305	410	36
25	.26584	.27576	3.6264	.96402	35
26	612	607	.6222	394	34
27	640	638	.6181	386	33
28	668	670	.6140	379	32
29	696	701	.6100	371	31
30	.26724	.27732	3.6059	.96363	30
31	752	764	.6018	355	29
32	780	795	.5978	347	28
33	808	826	.5937	340	27
34	836	858	.5897	332	26
35	.26864	.27889	3.5856	.96324	25
36	892	921	.5816	316	24
37	920	952	.5776	308	23
38	948	.27983	.5736	301	22
39	.26976	.28015	.5696	293	21
40	.27004	.28046	3.5656	.96285	20
41	032	077	.5616	277	19
42	060	109	.5576	269	18
43	088	140	.5536	261	17
44	116	172	.5497	253	16
45	.27144	.28203	3.5457	.96246	15
46	172	234	.5418	238	14
47	200	266	.5379	230	13
48	228	297	.5339	222	12
49	256	329	.5300	214	11
50	.27284	.28360	3.5261	.96206	10
51	312	391	.5222	198	9
52	340	423	.5183	190	8
53	368	454	.5144	182	7
54	396	486	.5105	174	6
55	.27424	.28517	3.5067	.96166	5
56	452	549	.5028	158	4
57	480	580	.4989	150	3
58	508	612	.4951	142	2
59	536	643	.4912	134	1
60	.27564	.28675	3.4874	.96126	0
	Cos	Ctn	Tan	Sin	′

74°

′	Sin	Tan	Ctn	Cos	
0	.27564	.28675	3.4874	.96126	60
1	592	706	.4836	118	59
2	620	738	.4798	110	58
3	648	769	.4760	102	57
4	676	801	.4722	094	56
5	.27704	.28832	3.4684	.96086	55
6	731	864	.4646	078	54
7	759	895	.4608	070	53
8	787	927	.4570	062	52
9	815	958	.4533	054	51
10	.27843	.28990	3.4495	.96046	50
11	871	.29021	.4458	037	49
12	899	053	.4420	029	48
13	927	084	.4383	021	47
14	955	116	.4346	013	46
15	.27983	.29147	3.4308	.96005	45
16	.28011	179	.4271	.95997	44
17	039	210	.4234	989	43
18	067	242	.4197	981	42
19	095	274	.4160	972	41
20	.28123	.29305	3.4124	.95964	40
21	150	337	.4087	956	39
22	178	368	.4050	948	38
23	206	400	.4014	940	37
24	234	432	.3977	931	36
25	.28262	.29463	3.3941	.95923	35
26	290	495	.3904	915	34
27	318	526	.3868	907	33
28	346	558	.3832	898	32
29	374	590	.3796	890	31
30	.28402	.29621	3.3759	.95882	30
31	429	653	.3723	874	29
32	457	685	.3687	865	28
33	485	716	.3652	857	27
34	513	748	.3616	849	26
35	.28541	.29780	3.3580	.95841	25
36	569	811	.3544	832	24
37	597	843	.3509	824	23
38	625	875	.3473	816	22
39	652	906	.3438	807	21
40	.28680	.29938	3.3402	.95799	20
41	708	.29970	.3367	791	19
42	736	.30001	.3332	782	18
43	764	033	.3297	774	17
44	792	065	.3261	766	16
45	.28820	.30097	3.3226	.95757	15
46	847	128	.3191	749	14
47	875	160	.3156	740	13
48	903	192	.3122	732	12
49	931	224	.3087	724	11
50	.28959	.30255	3.3052	.95715	10
51	.28987	287	.3017	707	9
52	.29015	319	.2983	698	8
53	042	351	.2948	690	7
54	070	382	.2914	681	6
55	.29098	.30414	3.2879	.95673	5
56	126	446	.2845	664	4
57	154	478	.2811	656	3
58	182	509	.2777	647	2
59	209	541	.2743	639	1
60	.29237	.30573	3.2709	.95630	0
	Cos	Ctn	Tan	Sin	′

73°

′	Sin	Tan	Ctn	Cos	
0	.29237	.30573	3.2709	.95630	60
1	265	605	.2675	622	59
2	293	637	.2641	613	58
3	321	669	.2607	605	57
4	348	700	.2573	596	56
5	.29376	.30732	3.2539	.95588	55
6	404	764	.2506	579	54
7	432	796	.2472	571	53
8	460	828	.2438	562	52
9	487	860	.2405	554	51
10	.29515	.30891	3.2371	.95545	50
11	543	923	.2338	536	49
12	571	955	.2305	528	48
13	599	.30987	.2272	519	47
14	626	.31019	.2238	511	46
15	.29654	.31051	3.2205	.95502	45
16	682	083	.2172	493	44
17	710	115	.2139	485	43
18	737	147	.2106	476	42
19	765	178	.2073	467	41
20	.29793	.31210	3.2041	.95459	40
21	821	242	.2008	450	39
22	849	274	.1975	441	38
23	876	306	.1943	433	37
24	904	338	.1910	424	36
25	.29932	.31370	3.1878	.95415	35
26	960	402	.1845	407	34
27	.29987	434	.1813	398	33
28	.30015	466	.1780	389	32
29	043	498	.1748	380	31
30	.30071	.31530	3.1716	.95372	30
31	098	562	.1684	363	29
32	126	594	.1652	354	28
33	154	626	.1620	345	27
34	182	658	.1588	337	26
35	.30209	.31690	3.1556	.95328	25
36	237	722	.1524	319	24
37	265	754	.1492	310	23
38	292	786	.1460	301	22
39	320	818	.1429	293	21
40	.30348	.31850	3.1397	.95284	20
41	376	882	.1366	275	19
42	403	914	.1334	266	18
43	431	946	.1303	257	17
44	459	.31978	.1271	248	16
45	.30486	.32010	3.1240	.95240	15
46	514	042	.1209	231	14
47	542	074	.1178	222	13
48	570	106	.1146	213	12
49	597	139	.1115	204	11
50	.30625	.32171	3.1084	.95195	10
51	653	203	.1053	186	9
52	680	235	.1022	177	8
53	708	267	.0991	168	7
54	736	299	.0961	159	6
55	.30763	.32331	3.0930	.95150	5
56	791	363	.0899	142	4
57	819	396	.0868	133	3
58	846	428	.0838	124	2
59	874	460	.0807	115	1
60	.30902	.32492	3.0777	.95106	0
	Cos	Ctn	Tan	Sin	′

72°

′	Sin	Tan	Ctn	Cos	
0	.30902	.32492	3.0777	.95106	60
1	929	524	.0746	097	59
2	957	556	.0716	088	58
3	.30985	588	.0686	079	57
4	.31012	621	.0655	070	56
5	.31040	.32653	3.0625	.95061	55
6	068	685	.0595	052	54
7	095	717	.0565	043	53
8	123	749	.0535	033	52
9	151	782	.0505	024	51
10	.31178	.32814	3.0475	.95015	50
11	206	846	.0445	.95006	49
12	233	878	.0415	.94997	48
13	261	911	.0385	988	47
14	289	943	.0356	979	46
15	.31316	.32975	3.0326	.94970	45
16	344	.33007	.0296	961	44
17	372	040	.0267	952	43
18	399	072	.0237	943	42
19	427	104	.0208	933	41
20	.31454	.33136	3.0178	.94924	40
21	482	169	.0149	915	39
22	510	201	.0120	906	38
23	537	233	.0090	897	37
24	565	266	.0061	888	36
25	.31593	.33298	3.0032	.94878	35
26	620	330	3.0003	869	34
27	648	363	2.9974	860	33
28	675	395	.9945	851	32
29	703	427	.9916	842	31
30	.31730	.33460	2.9887	.94832	30
31	758	492	.9858	823	29
32	786	524	.9829	814	28
33	813	557	.9800	805	27
34	841	589	.9772	795	26
35	.31868	.33621	2.9743	.94786	25
36	896	654	.9714	777	24
37	923	686	.9686	768	23
38	951	718	.9657	758	22
39	.31979	751	.9629	749	21
40	.32006	.33783	2.9600	.94740	20
41	034	816	.9572	730	19
42	061	848	.9544	721	18
43	089	881	.9515	712	17
44	116	913	.9487	702	16
45	.32144	.33945	2.9459	.94693	15
46	171	.33978	.9431	684	14
47	199	.34010	.9403	674	13
48	227	043	.9375	665	12
49	254	075	.9347	656	11
50	.32282	.34108	2.9319	.94646	10
51	309	140	.9291	637	9
52	337	173	.9263	627	8
53	364	205	.9235	618	7
54	392	238	.9208	609	6
55	.32419	.34270	2.9180	.94599	5
56	447	303	.9152	590	4
57	474	335	.9125	580	3
58	502	368	.9097	571	2
59	529	400	.9070	561	1
60	.32557	.34433	2.9042	.94552	0
	Cos	Ctn	Tan	Sin	′

71°

′	Sin	Tan	Ctn	Cos	
0	.32557	.34433	2.9042	.94552	60
1	584	465	.9015	542	59
2	612	498	.8987	533	58
3	639	530	.8960	523	57
4	667	563	.8933	514	56
5	.32694	.34596	2.8905	.94504	55
6	722	628	.8878	495	54
7	749	661	.8851	485	53
8	777	693	.8824	476	52
9	804	726	.8797	466	51
10	.32832	.34758	2.8770	.94457	50
11	859	791	.8743	447	49
12	887	824	.8716	438	48
13	914	856	.8689	428	47
14	942	889	.8662	418	46
15	.32969	.34922	2.8636	.94409	45
16	.32997	954	.8609	399	44
17	.33024	.34987	.8582	390	43
18	051	.35020	.8556	380	42
19	079	052	.8529	370	41
20	.33106	.35085	2.8502	.94361	40
21	134	118	.8476	351	39
22	161	150	.8449	342	38
23	189	183	.8423	332	37
24	216	216	.8397	322	36
25	.33244	.35248	2.8370	.94313	35
26	271	281	.8344	303	34
27	298	314	.8318	293	33
28	326	346	.8291	284	32
29	353	379	.8265	274	31
30	.33381	.35412	2.8239	.94264	30
31	408	445	.8213	254	29
32	436	477	.8187	245	28
33	463	510	.8161	235	27
34	490	543	.8135	225	26
35	.33518	.35576	2.8109	.94215	25
36	545	608	.8083	206	24
37	573	641	.8057	196	23
38	600	674	.8032	186	22
39	627	707	.8006	176	21
40	.33655	.35740	2.7980	.94167	20
41	682	772	.7955	157	19
42	710	805	.7929	147	18
43	737	838	.7903	137	17
44	764	871	.7878	127	16
45	.33792	.35904	2.7852	.94118	15
46	819	937	.7827	108	14
47	846	.35969	.7801	098	13
48	874	.36002	.7776	088	12
49	901	035	.7751	078	11
50	.33929	.36068	2.7725	.94068	10
51	956	101	.7700	058	9
52	.33983	134	.7675	049	8
53	.34011	167	.7650	039	7
54	038	199	.7625	029	6
55	.34065	.36232	2.7600	.94019	5
56	093	265	.7575	.94009	4
57	120	298	.7550	.93999	3
58	147	331	.7525	989	2
59	175	364	.7500	979	1
60	.34202	.36397	2.7475	.93969	0
	Cos	Ctn	Tan	Sin	′

70°

′	Sin	Tan	Ctn	Cos	
0	.40674	.44523	2.2460	.91355	60
1	700	558	.2443	343	59
2	727	593	.2425	331	58
3	753	627	.2408	319	57
4	780	662	.2390	307	56
5	.40806	.44697	2.2373	.91295	55
6	833	732	.2355	283	54
7	860	767	.2338	272	53
8	886	802	.2320	260	52
9	913	837	.2303	248	51
10	.40939	.44872	2.2286	.91236	50
11	966	907	.2268	224	49
12	.40992	942	.2251	212	48
13	.41019	.44977	.2234	200	47
14	045	.45012	.2216	188	46
15	.41072	.45047	2.2199	.91176	45
16	098	082	.2182	164	44
17	125	117	.2165	152	43
18	151	152	.2148	140	42
19	178	187	.2130	128	41
20	.41204	.45222	2.2113	.91116	40
21	231	257	.2096	104	39
22	257	292	.2079	092	38
23	284	327	.2062	080	37
24	310	362	.2045	068	36
25	.41337	.45397	2.2028	.91056	35
26	363	432	.2011	044	34
27	390	467	.1994	032	33
28	416	502	.1977	020	32
29	443	538	.1960	.91008	31
30	.41469	.45573	2.1943	.90996	30
31	496	608	.1926	984	29
32	522	643	.1909	972	28
33	549	678	.1892	960	27
34	575	713	.1876	948	26
35	.41602	.45748	2.1859	.90936	25
36	628	784	.1842	924	24
37	655	819	.1825	911	23
38	681	854	.1808	899	22
39	707	889	.1792	887	21
40	.41734	.45924	2.1775	.90875	20
41	760	960	.1758	863	19
42	787	.45995	.1742	851	18
43	813	.46030	.1725	839	17
44	840	065	.1708	826	16
45	.41866	.46101	2.1692	.90814	15
46	892	136	.1675	802	14
47	919	171	.1659	790	13
48	945	206	.1642	778	12
49	972	242	.1625	766	11
50	.41998	.46277	2.1609	.90753	10
51	.42024	312	.1592	741	9
52	051	348	.1576	729	8
53	077	383	.1560	717	7
54	104	418	.1543	704	6
55	.42130	.46454	2.1527	.90692	5
56	156	489	.1510	680	4
57	183	525	.1494	668	3
58	209	560	.1478	655	2
59	235	595	.1461	643	1
60	.42262	.46631	2.1445	.90631	0
	Cos	Ctn	Tan	Sin	′

65°

′	Sin	Tan	Ctn	Cos	
0	.42262	.46631	2.1445	.90631	60
1	288	666	.1429	618	59
2	315	702	.1413	606	58
3	341	737	.1396	594	57
4	367	772	.1380	582	56
5	.42394	.46808	2.1364	.90569	55
6	420	843	.1348	557	54
7	446	879	.1332	545	53
8	473	914	.1315	532	52
9	499	950	.1299	520	51
10	.42525	.46985	2.1283	.90507	50
11	552	.47021	.1267	495	49
12	578	056	.1251	483	48
13	604	092	.1235	470	47
14	631	128	.1219	458	46
15	.42657	.47163	2.1203	.90446	45
16	683	199	.1187	433	44
17	709	234	.1171	421	43
18	736	270	.1155	408	42
19	762	305	.1139	396	41
20	.42788	.47341	2.1123	.90383	40
21	815	377	.1107	371	39
22	841	412	.1092	358	38
23	867	448	.1076	346	37
24	894	483	.1060	334	36
25	.42920	.47519	2.1044	.90321	35
26	946	555	.1028	309	34
27	972	590	.1013	296	33
28	.42999	626	.0997	284	32
29	.43025	662	.0981	271	31
30	.43051	.47698	2.0965	.90259	30
31	077	733	.0950	246	29
32	104	769	.0934	233	28
33	130	805	.0918	221	27
34	156	840	.0903	208	26
35	.43182	.47876	2.0887	.90196	25
36	209	912	.0872	183	24
37	235	948	.0856	171	23
38	261	.47984	.0840	158	22
39	287	.48019	.0825	146	21
40	.43313	.48055	2.0809	.90133	20
41	340	091	.0794	120	19
42	366	127	.0778	108	18
43	392	163	.0763	095	17
44	418	198	.0748	082	16
45	.43445	.48234	2.0732	.90070	15
46	471	270	.0717	057	14
47	497	306	.0701	045	13
48	523	342	.0686	032	12
49	549	378	.0671	019	11
50	.43575	.48414	2.0655	.90007	10
51	602	450	.0640	.89994	9
52	628	486	.0625	981	8
53	654	521	.0609	968	7
54	680	557	.0594	956	6
55	.43706	.48593	2.0579	.89943	5
56	733	629	.0564	930	4
57	759	665	.0549	918	3
58	785	701	.0533	905	2
59	811	737	.0518	892	1
60	.43837	.48773	2.0503	.89879	0
	Cos	Ctn	Tan	Sin	′

64°

′	Sin	Tan	Ctn	Cos	
0	.43837	.48773	2.0503	.89879	60
1	863	809	.0488	867	59
2	889	845	.0473	854	58
3	916	881	.0458	841	57
4	942	917	.0443	828	56
5	.43968	.48953	2.0428	.89816	55
6	.43994	.48989	.0413	803	54
7	.44020	.49026	.0398	790	53
8	046	062	.0383	777	52
9	072	098	.0368	764	51
10	.44098	.49134	2.0353	.89752	50
11	124	170	.0338	739	49
12	151	206	.0323	726	48
13	177	242	.0308	713	47
14	203	278	.0293	700	46
15	.44229	.49315	2.0278	.89687	45
16	255	351	.0263	674	44
17	281	387	.0248	662	43
18	307	423	.0233	649	42
19	333	459	.0219	636	41
20	.44359	.49495	2.0204	.89623	40
21	385	532	.0189	610	39
22	411	568	.0174	597	38
23	437	604	.0160	584	37
24	464	640	.0145	571	36
25	.44490	.49677	2.0130	.89558	35
26	516	713	.0115	545	34
27	542	749	.0101	532	33
28	568	786	.0086	519	32
29	594	822	.0072	506	31
30	.44620	.49858	2.0057	.89493	30
31	646	894	.0042	480	29
32	672	931	.0028	467	28
33	698	.49967	2.0013	454	27
34	724	.50004	1.9999	441	26
35	.44750	.50040	1.9984	.89428	25
36	776	076	.9970	415	24
37	802	113	.9955	402	23
38	828	149	.9941	389	22
39	854	185	.9926	376	21
40	.44880	.50222	1.9912	.89363	20
41	906	258	.9897	350	19
42	932	295	.9883	337	18
43	958	331	.9868	324	17
44	.44984	368	.9854	311	16
45	.45010	.50404	1.9840	.89298	15
46	036	441	.9825	285	14
47	062	477	.9811	272	13
48	088	514	.9797	259	12
49	114	550	.9782	245	11
50	.45140	.50587	1.9768	.89232	10
51	166	623	.9754	219	9
52	192	660	.9740	206	8
53	218	696	.9725	193	7
54	243	733	.9711	180	6
55	.45269	.50769	1.9697	.89167	5
56	295	806	.9683	153	4
57	321	843	.9669	140	3
58	347	879	.9654	127	2
59	373	916	.9640	114	1
60	.45399	.50953	1.9626	.89101	0
	Cos	Ctn	Tan	Sin	′

63°

′	Sin	Tan	Ctn	Cos	
0	.45399	.50953	1.9626	.89101	60
1	425	.50989	.9612	087	59
2	451	.51026	.9598	074	58
3	477	063	.9584	061	57
4	503	099	.9570	048	56
5	.45529	.51136	1.9556	.89035	55
6	554	173	.9542	021	54
7	580	209	.9528	.89008	53
8	606	246	.9514	.88995	52
9	632	283	.9500	981	51
10	.45658	.51319	1.9486	.88968	50
11	684	356	.9472	955	49
12	710	393	.9458	942	48
13	736	430	.9444	928	47
14	762	467	.9430	915	46
15	.45787	.51503	1.9416	.88902	45
16	813	540	.9402	888	44
17	839	577	.9388	875	43
18	865	614	.9375	862	42
19	891	651	.9361	848	41
20	.45917	.51688	1.9347	.88835	40
21	942	724	.9333	822	39
22	968	761	.9319	808	38
23	.45994	798	.9306	795	37
24	.46020	835	.9292	782	36
25	.46046	.51872	1.9278	.88768	35
26	072	909	.9265	755	34
27	097	946	.9251	741	33
28	123	.51983	.9237	728	32
29	149	.52020	.9223	715	31
30	.46175	.52057	1.9210	.88701	30
31	201	094	.9196	688	29
32	226	131	.9183	674	28
33	252	168	.9169	661	27
34	278	205	.9155	647	26
35	.46304	.52242	1.9142	.88634	25
36	330	279	.9128	620	24
37	355	316	.9115	607	23
38	381	353	.9101	593	22
39	407	390	.9088	580	21
40	.46433	.52427	1.9074	.88566	20
41	458	464	.9061	553	19
42	484	501	.9047	539	18
43	510	538	.9034	526	17
44	536	575	.9020	512	16
45	.46561	.52613	1.9007	.88499	15
46	587	650	.8993	485	14
47	613	687	.8980	472	13
48	639	724	.8967	458	12
49	664	761	.8953	445	11
50	.46690	.52798	1.8940	.88431	10
51	716	836	.8927	417	9
52	742	873	.8913	404	8
53	767	910	.8900	390	7
54	793	947	.8887	377	6
55	.46819	.52985	1.8873	.88363	5
56	844	.53022	.8860	349	4
57	870	059	.8847	336	3
58	896	096	.8834	322	2
59	921	134	.8820	308	1
60	.46947	.53171	1.8807	.88295	0
	Cos	Ctn	Tan	Sin	′

62°

′	Sin	Tan	Ctn	Cos	
0	.52992	.62487	1.6003	.84805	60
1	.53017	527	.5993	789	59
2	041	568	.5983	774	58
3	066	608	.5972	759	57
4	091	649	.5962	743	56
5	.53115	.62689	1.5952	.84728	55
6	140	730	.5941	712	54
7	164	770	.5931	697	53
8	189	811	.5921	681	52
9	214	852	.5911	666	51
10	.53238	.62892	1.5900	.84650	50
11	263	933	.5890	635	49
12	288	.62973	.5880	619	48
13	312	.63014	.5869	604	47
14	337	055	.5859	588	46
15	.53361	.63095	1.5849	.84573	45
16	386	136	.5839	557	44
17	411	177	.5829	542	43
18	435	217	.5818	526	42
19	460	258	.5808	511	41
20	.53484	.63299	1.5798	.84495	40
21	509	340	.5788	480	39
22	534	380	.5778	464	38
23	558	421	.5768	448	37
24	583	462	.5757	433	36
25	.53607	.63503	1.5747	.84417	35
26	632	544	.5737	402	34
27	656	584	.5727	386	33
28	681	625	.5717	370	32
29	705	666	.5707	355	31
30	.53730	.63707	1.5697	.84339	30
31	754	748	.5687	324	29
32	779	789	.5677	308	28
33	804	830	.5667	292	27
34	828	871	.5657	277	26
35	.53853	.63912	1.5647	.84261	25
36	877	953	.5637	245	24
37	902	.63994	.5627	230	23
38	926	.64035	.5617	214	22
39	951	076	.5607	198	21
40	.53975	.64117	1.5597	.84182	20
41	.54000	158	.5587	167	19
42	024	199	.5577	151	18
43	049	240	.5567	135	17
44	073	281	.5557	120	16
45	.54097	.64322	1.5547	.84104	15
46	122	363	.5537	088	14
47	146	404	.5527	072	13
48	171	446	.5517	057	12
49	195	487	.5507	041	11
50	.54220	.64528	1.5497	.84025	10
51	244	569	.5487	.84009	9
52	269	610	.5477	.83994	8
53	293	652	.5468	978	7
54	317	693	.5458	962	6
55	.54342	.64734	1.5448	.83946	5
56	366	775	.5438	930	4
57	391	817	.5428	915	3
58	415	858	.5418	899	2
59	440	899	.5408	883	1
60	.54464	.64941	1.5399	.83867	0
	Cos	Ctn	Tan	Sin	′

57°

′	Sin	Tan	Ctn	Cos	
0	.54464	.64941	1.5399	.83867	60
1	488	.64982	.5389	851	59
2	513	.65024	.5379	835	58
3	537	065	.5369	819	57
4	561	106	.5359	804	56
5	.54586	.65148	1.5350	.83788	55
6	610	189	.5340	772	54
7	635	231	.5330	756	53
8	659	272	.5320	740	52
9	683	314	.5311	724	51
10	.54708	.65355	1.5301	.83708	50
11	732	397	.5291	692	49
12	756	438	.5282	676	48
13	781	480	.5272	660	47
14	805	521	.5262	645	46
15	.54829	.65563	1.5253	.83629	45
16	854	604	.5243	613	44
17	878	646	.5233	597	43
18	902	688	.5224	581	42
19	927	729	.5214	565	41
20	.54951	.65771	1.5204	.83549	40
21	975	813	.5195	533	39
22	.54999	854	.5185	517	38
23	.55024	896	.5175	501	37
24	048	938	.5166	485	36
25	.55072	.65980	1.5156	.83469	35
26	097	.66021	.5147	453	34
27	121	063	.5137	437	33
28	145	105	.5127	421	32
29	169	147	.5118	405	31
30	.55194	.66189	1.5108	.83389	30
31	218	230	.5099	373	29
32	242	272	.5089	356	28
33	266	314	.5080	340	27
34	291	356	.5070	324	26
35	.55315	.66398	1.5061	.83308	25
36	339	440	.5051	292	24
37	363	482	.5042	276	23
38	388	524	.5032	260	22
39	412	566	.5023	244	21
40	.55436	.66608	1.5013	.83228	20
41	460	650	.5004	212	19
42	484	692	.4994	195	18
43	509	734	.4985	179	17
44	533	776	.4975	163	16
45	.55557	.66818	1.4966	.83147	15
46	581	860	.4957	131	14
47	605	902	.4947	115	13
48	630	944	.4938	098	12
49	654	.66986	.4928	082	11
50	.55678	.67028	1.4919	.83066	10
51	702	071	.4910	050	9
52	726	113	.4900	034	8
53	750	155	.4891	017	7
54	775	197	.4882	.83001	6
55	.55799	.67239	1.4872	.82985	5
56	823	282	.4863	969	4
57	847	324	.4854	953	3
58	871	366	.4844	936	2
59	895	409	.4835	920	1
60	.55919	.67451	1.4826	.82904	0
	Cos	Ctn	Tan	Sin	′

56°

′	Sin	Tan	Ctn	Cos	
0	.55919	.67451	1.4826	.82904	60
1	943	493	.4816	887	59
2	968	536	.4807	871	58
3	.55992	578	.4798	855	57
4	.56016	620	.4788	839	56
5	.56040	.67663	1.4779	.82822	55
6	064	705	.4770	806	54
7	088	748	.4761	790	53
8	112	790	.4751	773	52
9	136	832	.4742	757	51
10	.56160	.67875	1.4733	.82741	50
11	184	917	.4724	724	49
12	208	.67960	.4715	708	48
13	232	.68002	.4705	692	47
14	256	045	.4696	675	46
15	.56280	.68088	1.4687	.82659	45
16	305	130	.4678	643	44
17	329	173	.4669	626	43
18	353	215	.4659	610	42
19	377	258	.4650	593	41
20	.56401	.68301	1.4641	.82577	40
21	425	343	.4632	561	39
22	449	386	.4623	544	38
23	473	429	.4614	528	37
24	497	471	.4605	511	36
25	.56521	.68514	1.4596	.82495	35
26	545	557	.4586	478	34
27	569	600	.4577	462	33
28	593	642	.4568	446	32
29	617	685	.4559	429	31
30	.56641	.68728	1.4550	.82413	30
31	665	771	.4541	396	29
32	689	814	.4532	380	28
33	713	857	.4523	363	27
34	736	900	.4514	347	26
35	.56760	.68942	1.4505	.82330	25
36	784	.68985	.4496	314	24
37	808	.69028	.4487	297	23
38	832	071	.4478	281	22
39	856	114	.4469	264	21
40	.56880	.69157	1.4460	.82248	20
41	904	200	.4451	231	19
42	928	243	.4442	214	18
43	952	286	.4433	198	17
44	.56976	329	.4424	181	16
45	.57000	.69372	1.4415	.82165	15
46	024	416	.4406	148	14
47	047	459	.4397	132	13
48	071	502	.4388	115	12
49	095	545	.4379	098	11
50	.57119	.69588	1.4370	.82082	10
51	143	631	.4361	065	9
52	167	675	.4352	048	8
53	191	718	.4344	032	7
54	215	761	.4335	.82015	6
55	.57238	.69804	1.4326	.81999	5
56	262	847	.4317	982	4
57	286	891	.4308	965	3
58	310	934	.4299	949	2
59	334	.69977	.4290	932	1
60	.57358	.70021	1.4281	.81915	0
	Cos	Ctn	Tan	Sin	′

′	Sin	Tan	Ctn	Cos	
0	.57358	.70021	1.4281	.81915	60
1	381	064	.4273	899	59
2	405	107	.4264	882	58
3	429	151	.4255	865	57
4	453	194	.4246	848	56
5	.57477	.70238	1.4237	.81832	55
6	501	281	.4229	815	54
7	524	325	.4220	798	53
8	548	368	.4211	782	52
9	572	412	.4202	765	51
10	.57596	.70455	1.4193	.81748	50
11	619	499	.4185	731	49
12	643	542	.4176	714	48
13	667	586	.4167	698	47
14	691	629	.4158	681	46
15	.57715	.70673	1.4150	.81664	45
16	738	717	.4141	647	44
17	762	760	.4132	631	43
18	786	804	.4124	614	42
19	810	848	.4115	597	41
20	.57833	.70891	1.4106	.81580	40
21	857	935	.4097	563	39
22	881	.70979	.4089	546	38
23	904	.71023	.4080	530	37
24	928	066	.4071	513	36
25	.57952	.71110	1.4063	.81496	35
26	976	154	.4054	479	34
27	.57999	198	.4045	462	33
28	.58023	242	.4037	445	32
29	047	285	.4028	428	31
30	.58070	.71329	1.4019	.81412	30
31	094	373	.4011	395	29
32	118	417	.4002	378	28
33	141	461	.3994	361	27
34	165	505	.3985	344	26
35	.58189	.71549	1.3976	.81327	25
36	212	593	.3968	310	24
37	236	637	.3959	293	23
38	260	681	.3951	276	22
39	283	725	.3942	259	21
40	.58307	.71769	1.3934	.81242	20
41	330	813	.3925	225	19
42	354	857	.3916	208	18
43	378	901	.3908	191	17
44	401	946	.3899	174	16
45	.58425	.71990	1.3891	.81157	15
46	449	.72034	.3882	140	14
47	472	078	.3874	123	13
48	496	122	.3865	106	12
49	519	167	.3857	089	11
50	.58543	.72211	1.3848	.81072	10
51	567	255	.3840	055	9
52	590	299	.3831	038	8
53	614	344	.3823	021	7
54	637	388	.3814	.81004	6
55	.58661	.72432	1.3806	.80987	5
56	684	477	.3798	970	4
57	708	521	.3789	953	3
58	731	565	.3781	936	2
59	755	610	.3772	919	1
60	.58779	.72654	1.3764	.80902	0
	Cos	Ctn	Tan	Sin	′

'	Sin	Tan	Ctn	Cos	
0	.58779	.72654	1.3764	.80902	60
1	802	699	.3755	885	59
2	826	743	.3747	867	58
3	849	788	.3739	850	57
4	873	832	.3730	833	56
5	.58896	.72877	1.3722	.80816	55
6	920	921	.3713	799	54
7	943	.72966	.3705	782	53
8	967	.73010	.3697	765	52
9	.58990	055	.3688	748	51
10	.59014	.73100	1.3680	.80730	50
11	037	144	.3672	713	49
12	061	189	.3663	696	48
13	084	234	.3655	679	47
14	108	278	.3647	662	46
15	.59131	.73323	1.3638	.80644	45
16	154	368	.3630	627	44
17	178	413	.3622	610	43
18	201	457	.3613	593	42
19	225	502	.3605	576	41
20	.59248	.73547	1.3597	.80558	40
21	272	592	.3588	541	39
22	295	637	.3580	524	38
23	318	681	.3572	507	37
24	342	726	.3564	489	36
25	.59365	.73771	1.3555	.80472	35
26	389	816	.3547	455	34
27	412	861	.3539	438	33
28	436	906	.3531	420	32
29	459	951	.3522	403	31
30	.59482	.73996	1.3514	.80386	30
31	506	.74041	.3506	368	29
32	529	086	.3498	351	28
33	552	131	.3490	334	27
34	576	176	.3481	316	26
35	.59599	.74221	1.3473	.80299	25
36	622	267	.3465	282	24
37	646	312	.3457	264	23
38	669	357	.3449	247	22
39	693	402	.3440	230	21
40	.59716	.74447	1.3432	.80212	20
41	739	492	.3424	195	19
42	763	538	.3416	178	18
43	786	583	.3408	160	17
44	809	628	.3400	143	16
45	.59832	.74674	1.3392	.80125	15
46	856	719	.3384	108	14
47	879	764	.3375	091	13
48	902	810	.3367	073	12
49	926	855	.3359	056	11
50	.59949	.74900	1.3351	.80038	10
51	972	946	.3343	021	9
52	.59995	.74991	.3335	.80003	8
53	.60019	.75037	.3327	.79986	7
54	042	082	.3319	968	6
55	.60065	.75128	1.3311	.79951	5
56	089	173	.3303	934	4
57	112	219	.3295	916	3
58	135	264	.3287	899	2
59	158	310	.3278	881	1
60	.60182	.75355	1.3270	.79864	0
	Cos	Ctn	Tan	Sin	'

53°

'	Sin	Tan	Ctn	Cos
0	.60182	.75355	1.3270	.79864
1	205	401	.3262	846
2	228	447	.3254	829
3	251	492	.3246	811
4	274	538	.3238	793
5	.60298	.75584	1.3230	.79776
6	321	629	.3222	758
7	344	675	.3214	741
8	367	721	.3206	723
9	390	767	.3198	706
10	.60414	.75812	1.3190	.79688
11	437	858	.3182	671
12	460	904	.3175	653
13	483	950	.3167	635
14	506	.75996	.3159	618
15	.60529	.76042	1.3151	.79600
16	553	088	.3143	583
17	576	134	.3135	565
18	599	180	.3127	547
19	622	226	.3119	530
20	.60645	.76272	1.3111	.79512
21	668	318	.3103	494
22	691	364	.3095	477
23	714	410	.3087	459
24	738	456	.3079	441
25	.60761	.76502	1.3072	.79424
26	784	548	.3064	406
27	807	594	.3056	388
28	830	640	.3048	371
29	853	686	.3040	353
30	.60876	.76733	1.3032	.79335
31	899	779	.3024	318
32	922	825	.3017	300
33	945	871	.3009	282
34	968	918	.3001	264
35	.60991	.76964	1.2993	.79247
36	.61015	.77010	.2985	229
37	038	057	.2977	211
38	061	103	.2970	193
39	084	149	.2962	176
40	.61107	.77196	1.2954	.79158
41	130	242	.2946	140
42	153	289	.2938	122
43	176	335	.2931	105
44	199	382	.2923	087
45	.61222	.77428	1.2915	.79069
46	245	475	.2907	051
47	268	521	.2900	033
48	291	568	.2892	.79016
49	314	615	.2884	.78998
50	.61337	.77661	1.2876	.78980
51	360	708	.2869	962
52	383	754	.2861	944
53	406	801	.2853	926
54	429	848	.2846	908
55	.61451	.77895	1.2838	.78891
56	474	941	.2830	873
57	497	.77988	.2822	855
58	520	.78035	.2815	837
59	543	082	.2807	819
60	.61566	.78129	1.2799	.78801
	Cos	Ctn	Tan	Sin

52°

'	Sin	Tan	Ctn	Cos	
0	.61566	.78129	1.2799	.78801	60
1	589	175	.2792	783	59
2	612	222	.2784	765	58
3	635	269	.2776	747	57
4	658	316	.2769	729	56
5	.61681	.78363	1.2761	.78711	55
6	704	410	.2753	694	54
7	726	457	.2746	676	53
8	749	504	.2738	658	52
9	772	551	.2731	640	51
10	.61795	.78598	1.2723	.78622	50
11	818	645	.2715	604	49
12	841	692	.2708	586	48
13	864	739	.2700	568	47
14	887	786	.2693	550	46
15	.61909	.78834	1.2685	.78532	45
16	932	881	.2677	514	44
17	955	928	.2670	496	43
18	.61978	.78975	.2662	478	42
19	.62001	.79022	.2655	460	41
20	.62024	.79070	1.2647	.78442	40
21	046	117	.2640	424	39
22	069	164	.2632	405	38
23	092	212	.2624	387	37
24	115	259	.2617	369	36
25	.62138	.79306	1.2609	.78351	35
26	160	354	.2602	333	34
27	183	401	.2594	315	33
28	206	449	.2587	297	32
29	229	496	.2579	279	31
30	.62251	.79544	1.2572	.78261	30
31	274	591	.2564	243	29
32	297	639	.2557	225	28
33	320	686	.2549	206	27
34	342	734	.2542	188	26
35	.62365	.79781	1.2534	.78170	25
36	388	829	.2527	152	24
37	411	877	.2519	134	23
38	433	924	.2512	116	22
39	456	.79972	.2504	098	21
40	.62479	.80020	1.2497	.78079	20
41	502	067	.2489	061	19
42	524	115	.2482	043	18
43	547	163	.2475	025	17
44	570	211	.2467	.78007	16
45	.62592	.80258	1.2460	.77988	15
46	615	306	.2452	970	14
47	638	354	.2445	952	13
48	660	402	.2437	934	12
49	683	450	.2430	916	11
50	.62706	.80498	1.2423	.77897	10
51	728	546	.2415	879	9
52	751	594	.2408	861	8
53	774	642	.2401	843	7
54	796	690	.2393	824	6
55	.62819	.80738	1.2386	.77806	5
56	842	786	.2378	788	4
57	864	834	.2371	769	3
58	887	882	.2364	751	2
59	909	930	.2356	733	1
60	.62932	.80978	1.2349	.77715	0
	Cos	Ctn	Tan	Sin	'

51°

'	Sin	Tan	Ctn	Cos	
0	.62932	.80978	1.2349	.77715	60
1	955	.81027	.2342	696	59
2	.62977	075	.2334	678	58
3	.63000	123	.2327	660	57
4	022	171	.2320	641	56
5	.63045	.81220	1.2312	.77623	55
6	068	268	.2305	605	54
7	090	316	.2298	586	53
8	113	364	.2290	568	52
9	135	413	.2283	550	51
10	.63158	.81461	1.2276	.77531	50
11	180	510	.2268	513	49
12	203	558	.2261	494	48
13	225	606	.2254	476	47
14	248	655	.2247	458	46
15	.63271	.81703	1.2239	.77439	45
16	293	752	.2232	421	44
17	316	800	.2225	402	43
18	338	849	.2218	384	42
19	361	898	.2210	366	41
20	.63383	.81946	1.2203	.77347	40
21	406	.81995	.2196	329	39
22	428	.82044	.2189	310	38
23	451	092	.2181	292	37
24	473	141	.2174	273	36
25	.63496	.82190	1.2167	.77255	35
26	518	238	.2160	236	34
27	540	287	.2153	218	33
28	563	336	.2145	199	32
29	585	385	.2138	181	31
30	.63608	.82434	1.2131	.77162	30
31	630	483	.2124	144	29
32	653	531	.2117	125	28
33	675	580	.2109	107	27
34	698	629	.2102	088	26
35	.63720	.82678	1.2095	.77070	25
36	742	727	.2088	051	24
37	765	776	.2081	033	23
38	787	825	.2074	.77014	22
39	810	874	.2066	.76996	21
40	.63832	.82923	1.2059	.76977	20
41	854	.82972	.2052	959	19
42	877	.83022	.2045	940	18
43	899	071	.2038	921	17
44	922	120	.2031	903	16
45	.63944	.83169	1.2024	.76884	15
46	966	218	.2017	866	14
47	.63989	268	.2009	847	13
48	.64011	317	.2002	828	12
49	033	366	.1995	810	11
50	.64056	.83415	1.1988	.76791	10
51	078	465	.1981	772	9
52	100	514	.1974	754	8
53	123	564	.1967	735	7
54	145	613	.1960	717	6
55	.64167	.83662	1.1953	.76698	5
56	190	712	.1946	679	4
57	212	761	.1939	661	3
58	234	811	.1932	642	2
59	256	860	.1925	623	1
60	.64279	.83910	1.1918	.76604	0
	Cos	Ctn	Tan	Sin	'

50°

′	Sin	Tan	Ctn	Cos	
0	.64279	.83910	1.1918	.76604	60
1	301	.83960	.1910	586	59
2	323	.84009	.1903	567	58
3	346	059	.1896	548	57
4	368	108	.1889	530	56
5	.64390	.84158	1.1882	.76511	55
6	412	208	.1875	492	54
7	435	258	.1868	473	53
8	457	307	.1861	455	52
9	479	357	.1854	436	51
10	.64501	.84407	1.1847	.76417	50
11	524	457	.1840	398	49
12	546	507	.1833	380	48
13	568	556	.1826	361	47
14	590	606	.1819	342	46
15	.64612	.84656	1.1812	.76323	45
16	635	706	.1806	304	44
17	657	756	.1799	286	43
18	679	806	.1792	267	42
19	701	856	.1785	248	41
20	.64723	.84906	1.1778	.76229	40
21	746	.84956	.1771	210	39
22	768	.85006	.1764	192	38
23	790	057	.1757	173	37
24	812	107	.1750	154	36
25	.64834	.85157	1.1743	.76135	35
26	856	207	.1736	116	34
27	878	257	.1729	097	33
28	901	308	.1722	078	32
29	923	358	.1715	059	31
30	.64945	.85408	1.1708	.76041	30
31	967	458	.1702	022	29
32	.64989	509	.1695	.76003	28
33	.65011	559	.1688	.75984	27
34	033	609	.1681	965	26
35	.65055	.85660	1.1674	.75946	25
36	077	710	.1667	927	24
37	100	761	.1660	908	23
38	122	811	.1653	889	22
39	144	862	.1647	870	21
40	.65166	.85912	1.1640	.75851	20
41	188	.85963	.1633	832	19
42	210	.86014	.1626	813	18
43	232	064	.1619	794	17
44	254	115	.1612	775	16
45	.65276	.86166	1.1606	.75756	15
46	298	216	.1599	738	14
47	320	267	.1592	719	13
48	342	318	.1585	700	12
49	364	368	.1578	680	11
50	.65386	.86419	1.1571	.75661	10
51	408	470	.1565	642	9
52	430	521	.1558	623	8
53	452	572	.1551	604	7
54	474	623	.1544	585	6
55	.65496	.86674	1.1538	.75566	5
56	518	725	.1531	547	4
57	540	776	.1524	528	3
58	562	827	.1517	509	2
59	584	878	.1510	490	1
60	.65606	.86929	1.1504	.75471	0
	Cos	Ctn	Tan	Sin	′

49°

′	Sin	Tan	Ctn	Cos	
0	.65606	.86929	1.1504	.75471	60
1	628	.86980	.1497	452	59
2	650	.87031	.1490	433	58
3	672	082	.1483	414	57
4	694	133	.1477	395	56
5	.65716	.87184	1.1470	.75375	55
6	738	236	.1463	356	54
7	759	287	.1456	337	53
8	781	338	.1450	318	52
9	803	389	.1443	299	51
10	.65825	.87441	1.1436	.75280	50
11	847	492	.1430	261	49
12	869	543	.1423	241	48
13	891	595	.1416	222	47
14	913	646	.1410	203	46
15	.65935	.87698	1.1403	.75184	45
16	956	749	.1396	165	44
17	.65978	801	.1389	146	43
18	.66000	852	.1383	126	42
19	022	904	.1376	107	41
20	.66044	.87955	1.1369	.75088	40
21	066	.88007	.1363	069	39
22	088	059	.1356	050	38
23	109	110	.1349	030	37
24	131	162	.1343	.75011	36
25	.66153	.88214	1.1336	.74992	35
26	175	265	.1329	973	34
27	197	317	.1323	953	33
28	218	369	.1316	934	32
29	240	421	.1310	915	31
30	.66262	.88473	1.1303	.74896	30
31	284	524	.1296	876	29
32	306	576	.1290	857	28
33	327	628	.1283	838	27
34	349	680	.1276	818	26
35	.66371	.88732	1.1270	.74799	25
36	393	784	.1263	780	24
37	414	836	.1257	760	23
38	436	888	.1250	741	22
39	458	940	.1243	722	21
40	.66480	.88992	1.1237	.74703	20
41	501	.89045	.1230	683	19
42	523	097	.1224	664	18
43	545	149	.1217	644	17
44	566	201	.1211	625	16
45	.66588	.89253	1.1204	.74606	15
46	610	306	.1197	586	14
47	632	358	.1191	567	13
48	653	410	.1184	548	12
49	675	463	.1178	528	11
50	.66697	.89515	1.1171	.74509	10
51	718	567	.1165	489	9
52	740	620	.1158	470	8
53	762	672	.1152	451	7
54	783	725	.1145	431	6
55	.66805	.89777	1.1139	.74412	5
56	827	830	.1132	392	4
57	848	883	.1126	373	3
58	870	935	.1119	353	2
59	891	.89988	.1113	334	1
60	.66913	.90040	1.1106	.74314	0
	Cos	Ctn	Tan	Sin	′

48°

′	Sin	Tan	Ctn	Cos	
0	.66913	.90040	1.1106	.74314	60
1	935	093	.1100	295	59
2	956	146	.1093	276	58
3	978	199	.1087	256	57
4	.66999	251	.1080	237	56
5	.67021	.90304	1.1074	.74217	55
6	043	357	.1067	198	54
7	064	410	.1061	178	53
8	086	463	.1054	159	52
9	107	516	.1048	139	51
10	.67129	.90569	1.1041	.74120	50
11	151	621	.1035	100	49
12	172	674	.1028	080	48
13	194	727	.1022	061	47
14	215	781	.1016	041	46
15	.67237	.90834	1.1009	.74022	45
16	258	887	.1003	.74002	44
17	280	940	.0996	.73983	43
18	301	.90993	.0990	963	42
19	323	.91046	.0983	944	41
20	.67344	.91099	1.0977	.73924	40
21	366	153	.0971	904	39
22	387	206	.0964	885	38
23	409	259	.0958	865	37
24	430	313	.0951	846	36
25	.67452	.91366	1.0945	.73826	35
26	473	419	.0939	806	34
27	495	473	.0932	787	33
28	516	526	.0926	767	32
29	538	580	.0919	747	31
30	.67559	.91633	1.0913	.73728	30
31	580	687	.0907	708	29
32	602	740	.0900	688	28
33	623	794	.0894	669	27
34	645	847	.0888	649	26
35	.67666	.91901	1.0881	.73629	25
36	688	.91955	.0875	610	24
37	709	.92008	.0869	590	23
38	730	062	.0862	570	22
39	752	116	.0856	551	21
40	.67773	.92170	1.0850	.73531	20
41	795	224	.0843	511	19
42	816	277	.0837	491	18
43	837	331	.0831	472	17
44	859	385	.0824	452	16
45	.67880	.92439	1.0818	.73432	15
46	901	493	.0812	413	14
47	923	547	.0805	393	13
48	944	601	.0799	373	12
49	965	655	.0793	353	11
50	.67987	.92709	1.0786	.73333	10
51	.68008	763	.0780	314	9
52	029	817	.0774	294	8
53	051	872	.0768	274	7
54	072	926	.0761	254	6
55	.68093	.92980	1.0755	.73234	5
56	115	.93034	.0749	215	4
57	136	088	.0742	195	3
58	157	143	.0736	175	2
59	179	197	.0730	155	1
60	.68200	.93252	1.0724	.73135	0
	Cos	Ctn	Tan	Sin	′

47°

′	Sin	Tan	Ctn	Cos	
0	.68200	.93252	1.0724	.73135	60
1	221	306	.0717	116	59
2	242	360	.0711	096	58
3	264	415	.0705	076	57
4	285	469	.0699	056	56
5	.68306	.93524	1.0692	.73036	55
6	327	578	.0686	.73016	54
7	349	633	.0680	.72996	53
8	370	688	.0674	976	52
9	391	742	.0668	957	51
10	.68412	.93797	1.0661	.72937	50
11	434	852	.0655	917	49
12	455	906	.0649	897	48
13	476	.93961	.0643	877	47
14	497	.94016	.0637	857	46
15	.68518	.94071	1.0630	.72837	45
16	539	125	.0624	817	44
17	561	180	.0618	797	43
18	582	235	.0612	777	42
19	603	290	.0606	757	41
20	.68624	.94345	1.0599	.72737	40
21	645	400	.0593	717	39
22	666	455	.0587	697	38
23	688	510	.0581	677	37
24	709	565	.0575	657	36
25	.68730	.94620	1.0569	.72637	35
26	751	676	.0562	617	34
27	772	731	.0556	597	33
28	793	786	.0550	577	32
29	814	841	.0544	557	31
30	.68835	.94896	1.0538	.72537	30
31	857	.94952	.0532	517	29
32	878	.95007	.0526	497	28
33	899	062	.0519	477	27
34	920	118	.0513	457	26
35	.68941	.95173	1.0507	.72437	25
36	962	229	.0501	417	24
37	.68983	284	.0495	397	23
38	.69004	340	.0489	377	22
39	025	395	.0483	357	21
40	.69046	.95451	1.0477	.72337	20
41	067	506	.0470	317	19
42	088	562	.0464	297	18
43	109	618	.0458	277	17
44	130	673	.0452	257	16
45	.69151	.95729	1.0446	.72236	15
46	172	785	.0440	216	14
47	193	841	.0434	196	13
48	214	897	.0428	176	12
49	235	.95952	.0422	156	11
50	.69256	.96008	1.0416	.72136	10
51	277	064	.0410	116	9
52	298	120	.0404	095	8
53	319	176	.0398	075	7
54	340	232	.0392	055	6
55	.69361	.96288	1.0385	.72035	5
56	382	344	.0379	.72015	4
57	403	400	.0373	.71995	3
58	424	457	.0367	974	2
59	445	513	.0361	954	1
60	.69466	.96569	1.0355	.71934	0
	Cos	Ctn	Tan	Sin	′

46°

′	Sin	Tan	Ctn	Cos	
0	.69466	.96569	1.0355	.71934	**60**
1	487	625	.0349	914	59
2	508	681	.0343	894	58
3	529	738	.0337	873	57
4	549	794	.0331	853	56
5	.69570	.96850	1.0325	.71833	**55**
6	591	907	.0319	813	54
7	612	.96963	.0313	792	53
8	633	.97020	.0307	772	52
9	654	076	.0301	752	51
10	.69675	.97133	1.0295	.71732	**50**
11	696	189	.0289	711	49
12	717	246	.0283	691	48
13	737	302	.0277	671	47
14	758	359	.0271	650	46
15	.69779	.97416	1.0265	.71630	**45**
16	800	472	.0259	610	44
17	821	529	.0253	590	43
18	842	586	.0247	569	42
19	862	643	.0241	549	41
20	.69883	.97700	1.0235	.71529	**40**
21	904	756	.0230	508	39
22	925	813	.0224	488	38
23	946	870	.0218	468	37
24	966	927	.0212	447	36
25	.69987	.97984	1.0206	.71427	**35**
26	.70008	.98041	.0200	407	34
27	029	098	.0194	386	33
28	049	155	.0188	366	32
29	070	213	.0182	345	31
30	.70091	.98270	1.0176	.71325	**30**
31	112	327	.0170	305	29
32	132	384	.0164	284	28
33	153	441	.0158	264	27
34	174	499	.0152	243	26
35	.70195	.98556	1.0147	.71223	**25**
36	215	613	.0141	203	24
37	236	671	.0135	182	23
38	257	728	.0129	162	22
39	277	786	.0123	141	21
40	.70298	.98843	1.0117	.71121	**20**
41	319	901	.0111	100	19
42	339	.98958	.0105	080	18
43	360	.99016	.0099	059	17
44	381	073	.0094	039	16
45	.70401	.99131	1.0088	.71019	**15**
46	422	189	.0082	.70998	14
47	443	247	.0076	978	13
48	463	304	.0070	957	12
49	484	362	.0064	937	11
50	.70505	.99420	1.0058	.70916	**10**
51	525	478	.0052	896	9
52	546	536	.0047	875	8
53	567	594	.0041	855	7
54	587	652	.0035	834	6
55	.70608	.99710	1.0029	.70813	**5**
56	628	768	.0023	793	4
57	649	826	.0017	772	3
58	670	884	.0012	752	2
59	690	.99942	.0006	731	1
60	.70711	1.0000	1.0000	.70711	**0**
	Cos	Ctn	Tan	Sin	′

45°

TABLE III

COMMON LOGARITHMS

OF THE

TRIGONOMETRIC FUNCTIONS

FROM

0° TO 90° AT INTERVALS OF ONE MINUTE

TO

FIVE DECIMAL PLACES

From each logarithm given, subtract 10

Table IIIa—Auxiliary Table of S and T for A in Minutes

$S = \log \sin A - \log A'$ and $T = \log \tan A - \log A'$

A'	$S+10$
0′ – 13′	6.46373
14′ – 42′	72
43′ – 58′	71
59′ – 71′	6.46370
72′ – 81′	69
82′ – 91′	68
92′ – 99′	6.46367
100′ – 107′	66
108′ – 115′	65
116′ – 121′	6.46364
122′ – 128′	63
129′ – 134′	62
135′ – 140′	6.46361
141′ – 146′	60
147′ – 151′	.59
152′ – 157′	6.46358
158′ – 162′	57
163′ – 167′	56
168′ – 171′	6.46355
172′ – 176′	54
177′ – 181′	53

A'	$T+10$
0′ – 26′	6.46373
27′ – 39′	74
40′ – 48′	75
49′ – 56′	6.46376
57′ – 63′	77
64′ – 69′	78
70′ – 74′	6.46379
75′ – 80′	80
81′ – 85′	81
86′ – 89′	6.46382
90′ – 94′	83
95′ – 98′	84
99′ – 102′	6.46385
103′ – 106′	86
107′ – 110′	87
111′ – 113′	6.46388
114′ – 117′	89
118′ – 120′	90
121′ – 124′	6.46391
125′ – 127′	92
128′ – 130′	93

A'	$T+10$
131′ – 133′	6.46394
134′ – 136′	95
137′ – 139′	96
140′ – 142′	6.46397
143′ – 145′	98
146′ – 148′	99
149′ – 150′	6.46400
151′ – 153′	01
154′ – 156′	02
157′ – 158′	6.46403
159′ – 161′	04
162′ – 163′	05
164′ – 166′	6.46406
167′ – 168′	07
169′ – 171′	08
172′ – 173′	6.46409
174′ – 175′	10
176′ – 178′	11
179′ – 180′	6.46412
181′ – 182′	13
183′ – 184′	14

For small angles: $\log \sin A = \log A' + S$ and $\log \tan A = \log A' + T$.
For angles near 90°: $\log \cos A = \log (90° - A)' + S$, $\log \text{ctn}\, A = \log (90° - A)' + T$ where A' = number of minutes in A, and $(90° - A)'$ = number of minutes in $90° - A$.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	′
						10.00 000	60
	6.46 373	30103	6.46 373	30103	13.53 627	10.00 000	59
	6.76 476	17609	6.76 476	17609	13.23 524	10.00 000	58
	6.94 085	12494	6.94 085	12494	13.05 915	10.00 000	57
	7.06 579	9691	7.06 579	9691	12.93 421	10.00 000	56
	7.16 270	7918	7.16 270	7918	12.83 730	10.00 000	55
	7.24 188	6694	7.24 188	6694	12.75 812	10.00 000	54
	7.30 882	5800	7.30 882	5800	12.69 118	10.00 000	53
	7.36 682	5115	7.36 682	5115	12.63 318	10.00 000	52
	7.41 797	4576	7.41 797	4576	12.58 203	10.00 000	51
	7.46 373	4139	7.46 373	4139	12.53 627	10.00 000	50
	7.50 512	3779	7.50 512	3779	12.49 488	10.00 000	49
	7.54 291	3476	7.54 291	3476	12.45 709	10.00 000	48
	7.57 767	3218	7.57 767	3219	12.42 233	10.00 000	47
	7.60 985	2997	7.60 986	2996	12.39 014	10.00 000	46
	7.63 982	2802	7.63 982	2803	12.36 018	10.00 000	45
	7.66 784	2633	7.66 785	2633	12.33 215	10.00 000	44
	7.69 417	2483	7.69 418	2482	12.30 582	9.99 999	43
	7.71 900	2348	7.71 900	2348	12.28 100	9.99 999	42
	7.74 248	2227	7.74 248	2228	12.25 752	9.99 999	41
	7.76 475	2119	7.76 476	2119	12.23 524	9.99 999	40
21	7.78 594	2021	7.78 595	2020	12.21 405	9.99 999	39
22	7.80 615	1930	7.80 615	1931	12.19 385	9.99 999	38
23	7.82 545	1848	7.82 546	1848	12.17 454	9.99 999	37
24	7.84 393	1773	7.84 394	1773	12.15 606	9.99 999	36
25	7.86 166	1704	7.86 167	1704	12.13 833	9.99 999	35
26	7.87 870	1639	7.87 871	1639	12.12 129	9.99 999	34
27	7.89 509	1579	7.89 510	1579	12.10 490	9.99 999	33
28	7.91 088	1524	7.91 089	1524	12.08 911	9.99 999	32
29	7.92 612	1472	7.92 613	1473	12.07 387	9.99 998	31
30	7.94 084	1424	7.94 086	1424	12.05 914	9.99 998	30
31	7.95 508	1379	7.95 510	1379	12.04 490	9.99 998	29
32	7.96 887	1336	7.96 889	1336	12.03 111	9.99 998	28
33	7.98 223	1297	7.98 225	1297	12.01 775	9.99 998	27
34	7.99 520	1259	7.99 522	1259	12.00 478	9.99 998	26
35	8.00 779	1223	8.00 781	1223	11.99 219	9.99 998	25
36	8.02 002	1190	8.02 004	1190	11.97 996	9.99 998	24
37	8.03 192	1158	8.03 194	1159	11.96 806	9.99 997	23
38	8.04 350	1128	8.04 353	1128	11.95 647	9.99 997	22
39	8.05 478	1100	8.05 481	1100	11.94 519	9.99 997	21
40	8.06 578	1072	8.06 581	1072	11.93 419	9.99 997	20
41	8.07 650	1046	8.07 653	1047	11.92 347	9.99 997	19
42	8.08 696	1022	8.08 700	1022	11.91 300	9.99 997	18
43	8.09 718	999	8.09 722	998	11.90 278	9.99 997	17
44	8.10 717	976	8.10 720	976	11.89 280	9.99 996	16
45	8.11 693	954	8.11 696	955	11.88 304	9.99 996	15
46	8.12 647	934	8.12 651	934	11.87 349	9.99 996	14
47	8.13 581	914	8.13 585	915	11.86 415	9.99 996	13
48	8.14 495	896	8.14 500	895	11.85 500	9.99 996	12
49	8.15 391	877	8.15 395	878	11.84 605	9.99 996	11
50	8.16 268	860	8.16 273	860	11.83 727	9.99 995	10
51	8.17 128	843	8.17 133	843	11.82 867	9.99 995	9
52	8.17 971	827	8.17 976	828	11.82 024	9.99 995	8
53	8.18 798	812	8.18 804	812	11.81 196	9.99 995	7
54	8.19 610	797	8.19 616	797	11.80 384	9.99 995	6
55	8.20 407	782	8.20 413	782	11.79 587	9.99 994	5
56	8.21 189	769	8.21 195	769	11.78 805	9.99 994	4
57	8.21 958	755	8.21 964	756	11.78 036	9.99 994	3
58	8.2[illegible] 713	743	8.22 720	742	11.77 280	9.99 994	2
59	8.[illegible]	730	8.23 462	730	11.76 538	9.99 994	1
60	[illegible]186		8.24 192		11.75 808	9.99 993	0
			L Ctn	c d	L Tan	L Sin	

Fo[illegible] o[illegible] or [illegible] ngents o[illegible] ngles les[illegible] th[illegible] [illegible]arithms or co[illegible] ugents [illegible]f [illegible]gles greater than 87°), s[illegible] Table III*a*, p. 45.

When the tabular differences are large, that method is usually be[illegible]ter. The proportional parts [illegible]ted for 1° and 2° in this table are sufficien[illegible] hen great accuracy is not req[illegible]ed, even if the ordinary method of interpolation is used.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	
0	8.24 186	717	8.24 192	718	11.75 808	9.99 993	60
1	8.24 903	706	8.24 910	706	11.75 090	9.99 993	59
2	8.25 609	695	8.25 616	696	11.74 384	9.99 993	58
3	8.26 304	684	8.26 312	684	11.73 688	9.99 993	57
4	8.26 988	673	8.26 996	673	11.73 004	9.99 992	56
5	8.27 661	663	8.27 669	663	11.72 331	9.99 992	55
6	8.28 324	653	8.28 332	654	11.71 668	9.99 992	54
7	8.28 977	644	8.28 986	643	11.71 014	9.99 992	53
8	8.29 621	634	8.29 629	634	11.70 371	9.99 992	52
9	8.30 255	624	8.30 263	625	11.69 737	9.99 991	51
10	8.30 879	616	8.30 888	617	11.69 112	9.99 991	50
11	8.31 495	608	8.31 505	607	11.68 495	9.99 991	49
12	8.32 103	599	8.32 112	599	11.67 888	9.99 990	48
13	8.32 702	590	8.32 711	591	11.67 289	9.99 990	47
14	8.33 292	583	8.33 302	584	11.66 698	9.99 990	46
15	8.33 875	575	8.33 886	575	11.66 114	9.99 990	45
16	8.34 450	568	8.34 461	568	11.65 539	9.99 989	44
17	8.35 018	560	8.35 029	561	11.64 971	9.99 989	43
18	8.35 578	553	8.35 590	553	11.64 410	9.99 989	42
19	8.36 131	547	8.36 143	546	11.63 857	9.99 989	41
20	8.36 678	539	8.36 689	540	11.63 311	9.99 988	40
21	8.37 217	533	8.37 229	533	11.62 771	9.99 988	39
22	8.37 750	526	8.37 762	527	11.62 238	9.99 988	38
23	8.38 276	520	8.38 289	520	11.61 711	9.99 987	37
24	8.38 796	514	8.38 809	514	11.61 191	9.99 987	36
25	8.39 310	508	8.39 323	509	11.60 677	9.99 987	35
26	8.39 818	502	8.39 832	502	11.60 168	9.99 986	34
27	8.40 320	496	8.40 334	496	11.59 666	9.99 986	33
28	8.40 816	491	8.40 830	491	11.59 170	9.99 986	32
29	8.41 307	485	8.41 321	486	11.58 679	9.99 985	31
30	8.41 792	480	8.41 807	480	11.58 193	9.99 985	30
31	8.42 272	474	8.42 287	475	11.57 713	9.99 985	29
32	8.42 746	470	8.42 762	470	11.57 238	9.99 984	28
33	8.43 216	464	8.43 232	464	11.56 768	9.99 984	27
34	8.43 680	459	8.43 696	460	11.56 304	9.99 984	26
35	8.44 139	455	8.44 156	455	11.55 844	9.99 983	25
36	8.44 594	450	8.44 611	450	11.55 389	9.99 983	24
37	8.45 044	445	8.45 061	446	11.54 939	9.99 983	23
38	8.45 489	441	8.45 507	441	11.54 493	9.99 982	22
39	8.45 930	436	8.45 948	437	11.54 052	9.99 982	21
40	8.46 366	433	8.46 385	432	11.53 615	9.99 982	20
41	8.46 799	427	8.46 817	428	11.53 183	9.99 981	19
42	8.47 226	424	8.47 245	424	11.52 755	9.99 981	18
43	8.47 650	419	8.47 669	420	11.52 331	9.99 981	17
44	8.48 069	416	8.48 089	416	11.51 911	9.99 980	16
45	8.48 485	411	8.48 505	412	11.51 495	9.99 980	15
46	8.48 896	408	8.48 917	408	11.51 083	9.99 979	14
47	8.49 304	404	8.49 325	404	11.50 675	9.99 979	13
48	8.49 708	400	8.49 729	401	11.50 271	9.99 979	12
49	8.50 108	396	8.50 130	397	11.49 870	9.99 978	11
50	8.50 504	393	8.50 527	393	11.49 473	9.99 978	10
51	8.50 897	390	8.50 920	390	11.49 080	9.99 977	9
52	8.51 287	386	8.51 310	386	11.48 690	9.99 977	8
53	8.51 673	382	8.51 696	383	11.48 304	9.99 977	7
54	8.52 055	379	8.52 079	380	11.47 921	9.99 976	6
55	8.52 434	376	8.52 459	376	11.47 541	9.99 976	5
56	8.52 810	373	8.52 835	373	11.47 165	9.99 975	4
57	8.53 183	369	8.53 208	370	11.46 792	9.99 975	3
58	8.53 552	367	8.53 578	367	11.46 422	9.99 974	2
59	8.53 919	363	8.53 945	363	11.46 055	9.99 974	1
60	8.54 282		8.54 308		11.45 692	9.99 974	0
	L Cos	d	L Ctn	c d	L Tan	L Sin	′

Prop. Pts.

	710	690	670	650
2	142	138	134	130
3	213	207	201	195
4	284	276	268	260
5	355	345	335	325
6	426	414	402	390
7	497	483	469	455
8	568	552	536	520
9	639	621	603	585
	630	620	610	600
2	126	124	122	120
3	189	186	183	180
4	252	248	244	240
5	315	310	305	300
6	378	372	366	360
7	441	434	427	420
8	504	496	488	480
9	567	558	549	540
	590	580	570	560
2	118	116	114	112
3	177	174	171	168
4	236	232	228	224
5	295	290	285	280
6	354	348	342	336
7	413	406	399	392
8	472	464	456	448
9	531	522	513	504
	550	540	530	520
2	110	108	106	104
3	165	162	159	156
4	220	216	212	208
5	275	270	265	260
6	330	324	318	312
7	385	378	371	364
8	440	432	424	416
9	495	486	477	468
	510	500	490	480
2	102	100	98	96
3	153	150	147	144
4	204	200	196	192
5	255	250	245	240
6	306	300	294	288
7	357	350	343	336
8	408	400	392	384
9	459	450	441	432
	470	460	450	440
2	94	92	90	88
3	141	138	135	132
4	188	184	180	176
5	235	230	225	220
6	282	276	270	264
7	329	322	315	308
8	376	368	360	352
9	423	414	405	396
	430	420	410	400
2	86	84	82	80
3	129	126	123	120
4	172	168	164	160
5	215	210	205	200
6	258	252	246	240
7	301	294	287	280
8	344	336	328	320
9	387	378	369	360
	390	380	370	360
2	78	76	74	72
3	117	114	111	108
4	156	152	148	144
5	195	190	185	180
6	234	228	222	216
7	273	266	259	252
8	312	304	296	288
9	351	342	333	324

Prop. Pts.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	′
0	8.54 282	360	8.54 308	361	11.45 692	9.99 974	60
1	8.54 642	357	8.54 669	358	11.45 331	9.99 973	59
2	8.54 999	355	8.55 027	355	11.44 973	9.99 973	58
3	8.55 354	351	8.55 382	352	11.44 618	9.99 972	57
4	8.55 705	349	8.55 734	349	11.44 266	9.99 972	56
5	8.56 054	346	8.56 083	346	11.43 917	9.99 971	55
6	8.56 400	343	8.56 429	344	11.43 571	9.99 971	54
7	8.56 743	341	8.56 773	341	11.43 227	9.99 970	53
8	8.57 084	337	8.57 114	338	11.42 886	9.99 970	52
9	8.57 421	336	8.57 452	336	11.42 548	9.99 969	51
10	8.57 757	332	8.57 788	333	11.42 212	9.99 969	50
11	8.58 089	330	8.58 121	330	11.41 879	9.99 968	49
12	8.58 419	328	8.58 451	328	11.41 549	9.99 968	48
13	8.58 747	325	8.58 779	326	11.41 221	9.99 967	47
14	8.59 072	323	8.59 105	323	11.40 895	9.99 967	46
15	8.59 395	320	8.59 428	321	11.40 572	9.99 967	45
16	8.59 715	318	8.59 749	319	11.40 251	9.99 966	44
17	8.60 033	316	8.60 068	316	11.39 932	9.99 966	43
18	8.60 349	313	8.60 384	314	11.39 616	9.99 965	42
19	8.60 662	311	8.60 698	311	11.39 302	9.99 964	41
20	8.60 973	309	8.61 009	310	11.38 991	9.99 964	40
21	8.61 282	307	8.61 319	307	11.38 681	9.99 963	39
22	8.61 589	305	8.61 626	305	11.38 374	9.99 963	38
23	8.61 894	302	8.61 931	303	11.38 069	9.99 962	37
24	8.62 196	301	8.62 234	301	11.37 766	9.99 962	36
25	8.62 497	298	8.62 535	299	11.37 465	9.99 961	35
26	8.62 795	296	8.62 834	297	11.37 166	9.99 961	34
27	8.63 091	294	8.63 131	295	11.36 869	9.99 960	33
28	8.63 385	293	8.63 426	292	11.36 574	9.99 960	32
29	8.63 678	290	8.63 718	291	11.36 282	9.99 959	31
30	8.63 968	288	8.64 009	289	11.35 991	9.99 959	30
31	8.64 256	287	8.64 298	287	11.35 702	9.99 958	29
32	8.64 543	284	8.64 585	285	11.35 415	9.99 958	28
33	8.64 827	283	8.64 870	284	11.35 130	9.99 957	27
34	8.65 110	281	8.65 154	281	11.34 846	9.99 956	26
35	8.65 391	279	8.65 435	280	11.34 565	9.99 956	25
36	8.65 670	277	8.65 715	278	11.34 285	9.99 955	24
37	8.65 947	276	8.65 993	276	11.34 007	9.99 955	23
38	8.66 223	274	8.66 269	274	11.33 731	9.99 954	22
39	8.66 497	272	8.66 543	273	11.33 457	9.99 954	21
40	8.66 769	270	8.66 816	271	11.33 184	9.99 953	20
41	8.67 039	269	8.67 087	269	11.32 913	9.99 952	19
42	8.67 308	267	8.67 356	268	11.32 644	9.99 952	18
43	8.67 575	266	8.67 624	266	11.32 376	9.99 951	17
44	8.67 841	263	8.67 890	264	11.32 110	9.99 951	16
45	8.68 104	263	8.68 154	263	11.31 846	9.99 950	15
46	8.68 367	260	8.68 417	261	11.31 583	9.99 949	14
47	8.68 627	259	8.68 678	260	11.31 322	9.99 949	13
48	8.68 886	258	8.68 938	258	11.31 062	9.99 948	12
49	8.69 144	256	8.69 196	257	11.30 804	9.99 948	11
50	8.69 400	254	8.69 453	255	11.30 547	9.99 947	10
51	8.69 654	253	8.69 708	254	11.30 292	9.99 946	9
52	8.69 907	252	8.69 962	252	11.30 038	9.99 946	8
53	8.70 159	250	8.70 214	251	11.29 786	9.99 945	7
54	8.70 409	249	8.70 465	249	11.29 535	9.99 944	6
55	8.70 658	247	8.70 714	248	11.29 286	9.99 944	5
56	8.70 905	246	8.70 962	246	11.29 038	9.99 943	4
57	8.71 151	244	8.71 208	245	11.28 792	9.99 942	3
58	8.71 395	243	8.71 453	244	11.28 547	9.99 942	2
59	8.71 638	242	8.71 697	243	11.28 303	9.99 941	1
60	8.71 880		8.71 940		11.28 060	9.99 940	0
	L Cos	d	L Ctn	c d	L Tan	L Sin	′

Prop. Pts.

	360	355	350	345	340	335	330	325	320	315	310	305
2	72	71.0	70	69.0	68	67.0	66	65.0	64	63.0	62	61.0
3	108	106.5	105	103.5	102	100.5	99	97.5	96	94.5	93	91.5
4	144	142.0	140	138.0	136	134.0	132	130.0	128	126.0	124	122.0
5	180	177.5	175	172.5	170	167.5	165	162.5	160	157.5	155	152.5
6	216	213.0	210	207.0	204	201.0	198	195.0	192	189.0	186	183.0
7	252	248.5	245	241.5	238	234.5	231	227.5	224	220.5	217	213.5
8	288	284.0	280	276.0	272	268.0	264	260.0	256	252.0	248	244.0
9	324	319.5	315	310.5	306	301.5	297	292.5	288	283.5	279	274.5

	300	295	290	285	280	275	270	265	260	255	250	245
2	60	59.0	58	57.0	56	55.0	54	53.0	52	51.0	50	49.0
3	90	88.5	87	85.5	84	82.5	81	79.5	78	76.5	75	73.5
4	120	118.0	116	114.0	112	110.0	108	106.0	104	102.0	100	98.0
5	150	147.5	145	142.5	140	137.5	135	132.5	130	127.5	125	122.5
6	180	177.0	174	171.0	168	165.0	162	159.0	156	153.0	150	147.0
7	210	206.5	203	199.5	196	192.5	189	185.5	182	178.5	175	171.5
8	240	236.0	232	228.0	224	220.0	216	212.0	208	204.0	200	196.0
9	270	265.5	261	256.5	252	247.5	243	238.5	234	229.5	225	220.5

Prop. Pts.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	
0	8.71 880	240	8.71 940	241	11.28 060	9.99 940	60
1	8.72 120	239	8.72 181	239	11.27 819	9.99 940	59
2	8.72 359	238	8.72 420	239	11.27 580	9.99 939	58
3	8.72 597	237	8.72 659	237	11.27 341	9.99 938	57
4	8.72 834	235	8.72 896	236	11.27 104	9.99 938	56
5	8.73 069	234	8.73 132	234	11.26 868	9.99 937	55
6	8.73 303	232	8.73 366	234	11.26 634	9.99 936	54
7	8.73 535	232	8.73 600	232	11.26 400	9.99 936	53
8	8.73 767	230	8.73 832	231	11.26 168	9.99 935	52
9	8.73 997	229	8.74 063	229	11.25 937	9.99 934	51
10	8.74 226	228	8.74 292	229	11.25 708	9.99 934	50
11	8.74 454	226	8.74 521	227	11.25 479	9.99 933	49
12	8.74 680	226	8.74 748	226	11.25 252	9.99 932	48
13	8.74 906	224	8.74 974	225	11.25 026	9.99 932	47
14	8.75 130	223	8.75 199	224	11.24 801	9.99 931	46
15	8.75 353	222	8.75 423	222	11.24 577	9.99 930	45
16	8.75 575	220	8.75 645	222	11.24 355	9.99 929	44
17	8.75 795	220	8.75 867	220	11.24 133	9.99 929	43
18	8.76 015	219	8.76 087	219	11.23 913	9.99 928	42
19	8.76 234	217	8.76 306	219	11.23 694	9.99 927	41
20	8.76 451	216	8.76 525	217	11.23 475	9.99 926	40
21	8.76 667	216	8.76 742	216	11.23 258	9.99 926	39
22	8.76 883	214	8.76 958	215	11.23 042	9.99 925	38
23	8.77 097	213	8.77 173	214	11.22 827	9.99 924	37
24	8.77 310	212	8.77 387	213	11.22 613	9.99 923	36
25	8.77 522	211	8.77 600	211	11.22 400	9.99 923	35
26	8.77 733	210	8.77 811	211	11.22 189	9.99 922	34
27	8.77 943	209	8.78 022	210	11.21 978	9.99 921	33
28	8.78 152	208	8.78 232	209	11.21 768	9.99 920	32
29	8.78 360	208	8.78 441	208	11.21 559	9.99 920	31
30	8.78 568	206	8.78 649	206	11.21 351	9.99 919	30
31	8.78 774	205	8.78 855	206	11.21 145	9.99 918	29
32	8.78 979	204	8.79 061	205	11.20 939	9.99 917	28
33	8.79 183	203	8.79 266	204	11.20 734	9.99 917	27
34	8.79 386	202	8.79 470	203	11.20 530	9.99 916	26
35	8.79 588	201	8.79 673	202	11.20 327	9.99 915	25
36	8.79 789	201	8.79 875	201	11.20 125	9.99 914	24
37	8.79 990	199	8.80 076	201	11.19 924	9.99 913	23
38	8.80 189	199	8.80 277	199	11.19 723	9.99 913	22
39	8.80 388	197	8.80 476	198	11.19 524	9.99 912	21
40	8.80 585	197	8.80 674	198	11.19 326	9.99 911	20
41	8.80 782	196	8.80 872	196	11.19 128	9.99 910	19
42	8.80 978	195	8.81 068	196	11.18 932	9.99 909	18
43	8.81 173	194	8.81 264	195	11.18 736	9.99 909	17
44	8.81 367	193	8.81 459	194	11.18 541	9.99 908	16
45	8.81 560	192	8.81 653	193	11.18 347	9.99 907	15
46	8.81 752	192	8.81 846	192	11.18 154	9.99 906	14
47	8.81 944	190	8.82 038	192	11.17 962	9.99 905	13
48	8.82 134	190	8.82 230	190	11.17 770	9.99 904	12
49	8.82 324	189	8.82 420	190	11.17 580	9.99 904	11
50	8.82 513	188	8.82 610	189	11.17 390	9.99 903	10
51	8.82 701	187	8.82 799	188	11.17 201	9.99 902	9
52	8.82 888	187	8.82 987	188	11.17 013	9.99 901	8
53	8.83 075	186	8.83 175	186	11.16 825	9.99 900	7
54	8.83 261	185	8.83 361	186	11.16 639	9.99 899	6
55	8.83 446	184	8.83 547	185	11.16 453	9.99 898	5
56	8.83 630	183	8.83 732	184	11.16 268	9.99 898	4
57	8.83 813	183	8.83 916	184	11.16 084	9.99 897	3
58	8.83 996	181	8.84 100	182	11.15 900	9.99 896	2
59	8.84 177	181	8.84 282	182	11.15 718	9.99 895	1
60	8.84 358		8.84 464		11.15 536	9.99 894	0
	L Cos	d	L Ctn	c d	L Tan	L Sin	′

Prop. Pts.

	240	235	230
2	48	47.0	46
3	72	70.5	69
4	96	94.0	92
5	120	117.5	115
6	144	141.0	138
7	168	164.5	161
8	192	188.0	184
9	216	211.5	207

	225	220	215
2	45.0	44.0	43.0
3	67.5	66.0	64.5
4	90.0	88.0	86.0
5	112.5	110.0	107.5
6	135.0	132.0	129.0
7	157.5	154.0	150.5
8	180.0	176.0	172.0
9	202.5	198.0	193.5

	213	211	208
2	42.6	42.2	41.6
3	63.9	63.3	62.4
4	85.2	84.4	83.2
5	106.5	105.5	104.0
6	127.8	126.6	124.8
7	149.1	147.7	145.6
8	170.4	168.8	166.4
9	191.7	189.9	187.2

	206	203	201
2	41.2	40.6	40.2
3	61.8	60.9	60.3
4	82.4	81.2	80.4
5	103.0	101.5	100.5
6	123.6	121.8	120.6
7	144.2	142.1	140.7
8	164.8	162.4	160.8
9	185.4	182.7	180.9

	199	197	195
2	39.8	39.4	39.0
3	59.7	59.1	58.5
4	79.6	78.8	78.0
5	99.5	98.5	97.5
6	119.4	118.2	117.0
7	139.3	137.9	136.5
8	159.2	157.6	156.0
9	179.1	177.3	175.5

	193	192	190
2	38.6	38.4	38.0
3	57.9	57.6	57.0
4	77.2	76.8	76.0
5	96.5	96.0	95.0
6	115.8	115.2	114.0
7	135.1	134.4	133.0
8	154.4	153.6	152.0
9	173.7	172.8	171.0

	188	186	184
2	37.6	37.2	36.8
3	56.4	55.8	55.2
4	75.2	74.4	73.6
5	94.0	93.0	92.0
6	112.8	111.6	110.4
7	131.6	130.2	128.8
8	150.4	148.8	147.2
9	169.2	167.4	165.6

	183	182	181
2	36.6	36.4	36.2
3	54.9	54.6	54.3
4	73.2	72.8	72.4
5	91.5	91.0	90.5
6	109.8	109.2	108.6
7	128.1	127.4	126.7
8	146.4	145.6	144.8
9	164.7	163.8	162.9

Prop. Pts.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	
0	8.84 358	181	8.84 464	182	11.15 536	9.99 894	60
1	8.84 539	179	8.84 646	180	11.15 354	9.99 893	59
2	8.84 718	179	8.84 826	180	11.15 174	9.99 892	58
3	8.84 897	178	8.85 006	179	11.14 994	9.99 891	57
4	8.85 075	177	8.85 185	178	11.14 815	9.99 891	56
5	8.85 252	177	8.85 363	177	11.14 637	9.99 890	55
6	8.85 429	176	8.85 540	177	11.14 460	9.99 889	54
7	8.85 605	175	8.85 717	176	11.14 283	9.99 888	53
8	8.85 780	175	8.85 893	176	11.14 107	9.99 887	52
9	8.85 955	173	8.86 069	174	11.13 931	9.99 886	51
10	8.86 128	173	8.86 243	174	11.13 757	9.99 885	50
11	8.86 301	173	8.86 417	174	11.13 583	9.99 884	49
12	8.86 474	171	8.86 591	172	11.13 409	9.99 883	48
13	8.86 645	171	8.86 763	172	11.13 237	9.99 882	47
14	8.86 816	171	8.86 935	171	11.13 065	9.99 881	46
15	8.86 987	169	8.87 106	171	11.12 894	9.99 880	45
16	8.87 156	169	8.87 277	170	11.12 723	9.99 879	44
17	8.87 325	169	8.87 447	169	11.12 553	9.99 879	43
18	8.87 494	167	8.87 616	169	11.12 384	9.99 878	42
19	8.87 661	168	8.87 785	168	11.12 215	9.99 877	41
20	8.87 829	166	8.87 953	167	11.12 047	9.99 876	40
21	8.87 995	166	8.88 120	167	11.11 880	9.99 875	39
22	8.88 161	165	8.88 287	166	11.11 713	9.99 874	38
23	8.88 326	164	8.88 453	165	11.11 547	9.99 873	37
24	8.88 490	164	8.88 618	165	11.11 382	9.99 872	36
25	8.88 654	163	8.88 783	165	11.11 217	9.99 871	35
26	8.88 817	163	8.88 948	163	11.11 052	9.99 870	34
27	8.88 980	162	8.89 111	163	11.10 889	9.99 869	33
28	8.89 142	162	8.89 274	163	11.10 726	9.99 868	32
29	8.89 304	160	8.89 437	161	11.10 563	9.99 867	31
30	8.89 464	161	8.89 598	162	11.10 402	9.99 866	30
31	8.89 625	159	8.89 760	160	11.10 240	9.99 865	29
32	8.89 784	159	8.89 920	160	11.10 080	9.99 864	28
33	8.89 943	159	8.90 080	160	11.09 920	9.99 863	27
34	8.90 102	158	8.90 240	159	11.09 760	9.99 862	26
35	8.90 260	157	8.90 399	158	11.09 601	9.99 861	25
36	8.90 417	157	8.90 557	158	11.09 443	9.99 860	24
37	8.90 574	156	8.90 715	157	11.09 285	9.99 859	23
38	8.90 730	155	8.90 872	157	11.09 128	9.99 858	22
39	8.90 885	155	8.91 029	156	11.08 971	9.99 857	21
40	8.91 040	155	8.91 185	155	11.08 815	9.99 856	20
41	8.91 195	154	8.91 340	155	11.08 660	9.99 855	19
42	8.91 349	153	8.91 495	155	11.08 505	9.99 854	18
43	8.91 502	153	8.91 650	153	11.08 350	9.99 853	17
44	8.91 655	152	8.91 803	154	11.08 197	9.99 852	16
45	8.91 807	152	8.91 957	153	11.08 043	9.99 851	15
46	8.91 959	151	8.92 110	152	11.07 890	9.99 850	14
47	8.92 110	151	8.92 262	152	11.07 738	9.99 848	13
48	8.92 261	150	8.92 414	151	11.07 586	9.99 847	12
49	8.92 411	150	8.92 565	151	11.07 435	9.99 846	11
50	8.92 561	149	8.92 716	150	11.07 284	9.99 845	10
51	8.92 710	149	8.92 866	150	11.07 134	9.99 844	9
52	8.92 859	148	8.93 016	149	11.06 984	9.99 843	8
53	8.93 007	147	8.93 165	148	11.06 835	9.99 842	7
54	8.93 154	147	8.93 313	149	11.06 687	9.99 841	6
55	8.93 301	147	8.93 462	147	11.06 538	9.99 840	5
56	8.93 448	146	8.93 609	147	11.06 391	9.99 839	4
57	8.93 594	146	8.93 756	147	11.06 244	9.99 838	3
58	8.93 740	145	8.93 903	146	11.06 097	9.99 837	2
59	8.93 885	145	8.94 049	146	11.05 951	9.99 836	1
60	8.94 030		8.94 195		11.05 805	9.99 834	0
	L Cos	d	L Ctn	c d	L Tan	L Sin	′

Prop. Pts.

	181	180	179
2	36.2	36.0	35.8
3	54.3	54.0	53.7
4	72.4	72.0	71.6
5	90.5	90.0	89.5
6	108.6	108.0	107.4
7	126.7	126.0	125.3
8	144.8	144.0	143.2
9	162.9	162.0	161.1

	177	175	173
2	35.4	35.0	34.6
3	53.1	52.5	51.9
4	70.8	70.0	69.2
5	88.5	87.5	86.5
6	106.2	105.0	103.8
7	123.9	122.5	121.1
8	141.6	140.0	138.4
9	159.3	157.5	155.7

	171	170	169
2	34.2	34.0	33.8
3	51.3	51.0	50.7
4	68.4	68.0	67.6
5	85.5	85.0	84.5
6	102.6	102.0	101.4
7	119.7	119.0	118.3
8	136.8	136.0	135.2
9	153.9	153.0	152.1

	167	165	163
2	33.4	33.0	32.6
3	50.1	49.5	48.9
4	66.8	66.0	65.2
5	83.5	82.5	81.5
6	100.2	99.0	97.8
7	116.9	115.5	114.1
8	133.6	132.0	130.4
9	150.3	148.5	146.7

	161	160	159
2	32.2	32.0	31.8
3	48.3	48.0	47.7
4	64.4	64.0	63.6
5	80.5	80.0	79.5
6	96.6	96.0	95.4
7	112.7	112.0	111.3
8	128.8	128.0	127.2
9	144.9	144.0	143.1

	157	155	153
2	31.4	31.0	30.6
3	47.1	46.5	45.9
4	62.8	62.0	61.2
5	78.5	77.5	76.5
6	94.2	93.0	91.8
7	109.9	108.5	107.1
8	125.6	124.0	122.4
9	141.3	139.5	137.7

	151	150	149
2	30.2	30.0	29.8
3	45.3	45.0	44.7
4	60.4	60.0	59.6
5	75.5	75.0	74.5
6	90.6	90.0	89.4
7	105.7	105.0	104.3
8	120.8	120.0	119.2
9	135.9	135.0	134.1

	147	145	144
2	29.4	29.0	28.8
3	44.1	43.5	43.2
4	58.8	58.0	57.6
5	73.5	72.5	72.0
6	88.2	87.0	86.4
7	102.9	101.5	100.8
8	117.6	116.0	115.2
9	132.3	130.5	129.6

Prop. Pts.

L Sin	d	L Tan	c d	L Ctn	L Cos	
3.94 030	.44	3.94 195	.45	11.05 805	).99 834	
3.94 174	.43	.94 340	.45	11.05 660	9.99 833	
3.94 317	.44	.94 485	.45	11.05 515	9.99 832	
3.94 461	.42	3.94 630	.43	11.05 370	.99 831	
3.94 603	.43	3.94 773	.44	11.05 227	9.99 830	
8.94 746	.41	.94 917	.43	11.05 083	9.99 829	
3.94 887	.42	3.95 060	.42	11.04 940	.99 828	
3.95 029	.41	8.95 202	.42	11.04 798	.99 827	
3.95 170	.40	3.95 344	142	11.04 656	9.99 825	
.95 310	.40	.95 486	141	11.04 514	9.99 824	
.95 450	139	.95 627	140	11.04 373	9.99 823	
8.95 589	139	8.95 767	141	11.04 233	9.99 822	
8.95 728	139	8.95 908	139	11.04 092	9.99 821	48
8.95 867	138	8.96 047	140	11.03 953	9.99 820	47
8.96 005	138	8.96 187	138	11.03 813	9.99 819	46
8.96 143	137	8.96 325	139	11.03 675	9.99 817	45
8.96 280	137	8.96 464	138	11.03 536	9.99 816	44
8.96 417	136	8.96 602	137	11.03 398	9.99 815	43
8.96 553	136	8.96 739	138	11.03 261	9.99 814	42
8.96 689	136	8.96 877	136	11.03 123	9.99 813	41
8.96 825	135	8.97 013	137	11.02 987	9.99 812	40
8.96 960	135	8.97 150	135	11.02 850	9.99 810	39
8.97 095	134	8.97 285	136	11.02 715	9.99 809	38
8.97 229	134	8.97 421	135	11.02 579	9.99 808	37
8.97 363	133	8.97 556	135	11.02 444	9.99 807	36
8.97 496	133	8.97 691	134	11.02 309	9.99 806	35
8.97 629	133	8.97 825	134	11.02 175	9.99 804	34
8.97 762	132	8.97 959	133	11.02 041	9.99 803	33
8.97 894	132	8.98 092	133	11.01 908	9.99 802	32
8.98 026	131	8.98 225	133	11.01 775	9.99 801	31
8.98 157	131	8.98 358	132	11.01 642	9.99 800	30
8.98 288	131	8.98 490	132	11.01 510	9.99 798	29
8.98 419	130	8.98 622	131	11.01 378	9.99 797	28
8.98 549	130	8.98 753	131	11.01 247	9.99 796	27
8.98 679	129	8.98 884	131	11.01 116	9.99 795	26
8.98 808	129	8.99 015	130	11.00 985	9.99 793	25
8.98 937	129	8.99 145	130	11.00 85[illegible]	9.99 79[illegible]	24
8.99 066	128	8.99 275	130	11.00 725	9.99 791	23
8.99 194	128	8.99 405	129	11.00 595	9.99 790	2[illegible]
8.99 322	128	8.99 534	128	11.00 466	9.99 788	21
8.99 450	127	8.99 662	129	11.00 338	9.99 787	20
8.99 577	127	8.99 791	128	11.00 209	9.99 786	19
8.99 704	126	8.99 919	127	11.00 081	9.99 785	18
8.99 830	126	9.00 046	128	10.99 954	9.99 783	1[illegible]
8.99 956	126	9.00 174	127	10.99 826	9.99 782	16
9.00 08[illegible]	125	9.00 301	126	10.99 699	9.99 781	15
9.00 207	125	9.00 427	126	10.99 57[illegible]	9.99 780	14
9.00 332	124	9.00 553	126	10.99 447	9.99 778	13
9.00 456	125	9.00 679	126	10.99 321	9.99 77[illegible]	12
9.00 581	12[illegible]	9.00 80[illegible]	125	10.99 195	9.99 776	11
9.00 704	124	9.00 930	125	10.99 070	9.99 775	10
9.00 828	123	9.01 055	124	10.98 94[illegible]	9.99 77[illegible]	9
9.00 951	123	9.01 179	124	10.98 821	9.99 772	8
9.01 074	122	9.01 30[illegible]	124	10.98 697	9.99 771	7
9.01 196	122	9.01 42[illegible]	123	10.98 573	9.99 769	6
9.01 318	122	9.01 550	123	10.98 450	9.99 768	5
9.01 440	12[illegible]	9.01 673	123	10.98 327	9.99 767	
9.01 561	12[illegible]	9.01 796	122	10.98 20[illegible]	9.99 765	
9.01 682	12[illegible]	9.01 918	122	10.98 082	9.99 764	
9.01 803	12[illegible]	9.02 040	122	10.97 96[illegible]	9.99 76[illegible]	
9.01 923		9.02 162		10.97 88[illegible]	9.99 76	
L Cos		L Ctn	c d	L Tan	L Sin	

Prop. Pts.

	143	142	141
2	28.6	28.4	28.2
3	42.9	42.6	42.3
4	57.2	56.8	56.4
5	71.5	71.0	70.5
6	85.8	85.2	84.6
7	100.1	99.4	98.7
8	114.4	113.6	112.8
9	128.7	127.8	126.9

	140	139	138
2	28.0	27.8	27.6
3	42.0	41.7	41.4
4	56.0	55.6	55.2
5	70.0	69.5	69.0
6	84.0	83.4	82.8
7	98.0	97.3	96.6
8	112.0	111.2	110.4
9	126.0	125.1	124.2

	137	136	135
2	27.4	27.2	27.0
3	41.1	40.8	40.5
4	54.8	54.4	54.0
5	68.5	68.0	67.5
6	82.2	81.6	81.0
7	95.9	95.2	94.5
8	109.6	108.8	108.0
9	123.3	122.4	121.5

	134	133	132
2	26.8	26.6	26.4
3	40.2	39.9	39.6
4	53.6	53.2	52.8
5	67.0	66.5	66.0
6	80.4	79.8	79.2
7	93.8	93.1	92.4
8	107.2	106.4	105.6
9	120.6	119.7	118.8

	131	130	129
2	26.2	26.0	25.8
3	39.3	39.0	38.7
4	52.4	52.0	51.6
5	65.5	65.0	64.5
6	78.6	78.0	77.4
7	91.7	91.0	90.3
8	104.8	104.0	103.2
9	117.9	117.0	116.1

	128	127	126
2	25.6	25.4	25.2
3	38.4	38.1	37.8
4	51.2	50.8	50.4
5	64.0	63.5	63.0
6	76.8	76.2	75.6
7	89.6	88.9	88.2
8	102.4	101.6	100.8
9	115.2	114.3	113.4

	125	124	123
2	25.0	24.8	24.6
3	37.5	37.2	36.9
4	50.0	49.6	49.2
5	62.5	62.0	61.5
6	75.0	74.4	73.8
7	87.5	86.8	86.1
8	100.0	99.2	98.4
9	112.5	111.6	110.7

	122	121	120
2	24.4	24.2	24.0
3	36.6	36.3	36.0
4	48.8	48.4	48.0
5	61.0	60.5	60.0
6	73.2	72.6	72.0
7	85.4	84.7	84.0
8	97.6	96.8	96.0
9	109.8	108.9	108.0

Prop. Pts.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	
0	9.01 923	120	9.02 162	121	10.97 838	9.99 761	60
1	9.02 043	120	9.02 283	121	10.97 717	9.99 760	59
2	9.02 163	120	9.02 404	121	10.97 596	9.99 759	58
3	9.02 283	119	9.02 525	120	10.97 475	9.99 757	57
4	9.02 402	118	9.02 645	121	10.97 355	9.99 756	56
5	9.02 520	119	9.02 766	119	10.97 234	9.99 755	55
6	9.02 639	118	9.02 885	120	10.97 115	9.99 753	54
7	9.02 757	117	9.03 005	119	10.96 995	9.99 752	53
8	9.02 874	118	9.03 124	118	10.96 876	9.99 751	52
9	9.02 992	117	9.03 242	119	10.96 758	9.99 749	51
10	9.03 109	117	9.03 361	118	10.96 639	9.99 748	50
11	9.03 226	116	9.03 479	118	10.96 521	9.99 747	49
12	9.03 342	116	9.03 597	117	10.96 403	9.99 745	48
13	9.03 458	116	9.03 714	118	10.96 286	9.99 744	47
14	9.03 574	116	9.03 832	116	10.96 168	9.99 742	46
15	9.03 690	115	9.03 948	117	10.96 052	9.99 741	45
16	9.03 805	115	9.04 065	116	10.95 935	9.99 740	44
17	9.03 920	114	9.04 181	116	10.95 819	9.99 738	43
18	9.04 034	115	9.04 297	116	10.95 703	9.99 737	42
19	9.04 149	113	9.04 413	115	10.95 587	9.99 736	41
20	9.04 262	114	9.04 528	115	10.95 472	9.99 734	40
21	9.04 376	114	9.04 643	115	10.95 357	9.99 733	39
22	9.04 490	113	9.04 758	115	10.95 242	9.99 731	38
23	9.04 603	112	9.04 873	114	10.95 127	9.99 730	37
24	9.04 715	113	9.04 987	114	10.95 013	9.99 728	36
25	9.04 828	112	9.05 101	113	10.94 899	9.99 727	35
26	9.04 940	112	9.05 214	114	10.94 786	9.99 726	34
27	9.05 052	112	9.05 328	113	10.94 672	9.99 724	33
28	9.05 164	111	9.05 441	112	10.94 559	9.99 723	32
29	9.05 275	111	9.05 553	113	10.94 447	9.99 721	31
30	9.05 386	111	9.05 666	112	10.94 334	9.99 720	30
31	9.05 497	110	9.05 778	112	10.94 222	9.99 718	29
32	9.05 607	110	9.05 890	112	10.94 110	9.99 717	28
33	9.05 717	110	9.06 002	111	10.93 998	9.99 716	27
34	9.05 827	110	9.06 113	111	10.93 887	9.99 714	26
35	9.05 937	109	9.06 224	111	10.93 776	9.99 713	25
36	9.06 046	109	9.06 335	110	10.93 665	9.99 711	24
37	9.06 155	109	9.06 445	111	10.93 555	9.99 710	23
38	9.06 264	108	9.06 556	110	10.93 444	9.99 708	22
39	9.06 372	109	9.06 666	109	10.93 334	9.99 707	21
40	9.06 481	108	9.06 775	110	10.93 225	9.99 705	20
41	9.06 589	107	9.06 885	109	10.93 115	9.99 704	19
42	9.06 696	108	9.06 994	109	10.93 006	9.99 702	18
43	9.06 804	107	9.07 103	108	10.92 897	9.99 701	17
44	9.06 911	107	9.07 211	109	10.92 789	9.99 699	16
45	9.07 018	106	9.07 320	108	10.92 680	9.99 698	15
46	9.07 124	107	9.07 428	108	10.92 572	9.99 696	14
47	9.07 231	106	9.07 536	107	10.92 464	9.99 695	13
48	9.07 337	105	9.07 643	108	10.92 357	9.99 693	12
49	9.07 442	106	9.07 751	107	10.92 249	9.99 692	11
50	9.07 548	105	9.07 858	106	10.92 142	9.99 690	10
51	9.07 653	105	9.07 964	107	10.92 036	9.99 689	9
52	9.07 758	105	9.08 071	106	10.91 929	9.99 687	8
53	9.07 863	105	9.08 177	106	10.91 823	9.99 686	7
54	9.07 968	104	9.08 283	106	10.91 717	9.99 684	6
55	9.08 072	104	9.08 389	106	10.91 611	9.99 683	5
56	9.08 176	104	9.08 495	105	10.91 505	9.99 681	4
57	9.08 280	103	9.08 600	105	10.91 400	9.99 680	3
58	9.08 383	103	9.08 705	105	10.91 295	9.99 678	2
59	9.08 486	103	9.08 810	104	10.91 190	9.99 677	1
60	9.08 589		9.08 914		10.91 086	9.99 675	0
	L Cos	d	L Ctn	c d	L Tan	L Sin	′

Prop. Pts.

	121	120	119
2	24.2	24.0	23.8
3	36.3	36.0	35.7
4	48.4	48.0	47.6
5	60.5	60.0	59.5
6	72.6	72.0	71.4
7	84.7	84.0	83.3
8	96.8	96.0	95.2
9	108.9	108.0	107.1

	118	117	116
2	23.6	23.4	23.2
3	35.4	35.1	34.8
4	47.2	46.8	46.4
5	59.0	58.5	58.0
6	70.8	70.2	69.6
7	82.6	81.9	81.2
8	94.4	93.6	92.8
9	106.2	105.3	104.4

	115	114	113
2	23.0	22.8	22.6
3	34.5	34.2	33.9
4	46.0	45.6	45.2
5	57.5	57.0	56.5
6	69.0	68.4	67.8
7	80.5	79.8	79.1
8	92.0	91.2	90.4
9	103.5	102.6	101.7

	112	111	110
2	22.4	22.2	22.0
3	33.6	33.3	33.0
4	44.8	44.4	44.0
5	56.0	55.5	55.0
6	67.2	66.6	66.0
7	78.4	77.7	77.0
8	89.6	88.8	88.0
9	100.8	99.9	99.0

	109	108	107
2	21.8	21.6	21.4
3	32.7	32.4	32.1
4	43.6	43.2	42.8
5	54.5	54.0	53.5
6	65.4	64.8	64.2
7	76.3	75.6	74.9
8	87.2	86.4	85.6
9	98.1	97.2	96.3

	106	105	104
2	21.2	21.0	20.8
3	31.8	31.5	31.2
4	42.4	42.0	41.6
5	53.0	52.5	52.0
6	63.6	63.0	62.4
7	74.2	73.5	72.8
8	84.8	84.0	83.2
9	95.4	94.5	93.6

From the top:

For **6°+** or **186°+**, read as printed; for **96°+** or **276°+**, read co-function.

From the bottom:

For **83°+** or **263°+**, read as printed; for **173°+** or **353°+**, read co-function.

Prop. Pts.

83° Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	c d	L Ctn	L Cos	′
	9.08 589	103	9.08 914	105	10.91 086	9.99 675	60
	9.08 692	103	9.09 019	104	10.90 981	9.99 674	59
	9.08 795	102	9.09 123	104	10.90 877	9.99 672	58
	9.08 897	102	9.09 227	103	10.90 773	9.99 670	57
	9.08 999	102	9.09 330	104	10.90 670	9.99 669	56
	9.09 101	101	9.09 434	103	10.90 566	9.99 667	55
	9.09 202	102	9.09 537	103	10.90 463	9.99 666	54
	9.09 304	101	9.09 640	102	10.90 360	9.99 664	53
	9.09 405	101	9.09 742	103	10.90 258	9.99 663	52
	9.09 506	100	9.09 845	102	10.90 155	9.99 661	51
10	9.09 606	101	9.09 947	102	10.90 053	9.99 659	50
11	9.09 707	100	9.10 049	101	10.89 951	9.99 658	49
12	9.09 807	100	9.10 150	102	10.89 850	9.99 656	48
13	9.09 907	99	9.10 252	101	10.89 748	9.99 655	47
14	9.10 006	100	9.10 353	101	10.89 647	9.99 653	46
15	9.10 106	99	9.10 454	101	10.89 546	9.99 651	45
16	9.10 205	99	9.10 555	101	10.89 445	9.99 650	44
17	9.10 304		9.10 656	100	10.89 344	9.99 648	43
18	9.10 402		9.10 756	100	10.89 244	9.99 647	42
19	9.10 501		9.10 856	100	10.89 144	9.99 645	41
20	9.10 599		9.10 956	100	10.89 044	9.99 643	40
21	9.10 697		9.11 056	99	10.88 944	9.99 642	39
22	9.10 795		9.11 155	99	10.88 845	9.99 640	38
23	9.10 893		9.11 254	99	10.88 746	9.99 638	37
24	9.10 990		9.11 353	99	10.88 647	9.99 637	36
25	9.11 087		9.11 452	99	10.88 548	9.99 635	35
26	9.11 184		9.11 551	98	10.88 449	9.99 633	34
27	9.11 281		9.11 649	98	10.88 351	9.99 632	33
28	9.11 377		9.11 747	98	10.88 253	9.99 630	32
29	9.11 474		9.11 845	98	10.88 155	9.99 629	31
30	9.11 570		9.11 943	97	10.88 057	9.99 627	30
31	9.11 666		9.12 040	98	10.87 960	9.99 625	29
32	9.11 761		9.12 138	97	10.87 862	9.99 624	28
33	9.11 857		9.12 235	97	10.87 765	9.99 622	27
34	9.11 952		9.12 332	96	10.87 668	9.99 620	26
35	9.12 047		9.12 428	97	10.87 572	9.99 618	25
36	9.12 142		9.12 525	96	10.87 475	9.99 617	24
37	9.12 236		9.12 621	96	10.87 379	9.99 615	23
38	9.12 331		9.12 717	96	10.87 283	9.99 613	22
39	9.12 425		9.12 813	96	10.87 187	9.99 612	21
40	9.12 519		9.12 909	95	10.87 091	9.99 610	20
41	9.12 612		9.13 004	95	10.86 996	9.99 608	19
42	9.12 706		9.13 099	95	10.86 901	9.99 607	18
43	9.12 799		9.13 194	95	10.86 806	9.99 605	17
44	9.12 892		9.13 289	95	10.86 711	9.99 603	16
45	9.12 985		9.13 384	94	10.86 616	9.99 601	15
46	9.13 078		9.13 478	95	10.86 522	9.99 600	14
47	9.13 171		9.13 573	94	10.86 427	9.99 598	13
48	9.13 263		9.13 667	94	10.86 333	9.99 596	12
49	9.13 355		9.13 761	93	10.86 239	9.99 595	11
50	9.13 447		9.13 854	94	10.86 146	9.99 593	10
51	9.13 539		9.13 948	93	10.86 052	9.99 591	9
52	9.13 630		9.14 041	93	10.85 959	9.99 589	8
53	9.13 722		9.14 134	93	10.85 866	9.99 588	7
54	9.13 813		9.14 227	93	10.85 773	9.99 586	6
55	9.13 904		9.14 320	92	10.85 680	9.99 584	5
56	9.13 994		9.14 412	92	10.85 588	9.99 582	4
57	9.14 085		9.14 504	93	10.85 496	9.99 581	3
58	9.14 175		9.14 597	91	10.85 403	9.99 579	2
59	9.14 266		9.14 688	92	10.85 312	9.99 577	1
60	9.14 356		9.14 780		10.85 220	9.99 575	
	L Cos		L Ctn	c d	L Tan	L Sin	′

Prop. Pts.

105	104	103
21.0	20.8	20.6
31.5	31.2	30.9
42.0	41.6	41.2
52.5	52.0	51.5
63.0	62.4	61.8
73.5	72.8	72.1
84.0	83.2	82.4
94.5	93.6	92.7

102	101	99
20.4	20.2	19.8
30.6	30.3	29.7
40.8	40.4	39.6
51.0	50.5	49.5
61.2	60.6	59.4
71.4	70.7	69.3
81.6	80.8	79.2
91.8	90.9	89.1

	97	96
19.6	19.4	19.2
29.4	29.1	28.8
39.2	38.8	38.4
49.0	48.5	48.0
58.8	58.2	57.6
68.6	67.9	67.2
78.4	77.6	76.8
88.2	87.3	86.4

95	94	93
19.0	18.8	18.6
28.5	28.2	27.9
38.0	37.6	37.2
47.5	47.0	46.5
57.0	56.4	55.8
66.5	65.8	65.1
76.0	75.2	74.4
85.5	84.6	83.7

92	91	90
18.4	18.2	18.0
27.6	27.3	27.0
36.8	36.4	36.0
46.0	45.5	45.0
55.2	54.6	54.0
64.4	63.7	63.0
3.6	72.8	72.0
2.8	81.9	81.0

From the top:

For 7°+ or 187°+, read as printed; for 97°+ or 277°+, read co-function.

From the bottom:

For 82°+ or 262°+, read as printed; for 172°+ or 352°+, read co-function.

Prop. Pts.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	′
	9.14 35		9.14 78	92	10.85 22	9.99 57	
	9.14 44		9.14 87	9	10.85 12	9.99 57	
	9.14 53		9.14 96	91	10.85 03	9.99 57	
	9.14 624		9.15 05	91	10.84 94	9.99 57	
	9.14 714		9.15 14	91	10.84 85	9.99 56	
	9.14 80		9.15 23	91	10.84 76	9.99 56	
	9.14 89		9.15 32	90	10.84 67	9.99 56	
	9.14 98	8	9.15 41	91	10.84 58	9.99 56	
	9.15 06	88	9.15 50	90	10.84 492	9.99 56	
	9.15 157	88	9.15 598	90	10.84 402	9.99 55	
1	9.15 245	88	9.15 688	89	10.84 312	9.99 557	
1	9.15 333	88	9.15 777	90	10.84 223	9.99 55	
1	9.15 421	87	9.15 867	89	10.84 133	9.99 554	
1	9.15 508	88	9.15 956	90	10.84 044	9.99 552	
14	9.15 596	87	9.16 046	89	10.83 954	9.99 55	46
15	9.15 683	87	9.16 135	89	10.83 865	9.99 54	45
16	9.15 770	87	9.16 224	88	10.83 776	9.99 546	4
1	9.15 857	87	9.16 312	89	10.83 688	9.99 545	43
1	9.15 944	86	9.16 401	88	10.83 599	9.99 54	42
19	9.16 030	86	9.16 489	88	10.83 511	9.99 541	41
20	9.16 116	87	9.16 577	88	10.83 423	9.99 539	40
21	9.16 203	86	9.16 665	88	10.83 335	9.99 537	39
22	9.16 289	85	9.16 753	88	10.83 247	9.99 535	38
23	9.16 374	86	9.16 841	87	10.83 159	9.99 533	37
24	9.16 460	85	9.16 928	88	10.83 07	9.99 532	36
25	9.16 545	86	9.17 016	87	10.82 984	9.99 530	35
26	9.16 631	85	9.17 103	87	10.82 897	9.99 528	34
2	9.16 716	85	9.17 190	87	10.82 810	9.99 526	33
28	9.16 801	85	9.17 277	86	10.82 723	9.99 524	32
29	9.16 886	84	9.17 363	87	10.82 637	9.99 522	31
30	9.16 970	85	9.17 450	86	10.82 550	9.99 520	30
31	9.17 055	84	9.17 536	86	10.82 464	9.99 518	29
3	9.17 139	84	9.17 622	86	10.82 378	9.99 517	28
33	9.17 223	84	9.17 708	86	10.82 292	9.99 515	27
34	9.17 307	84	9.17 794	86	10.82 206	9.99 513	26
35	9.17 391	83	9.17 880	85	10.82 120	9.99 511	25
36	9.17 474	84	9.17 965	86	10.82 035	.99 509	24
37	9.17 558	83	9.18 051	85	10.81 949	.99 507	23
38	9.17 641	83	9.18 136	85	10.81 864	.99 505	22
39	9.17 724	83	.18 221	85	0.81 779	.99 503	
40	9.17 807	83	9.18 306	85	10.81 694	9.99 501	
41	.17 890	83	.18 391	84	0.81 609	.99 499	
42	9.17 973	82	9.18 475	85	0.81 525	.99 497	
43	.18 055	82	9.18 560	84	0.81 440	.99 495	
44	.18 137	83	.18 644	84	0.81 356	.99 494	
45	9.18 220	82	.18 728	84	0.81 272	.99 492	
46	9.18 302	81	.18 812	34	0.81 188	.99 490	
47	9.18 383	82	.18 896	33	0.81 104	.99 488	
48	.18 465	82	.18 979	84	0.81 021	.99 486	
9	9.18 547	31	.19 063	33	0.80 937	.99 484	
0	9.18 628	81	.19 146	33	0.80 854	.99 482	
1	.18 709	81	.19 229	33	0.80 771	.99 480	
2	.18 790	31	.19 312	83	0.80 688	.99 478	
3	.18 871	31	.19 395	33	0.80 605	.99 476	
54	.18 952	1	.19 478	33	0.80 522	.99 474	
5	.19 033	80	.19 561	32	0.80 439	.99 472	
6	.19 113	30	.19 643	32	0.80 357	.99 470	
7	.19 193	80	.19 725	2	0.80 275	.99 468	
8	.19 273	30	.19 807	2	0.80 193	.99 466	
9	.19 353	30	.19 889	32	0.80 111	.99 464	
0	.19 433		.19 971		0.80 029	.99 462	
	L Cos	d	L Ctn		Tan	L Sin	

Prop. Pts.

92	91	90
18.4	18.2	18.0
27.6	27.3	27.0
36.8	36.4	36.0
46.0	45.5	45.0
55.2	54.6	54.0
64.4	63.7	63.0
73.6	72.8	72.0
82.8	81.9	81.0

89	88	87
17.8	17.6	17.4
26.7	26.4	26.1
35.6	35.2	34.8
44.5	44.0	43.5
53.4	52.8	52.2
62.3	61.6	60.9
71.2	70.4	69.6
80.1	79.2	78.3

86	85	84
17.2	17.0	16.8
25.8	25.5	25.2
34.4	34.0	33.6
43.0	42.5	42.0
51.6	51.0	50.4
60.2	59.5	58.8
68.8	68.0	67.2
77.4	76.5	75.6

83	82	81
16.6	16.4	16.2
24.9	24.6	24.3
33.2	32.8	32.4
41.5	41.0	40.5
49.8	49.2	48.6
58.1	57.4	56.7
66.4	65.6	64.8
74.7	73.8	72.9

From the top:

For **8°+** or **188°+**, ead as printed; for **98°+** or **278°+**, read o-function.

From the bottom:

For **81°+** or **261°+**, ead as printed; for **71°+** or **351°+**, read o-function.

Prop. Pts.

L Sin	L Tan	cd	L Ctn	L Cos
9.19 433	9.19 971		10.80 029	9.99 462
9.19 513	9.20 053		10.79 947	9.99 460
9.19 592	9.20 134		10.79 866	9.99 458
9.19 672	9.20 216		10.79 784	9.99 456
9.19 751	9.20 297		10.79 703	9.99 454
9.19 830	9.20 378		10.79 622	9.99 452
9.19 909	9.20 459		10.79 541	9.99 450
9.19 988	9.20 540		10.79 460	9.99 448
9.20 067	9.20 621		10.79 379	9.99 446
9.20 145	9.20 701		10.79 299	9.99 444
9.20 223	9.20 782		10.79 218	9.99 442
9.20 302	9.20 862		10.79 138	9.99 440
9.20 380	9.20 942		10.79 058	9.99 438
9.20 458	9.21 022		10.78 978	9.99 436
9.20 535	9.21 102		10.78 898	9.99 434
9.20 613	9.21 182		10.78 818	9.99 432
9.20 691	9.21 261		10.78 739	9.99 429
9.20 768	9.21 341		10.78 659	9.99 427
9.20 845	9.21 420		10.78 580	9.99 425
9.20 922	9.21 499		10.78 501	9.99 423
9.20 999	9.21 578		10.78 422	9.99 421
9.21 076	9.21 657		10.78 343	9.99 419
9.21 153	9.21 736		10.78 264	9.99 417
9.21 229	9.21 814		10.78 186	9.99 415
9.21 306	9.21 893		10.78 107	9.99 413
9.21 382	9.21 971		10.78 029	9.99 411
9.21 458	9.22 049		10.77 951	9.99 409
9.21 534	9.22 127		10.77 873	9.99 407
9.21 610	9.22 205		10.77 795	9.99 404
9.21 685	9.22 283		10.77 717	9.99 402
9.21 761	9.22 361		10.77 639	9.99 400
9.21 836	9.22 438		10.77 562	9.99 398
9.21 912	9.22 516		10.77 484	9.99 396
9.21 987	9.22 593		10.77 407	9.99 394
9.22 062	9.22 670		10.77 330	9.99 392
9.22 137	9.22 747		10.77 253	9.99 390
9.22 211	9.22 824		10.77 176	9.99 388
9.22 286	9.22 901		10.77 099	9.99 385
9.22 361	9.22 977		10.77 023	9.99 383
9.22 435	9.23 054		10.76 946	9.99 381
9.22 509	9.23 130		10.76 870	9.99 379
9.22 583	9.23 206		10.76 794	9.99 377
9.22 657	9.23 283		10.76 717	9.99 375
9.22 731	9.23 359		10.76 641	9.99 372
9.22 805	9.23 435		10.76 565	9.99 370
9.22 878	9.23 510		10.76 490	9.99 368
9.22 952	9.23 586		10.76 414	9.99 366
9.23 025	9.23 661		10.76 339	9.99 364
9.23 098	9.23 737		10.76 263	9.99 362
9.23 171	9.23 812		10.76 188	9.99 359
9.23 244	9.23 887		10.76 113	9.99 357
9.23 317	9.23 962		10.76 038	9.99 355
9.23 390	9.24 037		10.75 963	9.99 353
9.23 462	9.24 112		10.75 888	9.99 351
9.23 535	9.24 186		10.75 814	9.99 348
9.23 607	9.24 261		10.75 739	9.99 346
9.23 679	9.24 335		10.75 665	9.99 344
9.23 752	9.24 410		10.75 590	9.99 342
9.23 823	9.24 484		10.75 516	9.99 340
9.23 895	9.24 558		10.75 442	9.99 337
9.23 967	9.24 632		10.75 368	9.99 335
L Cos	L Ctn		L Tan	L Sin

Prop. Pts.

82	81	80
16.4	16.2	16.0
24.6	24.3	24.0
32.8	32.4	32.0
41.0	40.5	40.0
49.2	48.6	48.0
57.4	56.7	56.0
65.6	64.8	64.0
73.8	72.9	72.0

79	78	77
15.8	15.6	15.4
23.7	23.4	23.1
31.6	31.2	30.8
39.5	39.0	38.5
47.4	46.8	46.2
55.3	54.6	53.9
63.2	62.4	61.6
71.1	70.2	69.3

76	75	74
15.2	15.0	14.8
22.8	22.5	22.2
30.4	30.0	29.6
38.0	37.5	37.0
45.6	45.0	44.4
53.2	52.5	51.8
60.8	60.0	59.2
68.4	67.5	66.6

73	72	71
14.6	14.4	14.2
21.9	21.6	21.3
29.2	28.8	28.4
36.5	36.0	35.5
43.8	43.2	42.6
51.1	50.4	49.7
58.4	57.6	56.8
65.7	64.8	63.9

From the top:

For **9°+**, or **189°+**, read as printed; for **99°+** or **279°+**, read co-function.

From the bottom:

For **80°+** or **260°+**, read as printed; for **170°+** or **350°+**, read co-function.

Prop. Pts.

80° — Logarithms of Trigonometric Functions

	L Sin	d	L Tan	cd	L Ctn	L Cos	
	9.23 96'		9.24 632	74	10.75 368	9.99 33.	6(
	9.24 03!		9.24 706	73	10.75 29·	9.99 33:	5'
	9.24 11(		9.24 77!	74	10.75 221	9.99 33	5.
	9.24 18_		9.24 853	7:	10.75 147	9.99 32ε	5'
	9.24 253		9.24 926	74	10.75 074	9.99 326	5'
	9.24 324		9.25 000	73	10.75 000	9.99 324	5ε
	9.24 39:		9.25 073	73	10.74 927	9.99 322	54
	9.24 46		9.25 146	73	10.74 854	9.99 319	5:
	9.24 53		9.25 219	73	10.74 781	9.99 317	52
	9.24 60'		9.25 292	73	10.74 708	9.99 31ε	5
10	9.24 677		9.25 365	7:	10.74 635	9.99 313	**50**
11	9.24 74		9.25 437	73	10.74 563	9.99 310	4ς
12	9.24 81		9.25 510	72	10.74 490	9.99 308	4ε
13	9.24 88		9.25 582	73	10.74 418	9.99 306	4.
14	9.24 958		9.25 655	72	10.74 345	9.99 304	46
15	9.25 028		9.25 727	72	10.74 273	9.99 301	**45**
16	9.25 098		9.25 799	72	10.74 201	9.99 299	44
17	9.25 168		9.25 871	72	10.74 129	9.99 297	43
18	9.25 237		9.25 943	72	10.74 057	9.99 294	42
19	9.25 307		9.26 01	71	10.73 985	9.99 29:	41
20	9.25 376		9.26 086	72	10.73 914	9.99 290	**40**
21	9.25 445	69	9.26 158	71	10.73 84:	9.99 288	39
22	9.25 514		9.26 229	72	10.73 771	9.99 285	38
23	9.25 583	69	9.26 301	71	10.73 699	9.99 283	37
24	9.25 652		9.26 37:	71	10.73 628	9.99 281	36
25	9.25 721		9.26 443	71	10.73 557	9.99 278	**35**
26	9.25 790		9.26 514	71	10.73 486	9.99 276	34
27	9.25 858		9.26 585	70	10.73 415	9.99 274	33
28	9.25 927	68	9.26 655	71	10.73 345	9.99 271	3.
29	9.25 995		9.26 726	71	10.73 274	9.99 269	31
30	9.26 063		9.26 797	70	10.73 203	9.99 267	**30**
31	9.26 131		9.26 867	70	10.73 133	9.99 264	29
32	9.26 199		9.26 937	71	10.73 063	9.99 26:	28
33	9.26 267		9.27 008	70	10.72 992	9.99 260	27
34	9.26 335		9.27 078	70	10.72 922	9.99 257	26
35	9.26 403		9.27 148	70	10.72 852	9.99 255	**25**
36	9.26 470		9.27 218	70	10.72 782	9.99 252	24
37	9.26 538		9.27 288	69	10.72 712	9.99 250	23
38	9.26 605		9.27 357	70	10.72 643	9.99 248	2
39	9.26 672		9.27 427	69	10.72 573	9.99 245	21
40	9.26 739		9.27 496	70	10.72 504	9.99 243	**20**
41	9.26 806		9.27 566	69	10.72 434	9.99 241	19
42	9.26 873		9.27 635	69	10.72 365	9.99 238	18
43	9.26 940		9.27 704	69	10.72 296	9.99 236	17
44	9.27 007		9.27 773	69	10.72 227	9.99 233	16
45	9.27 073		9.27 842	69	10.72 158	9.99 231	**15**
46	9.27 140		9.27 911	69	10.72 089	9.99 229	14
47	9.27 206		9.27 980	69	10.72 020	9.99 226	13
48	9.27 273		9.28 049	68	10.71 951	9.99 224	12
49	9.27 339		9.28 117	69	10.71 883	9.99 221	11
50	9.27 405		9.28 186	68	10.71 814	9.99 219	**10**
51	9.27 471		9.28 254	69	10.71 746	).99 217	
52	9.27 537		9.28 323	68	10.71 677	'.99 214	
53	9.27 602		9.28 391	68	10.71 609	'.99 212	
54	9.27 668		9.28 459	68	10.71 541	'.99 209	
	9.27 734		9.28 527	68	10.71 473	9.99 207	
	9.27 799		9.28 595	67	0.71 405	'.99 204	
	).27 864		'.28 662	68	10.71 338	'.99 202	
	9.27 930		'.28 730	68	0.71 270	'.99 200	
	9.27 995		9.28 798	67	0.71 202	'.99 197	
	).28 060		.28 865		0.71 135	.99 195	
	L Cos	d	L Ctn		L Tan	L Sin	

Prop. Pts.

74	73	72
14.8	14.6	14.4
22.2	21.9	21.6
29.6	29.2	28.8
37.0	36.5	36.0
44.4	43.8	43.2
51.8	51.1	50.4
59.2	58.4	57.6
66.6	65.7	64.8

71	70	69
14.2	14.0	13.8
21.3	21.0	20.7
28.4	28.0	27.6
35.5	35.0	34.5
42.6	42.0	41.4
49.7	49.0	48.3
56.8	56.0	55.2
63.9	63.0	62.1

	68	67	
2	13.6	13.4	13.:
3	20.4	20.1	19.8
4	27.2	26.8	26.4
5	34.0	33.5	33.0
6	40.8	40.2	39.6
7	47.6	46.9	46.2
8	54.4	53.6	52.8
9	61.2	60.3	59.4

65	3	
13.0	0.6	0.4
19.5	0.9	0.6
26.0	1.2	0.8
32.5	1.5	1.0
39.0	1.8	1.2
45.5	2.1	1.4
52.0	2.4	1.6
58.5	2.7	1.8

From the top:

For **10°+** or **190°+**, read as printed; for **100°+** or **280°+**, read co-function.

From the bottom:

For **79°+** or **259°+**, 'ead as printed; for .**69°+** or **349°+**, read :o-function.

Prop. Pts.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	′
	9.28 060	65	9.28 865		10.71 135	9.99 195	
	9.28 125	65	9.28 933		10.71 067	9.99 192	
	9.28 190	64	9.29 000		10.71 000	9.99 190	
	9.28 254	65	9.29 067		10.70 933	9.99 187	
	9.28 319	65	9.29 134		10.70 866	9.99 185	
	9.28 384	64	9.29 201		10.70 799	9.99 182	55
	9.28 448	64	9.29 268		10.70 732	9.99 180	54
	9.28 512	65	9.29 335		10.70 665	9.99 177	53
	9.28 577	64	9.29 402		10.70 598	9.99 175	52
	9.28 641	64	9.29 468		10.70 532	9.99 172	51
10	9.28 705	64	9.29 535		10.70 465	9.99 170	50
11	9.28 769	64	9.29 601		10.70 399	9.99 167	49
12	9.28 833	63	9.29 668		10.70 332	9.99 165	48
13	9.28 896	64	9.29 734		10.70 266	9.99 162	47
14	9.28 960	64	9.29 800		10.70 200	9.99 160	46
15	9.29 024	63	9.29 866		10.70 134	9.99 157	45
16	9.29 087	63	9.29 932		10.70 068	9.99 155	44
17	9.29 150	64	9.29 998		10.70 002	9.99 152	43
18	9.29 214	63	9.30 064		10.69 936	9.99 150	42
19	9.29 277	63	9.30 130		10.69 870	9.99 147	41
20	9.29 340	63	9.30 195		10.69 805	9.99 145	40
21	9.29 403	63	9.30 261		10.69 739	9.99 142	39
22	9.29 466	63	9.30 326		10.69 674	9.99 140	38
23	9.29 529	62	9.30 391		10.69 609	9.99 137	37
24	9.29 591	63	9.30 457		10.69 543	9.99 135	36
25	9.29 654	62	9.30 522		10.69 478	9.99 132	35
26	9.29 716	63	9.30 587		10.69 413	9.99 130	34
27	9.29 779	62	9.30 652		10.69 348	9.99 127	33
28	9.29 841	62	9.30 717		10.69 283	9.99 124	32
29	9.29 903	63	9.30 782		10.69 218	9.99 122	31
30	9.29 966	62	9.30 846		10.69 154	9.99 119	30
31	9.30 028	62	9.30 911		10.69 089	9.99 117	29
32	9.30 090	61	9.30 975		10.69 025	9.99 114	28
33	9.30 151	62	9.31 040		10.68 960	9.99 112	27
34	9.30 213	62	9.31 104		10.68 896	9.99 109	26
35	9.30 275	61	9.31 168		10.68 832	9.99 106	25
36	9.30 336	62	9.31 233		10.68 767	9.99 104	24
37	9.30 398	61	9.31 297		10.68 703	9.99 101	23
38	9.30 459	62	9.31 361		10.68 639	9.99 099	22
39	9.30 521	61	9.31 425		10.68 575	9.99 096	21
40	9.30 582	61	9.31 489		10.68 511	9.99 09[illegible]	20
41	9.30 643	61	9.31 552		10.68 448	9.99 091	19
42	9.30 704	61	9.31 616		10.68 384	9.99 088	18
43	9.30 765	61	9.31 679		10.68 321	9.99 086	17
44	9.30 826	61	9.31 743		10.68 257	9.99 083	16
45	9.30 887	60	9.31 806		10.68 194	9.99 080	15
46	9.30 947	61	9.31 870		10.68 130	9.99 078	14
47	9.31 008	60	9.31 933		10.68 067	9.99 075	13
48	9.31 068	61	9.31 996		10.68 004	9.99 07[illegible]	12
49	9.31 129	60	9.32 059		10.67 941	9.99 070	11
50	9.31 189	61	9.32 122		10.67 878	9.99 067	10
51	9.31 250	60	9.32 185		10.67 815	9.99 064	9
52	9.31 310	60	9.32 248		10.67 752	9.99 062	
53	9.31 370	60	9.32 311		10.67 689	9.99 059	
54	9.31 430	60	9.32 373		10.67 627	9.99 056	
55	9.31 490	59	9.32 436		10.67 564	9.99 054	
56	9.31 549	60	9.32 498		10.67 502	9.99 051	
57	9.31 609	60	9.32 561		10.67 439	9.99 048	
58	9.31 669	59	9.32 623		10.67 377	9.99 046	
59	9.31 728	60	9.32 685		10.67 315	9.99 043	
60	9.31 788		9.32 747		10.67 253	9.99 040	
	L Cos		L Ctn	c d	L Tan	L Sin	d

Prop. Pts.

68	67	66
13.6	13.4	13.2
20.4	20.1	19.8
27.2	26.8	26.4
34.0	33.5	33.0
40.8	40.2	39.6
47.6	46.9	46.2
54.4	53.6	52.8
61.2	60.3	59.4

65	64	63
13.0	12.8	12.6
19.5	19.2	18.9
26.0	25.6	25.2
32.5	32.0	31.5
39.0	38.4	37.8
45.5	44.8	44.1
52.0	51.2	50.4
58.5	57.6	56.7

62	61	60
12.4	12.2	12.0
18.6	18.3	18.0
24.8	24.4	24.0
31.0	30.5	30.0
37.2	36.6	36.0
43.4	42.7	42.0
49.6	48.8	48.0
55.8	54.9	54.0

59	3	
11.8	0.6	0.4
17.7	0.9	0.6
23.6	1.2	0.8
29.5	1.5	1.0
35.4	1.8	1.2
41.3	2.1	1.4
47.2	2.4	1.6
53.1	2.7	1.8

From the top:
For **11°+** or **191°+**, read as printed; for **101°+** or **281°+**, read co-function.

From the bottom:
For **78°+** or **258°+**, read as printed; for **168°+** or **348°+**, read co-function.

Prop. Pts.

	Sin	L Tan	c	L Ctn	L Co:	d
	9.31 78	9.32 74'		10.67 253	9.99 04	60
	9.31 84	9.32 81		10.67 190	9.99 03	59
	9.31 90	9.32 87:		10.67 128	9.99 03	58
	9.31 96	9.32 93		10.67 067	9.99 03	57
	9.32 02	9.32 99.		10.67 005	9.99 03	56
	9.32 08	9.33 05'		10.66 943	9.99 02	55
	9.32 14	9.33 11		10.66 881	9.99 0_	54
	9.32 20	9.33 18		10.66 820	9.99 02.	53
	9.32 26	9.33 24:		10.66 758	9.99 01	52
	9.32 31	9.33 30:		10.66 697	9.99 01	51
10	9.32 37:	9.33 36.		10.66 635	9.99 01.	50
11	9.32 43	9.33 42		10.66 574	9.99 01	49
12	9.32 49	9.33 48'		10.66 513	9.99 00	48
13	9.32 55	9.33 54		10.66 452	9.99 00	47
14	9.32 61:	9.33 60'		10.66 391	9.99 00.	46
15	9.32 67(	9.33 67		10.66 330	9.99 00	45
16	9.32 72	9.33 73		10.66 269	9.98 99	44
17	9.32 78(	9.33 792		10.66 208	9.98 99	43
18	9.32 844	9.33 85:		10.66 147	9.98 99	42
19	9.32 902	9.33 913		10.66 087	9.98 98	41
20	9.32 96(	9.33 974		10.66 026	9.98 986	40
21	9.33 01	9.34 03		10.65 966	9.98 98:	39
22	9.33 075	9.34 09.		10.65 905	9.98 98	38
23	9.33 133	9.34 15.		10.65 845	9.98 97_	37
24	9.33 190	9.34 215		10.65 785	9.98 975	36
25	9.33 248	9.34 276		10.65 724	9.98 972	35
26	9.33 305	9.34 336		10.65 664	9.98 96'	34
27	9.33 36:	9.34 396		10.65 604	9.98 96'.	33
28	9.33 420	9.34 456		10.65 544	9.98 964	32
29	9.33 477	9.34 516		10.65 484	9.98 96	31
30	9.33 534	9.34 576		10.65 424	9.98 958	30
31	9.33 591	9.34 635		10.65 365	9.98 955	29
32	9.33 647	9.34 695		10.65 305	9.98 953	28
33	9.33 704	.34 755		10.65 245	9.98 950	27
34	9.33 761	9.34 814		10.65 186	9.98 94	26
35	9.33 818	.34 874		10.65 126	9.98 944	25
36	9.33 874	.34 933		10.65 067	9.98 941	24
37	9.33 931	).34 992	59	.0.65 008	9.98 938	23
38	9.33 987	.35 051		.0.64 949	9.98 936	22
39	9.34 043	.35 111	60	.0.64 889	9.98 933	21
40	9.34 100	.35 170		.0.64 830	9.98 930	20
41	9.34 156	.35 229		10.64 771	9.98 927	19
42	9.34 212	.35 288		.0.64 712	9.98 924	18
43	9.34 268	.35 347		.0.64 653	9.98 921	17
44	9.34 324	.35 405		.0.64 595	9.98 919	16
45	9.34 380	.35 464		10.64 536	9.98 916	15
46	9.34 436	.35 523		.0.64 477	9.98 913	14
47	9.34 491	.35 581		.0.64 419	9.98 910	13
48	9.34 547	.35 640		.0.64 360	9.98 907	12
49	9.34 602	.35 698		.0.64 302	9.98 904	11
50	9.34 658	.35 757		0.64 243	9.98 901	10
51	9.34 713	.35 815		0.64 185	9.98 898	
52	9.34 769	.35 873		0.64 127	9.98 896	
53	9.34 824	.35 931		0.64 069	9.98 893	
54	9.34 879	.35 989		0.64 011	9.98 890	
55	9.34 934	.36 047		.0.63 953	9.98 887	
56	9.34 989	.36 105		.0.63 895	9.98 884	
57	9.35 044	.36 163		0.63 837	9.98 881	
58	9.35 099	.36 221		0.63 779	9.98 878	
59	9.35 154	.36 279		0.63 721	9.98 875	
60	9.35 209	.36 336		.0.63 664	9.98 872	
	L Cos	Ctn		Tan	L Sin	

Prop. Pts.

63		61
12.6	12.4	12.2
18.9	18.6	18.3
25.2	24.8	24.4
31.5	31.0	30.5
37.8	37.2	36.6
44.1	43.4	42.7
50.4	49.6	48.8
56.7	55.8	54.9

60	59	58
12.0	11.8	11.6
18.0	17.7	17.4
24.0	23.6	23.2
30.0	29.5	29.0
36.0	35.4	34.8
42.0	41.3	40.6
48.0	47.2	46.4
54.0	53.1	52.2

	57	56
2	11.4	11.2
3	17.1	16.8
4	22.8	22.4
5	28.5	28.0
6	34.2	33.6
7	39.9	39.2
8	45.6	44.8
9	51.3	50.4

55		
11.0	0.6	0.4
16.5	0.9	0.6
22.0	1.2	0.8
27.5	1.5	1.0
33.0	1.8	1.2
38.5	2.1	1.4
44.0	2.4	1.6
49.5	2.7	1.8

From the top:

For **12°+** or **192°+**, read as printed; for **102°+** or **282°+**, read co-function.

From the bottom:

For **77°+** or **257°+**, read as printed; for **167°+** or **347°+**, read co-function.

. Pts

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
	9.35 209		9.36 336		10.63 664	9.98 872		60
	9.35 263		9.36 394		10.63 606	9.98 869		59
	9.35 318		9.36 452		10.63 548	9.98 867		58
	9.35 373		9.36 509		10.63 491	9.98 864		57
	9.35 427		9.36 566		10.63 434	9.98 861		56
	9.35 481		9.36 624		10.63 376	9.98 858		55
	9.35 536		9.36 681		10.63 319	9.98 855		54
	9.35 590		9.36 738		10.63 262	9.98 852		53
	9.35 644		9.36 795		10.63 205	9.98 849		52
	9.35 698		9.36 852		10.63 148	9.98 846		51
	9.35 752		9.36 909		10.63 091	9.98 843		50
	9.35 806		9.36 966		10.63 034	9.98 840		49
	9.35 860		9.37 023		10.62 977	9.98 837		48
	9.35 914		9.37 080		10.62 920	9.98 834		47
	9.35 968		9.37 137		10.62 863	9.98 831		46
	9.36 022		9.37 193		10.62 807	9.98 828		45
	9.36 075		9.37 250		10.62 750	9.98 825		44
	9.36 129		9.37 306		10.62 694	9.98 822		43
	9.36 182		9.37 363		10.62 637	9.98 819		42
	9.36 236		9.37 419		10.62 581	9.98 816		41
	9.36 289		9.37 476		10.62 524	9.98 813		40
	9.36 342		9.37 532		10.62 468	9.98 810		39
	9.36 395		9.37 588		10.62 412	9.98 807		38
	9.36 449		9.37 644		10.62 356	9.98 804		37
	9.36 502		9.37 700		10.62 300	9.98 801		36
	9.36 555		9.37 756		10.62 244	9.98 798		35
	9.36 608		9.37 812		10.62 188	9.98 795		34
	9.36 660		9.37 868		10.62 132	9.98 792		33
	9.36 713		9.37 924		10.62 076	9.98 789		32
	9.36 766		9.37 980		10.62 020	9.98 786		31
	9.36 819		9.38 035		10.61 965	9.98 783		30
	9.36 871		9.38 091		10.61 909	9.98 780		29
	9.36 924		9.38 147		10.61 853	9.98 777		28
	9.36 976		9.38 202		10.61 798	9.98 774		27
	9.37 028		9.38 257		10.61 743	9.98 771		26
	9.37 081		9.38 313		10.61 687	9.98 768		25
	9.37 133		9.38 368		10.61 632	9.98 765		24
	9.37 185		9.38 423		10.61 577	9.98 762		23
	9.37 237		9.38 479		10.61 521	9.98 759		22
	9.37 289		9.38 534		10.61 466	9.98 756		21
	9.37 341		9.38 589		10.61 411	9.98 753		20
	9.37 393		9.38 644		10.61 356	9.98 750		19
	9.37 445		9.38 699		10.61 301	9.98 746		18
	9.37 497		9.38 754		10.61 246	9.98 743		17
	9.37 549		9.38 808		10.61 192	9.98 740		16
	9.37 600		9.38 863		10.61 137	9.98 737		15
	9.37 652		9.38 918		10.61 082	9.98 734		14
	9.37 703		9.38 972		10.61 028	9.98 731		13
	9.37 755		9.39 027		10.60 973	9.98 728		12
	9.37 806		9.39 082		10.60 918	9.98 725		11
	9.37 858		9.39 136		10.60 864	9.98 722		10
	9.37 909		9.39 190		10.60 810	9.98 719		
	9.37 960		9.39 245		10.60 755	9.98 715		
	9.38 011		9.39 299		10.60 701	9.98 712		
	9.38 062		9.39 353		10.60 647	9.98 709		
	9.38 113		9.39 407		10.60 593	9.98 706		
	9.38 164		9.39 461		10.60 539	9.98 703		
	9.38 215		9.39 515		10.60 485	9.98 700		
	9.38 266		9.39 569		10.60 431	9.98 697		
	9.38 317		9.39 623		10.60 377	9.98 694		
	9.38 368		9.39 677		10.60 323	9.98 690		
	L Cos	d	L Ctn	c d	L Tan	L Sin		

Prop. Pts.

58	57	56
11.6	11.4	11.2
17.4	17.1	16.8
23.2	22.8	22.4
29.0	28.5	28.0
34.8	34.2	33.6
40.6	39.9	39.2
46.4	45.6	44.8
52.2	51.3	50.4

55	54	53
11.0	10.8	10.6
16.5	16.2	15.9
22.0	21.6	21.2
27.5	27.0	26.5
33.0	32.4	31.8
38.5	37.8	37.1
44.0	43.2	42.4
49.5	48.6	47.7

	52	51
2	10.4	10.2
3	15.6	15.3
4	20.8	20.4
5	26.0	25.5
6	31.2	30.6
7	36.4	35.7
8	41.6	40.8
9	46.8	45.9

4	3	2
0.8	0.6	0.4
1.2	0.9	0.6
1.6	1.2	0.8
2.0	1.5	1.0
2.4	1.8	1.2
2.8	2.1	1.4
3.2	2.4	1.6
3.6	2.7	1.8

From the top:

For **13°+** or **193°+**, read as printed; for **103°+** or **283°+**, read co-function.

From the bottom:

For **76°+** or **256°+**, read as printed; for **166°+** or **346°+**, read co-function.

Prop. Pts.

	L Sin	d	L Tan	c d	L Ctn	L Cos	
	9.38 36	5	9.39 67		10.60 32	9.98 69	6
	9.38 41	5	9.39 73		10.60 26	9.98 68	5
	9.38 46	50	9.39 78		10.60 21	9.98 684	5
	9.38 51	5	9.39 83		10.60 16	9.98 68	5
	9.38 57	50	9.39 89		10.60 10	9.98 67	5
	9.38 62	50	9.39 94		10.60 05	9.98 67	5
	9.38 67	5	9.39 99		10.60 00	9.98 67	5
	9.38 72	50	9.40 052		10.59 94	9.98 66	5
	9.38 77	50	9.40 10		10.59 894	9.98 66	5
	9.38 82	50	9.40 159	53	10.59 84	9.98 66	5
1	9.38 87	50	9.40 212	54	10.59 78	9.98 65	50
1	9.38 92	50	9.40 266	53	10.59 734	9.98 656	4
1	9.38 97	50	9.40 319	53	10.59 68	9.98 652	4
1	9.39 02	50	9.40 372	53	10.59 628	9.98 64	4
1	9.39 071	50	9.40 425	53	10.59 575	9.98 646	46
1	9.39 121	49	9.40 478	53	10.59 522	9.98 643	45
1	9.39 170	50	9.40 531	53	10.59 46	9.98 640	44
1	9.39 220	50	9.40 584	52	10.59 416	9.98 636	43
1	9.39 270	49	9.40 636	53	10.59 364	9.98 633	42
19	9.39 319	50	9.40 689	53	10.59 311	9.98 630	41
20	9.39 369	49	9.40 742	53	10.59 258	9.98 627	40
2	9.39 418	49	9.40 795	52	10.59 205	9.98 623	39
22	9.39 467	50	9.40 847	53	10.59 153	9.98 620	38
23	9.39 517	49	9.40 900	52	10.59 100	9.98 617	3
2	9.39 566	49	9.40 952	53	10.59 048	9.98 614	36
25	9.39 615	49	9.41 005	52	10.58 99	9.98 610	3
26	9.39 664	49	9.41 057	52	10.58 943	9.98 607	34
27	9.39 713	49	9.41 109	52	10.58 891	9.98 604	33
28	9.39 762	49	9.41 161	53	10.58 839	9.98 601	32
29	9.39 811	49	9.41 214	52	10.58 786	9.98 59	31
30	9.39 860	49	9.41 266	52	10.58 734	9.98 594	30
31	9.39 909	49	9.41 318	52	10.58 682	9.98 591	29
3	9.39 958	48	9.41 370	52	10.58 630	9.98 588	28
33	9.40 006	49	9.41 422	52	10.58 578	9.98 584	27
34	9.40 055	48	9.41 474	52	10.58 526	9.98 581	26
35	9.40 103	49	9.41 526	52	10.58 474	9.98 578	25
36	9.40 152	48	9.41 578	51	10.58 422	9.98 574	24
3	9.40 200	49	9.41 629	52	10.58 371	.98 571	23
38	9.40 249	48	9.41 681	52	10.58 319	9.98 568	22
39	9.40 297	49	9.41 733	51	10.58 267	9.98 565	1
40	9.40 346	48	9.41 784	52	10.58 216	9.98 561	20
41	9.40 394	48	9.41 836	51	10.58 164	.98 558	
4	9.40 442	48	.41 887	52	10.58 113	.98 555	
4	9.40 490	48	9.41 939	51	10.58 061	.98 551	
44	9.40 538	48	.41 990	1	.0.58 010	.98 548	
45	9.40 586	48	.42 041	52	.0.57 959	.98 545	
46	9.40 634	48	.42 093	1	.0.57 907	.98 541	
47	9.40 682	48	.42 144	1	.0.57 856	.98 538	
48	9.40 730	48	.42 195	1	.0.57 805	.98 535	
49	9.40 778	7	.42 246	1	.0.57 754	.98 531	
50	.40 825	48	.42 297		.0.57 703	.98 528	
51	9.40 873	48	.42 348		.0.57 652	.98 525	
52	9.40 921	47	.42 399		.0.57 601	.98 521	
53	.40 968	48	.42 450		.0.57 550	.98 518	
54	.41 016	47	.42 501		.0.57 499	.98 515	
55	.41 063	8	.42 552		.0.57 448	.98 511	
56	.41 111	7	.42 603		.0.57 397	.98 508	
57	.41 158	7	.42 653		.0.57 347	.98 505	
58	.41 205	7	.42 704		.0.57 296	.98 501	
59	.41 252	48	.42 755		.0.57 245	.98 498	
60	.41 300		.42 805		.0.57 195	.98 494	
	L Cos	d	L Ctn	c d	L Tan	L Sin	

Prop. Pts.

54	53	52
10.8	10.6	10.
16.2	15.9	15.
21.6	21.2	20.
27.0	26.5	26.
32.4	31.8	31.
37.8	37.1	36.
43.2	42.4	41.
48.6	47.7	46.

51	50	
10.2	10.0	
15.3	15.0	14.7
20.4	20.0	19.6
25.5	25.0	24.5
30.6	30.0	29.4
35.7	35.0	34.3
40.8	40.0	39.2
45.9	45.0	44.1

	48	47
2	9.6	9.4
3	14.4	14.1
4	19.2	18.8
5	24.0	23.5
6	28.8	28.2
7	33.6	32.9
8	38.4	37.6
9	43.2	42.3

0.8	0.6
1.2	0.9
1.6	1.2
2.0	1.5
2.4	1.8
2.8	2.1
3.2	2.4
3.6	2.7

From the top:

For **14°+** or **194°+**, ead as printed; for **.04°+** or **284°+**, read o-function.

From the bottom:

For **75°+** or **255°+**, ead as printed; for **.65°+** or **345°+**, read o-function.

Prop. Pts.

75° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	c d	L Ctn	L Cos	
0	9.41 300	47	9.42 805	51	10.57 195	9.98 494	
1	9.41 347	47	9.42 856	50	10.57 144	9.98 491	
2	9.41 394	47	9.42 906	51	10.57 094	9.98 488	
3	9.41 441	47	9.42 957	50	10.57 043	9.98 484	
4	9.41 488	47	9.43 007	50	10.56 993	9.98 481	
5	9.41 535	47	9.43 057	51	10.56 943	9.98 477	55
6	9.41 582	46	9.43 108	50	10.56 892	9.98 474	54
7	9.41 628	47	9.43 158	50	10.56 842	9.98 471	53
8	9.41 675	47	9.43 208	50	10.56 792	9.98 467	52
9	9.41 722	46	9.43 258	50	10.56 742	9.98 464	51
10	9.41 768	47	9.43 308	50	10.56 692	9.98 460	50
11	9.41 815	46	9.43 358	50	10.56 642	9.98 457	49
12	9.41 861	47	9.43 408	50	10.56 592	9.98 453	48
13	9.41 908	46	9.43 458	50	10.56 542	9.98 450	47
14	9.41 954	47	9.43 508	50	10.56 492	9.98 447	46
15	9.42 001	46	9.43 558	49	10.56 442	9.98 443	45
16	9.42 047	46	9.43 607	50	10.56 393	9.98 440	44
17	9.42 093	47	9.43 657	50	10.56 343	9.98 436	43
18	9.42 140	46	9.43 707	49	10.56 293	9.98 433	42
19	9.42 186	46	9.43 756	50	10.56 244	9.98 429	41
20	9.42 232	46	9.43 806	49	10.56 194	9.98 426	40
21	9.42 278	46	9.43 855	50	10.56 145	9.98 422	39
22	9.42 324	46	9.43 905	49	10.56 095	9.98 419	38
23	9.42 370	46	9.43 954	50	10.56 046	9.98 415	37
24	9.42 416	45	9.44 004	49	10.55 996	9.98 412	36
25	9.42 461	46	9.44 053	49	10.55 947	9.98 409	35
26	9.42 507	46	9.44 102	49	10.55 898	9.98 405	34
27	9.42 553	46	9.44 151	50	10.55 849	9.98 402	33
28	9.42 599	45	9.44 201	49	10.55 799	9.98 398	32
29	9.42 644	46	9.44 250	49	10.55 750	9.98 395	31
30	9.42 690	45	9.44 299	49	10.55 701	9.98 391	30
31	9.42 735	46	9.44 348	49	10.55 65	9.98 388	29
32	9.42 781	45	9.44 397	49	10.55 603	9.98 384	28
33	9.42 826	46	9.44 446	49	10.55 554	9.98 381	2
34	9.42 87	45	9.44 495	49	10.55 505	9.98 377	26
35	9.42 917	45	9.44 544	48	10.55 456	9.98 373	25
36	9.42 962	46	9.44 59	49	10.55 408	9.98 370	24
37	9.43 008	45	9.44 641	49	10.55 359	9.98 366	23
38	9.43 053	45	9.44 690	48	10.55 310	9.98 363	22
39	9.43 098	45	9.44 738	49	10.55 262	9.98 359	21
40	9.43 143	45	9.44 787	49	10.55 213	9.98 356	20
41	9.43 188	45	9.44 836	48	10.55 164	9.98 35	19
42	9.43 233	45	9.44 884	49	10.55 116	9.98 349	18
43	9.43 278	45	9.44 933	48	10.55 067	9.98 345	1
44	9.43 32	44	9.44 981	48	10.55 019	9.98 342	16
45	9.43 367	45	9.45 029	49	10.54 971	9.98 338	15
46	9.43 412	45	9.45 078	48	10.54 922	9.98 334	1
47	9.43 45	45	9.45 126	48	10.54 874	9.98 331	1
48	9.43 502	44	9.45 174	48	10.54 826	9.98 327	12
49	9.43 546	45	9.45 222	49	10.54 778	9.98 324	1
50	9.43 591	44	9.45 271	48	10.54 729	9.98 320	10
51	9.43 635	45	9.45 31	48	10.54 68	9.98 317	
52	9.43 680	44	9.45 367	48	10.54 633	9.98 313	
53	9.43 724	45	9.45 415	48	10.54 585	9.98 309	
54	9.43 769	44	9.45 463	48	10.54 53	9.98 306	
55	9.43 813	44	9.45 51	48	10.54 489	9.98 302	
56	9.43 85	44	9.45 559	47	10.54 44	9.98 299	
5	9.43 90	45	9.45 60	48	10.54 394	9.98 29	
5	9.43 94	44	9.45 654	48	10.54 34	9.98 29	
5	9.43 99	44	9.45 702	48	10.54 29	9.98 28	
	9.44 03		9.45 75		10.54 25	9.98 28	
	L Cos		Ctn	c	L Tan	L Sin	

Prop. Pts.

	51	50	49
	10.2	10.0	9.8
	15.3	15.0	14.7
	20.4	20.0	19.6
	25.5	25.0	24.5
	30.6	30.0	29.4
	35.7	35.0	34.3
	40.8	40.0	39.2
	45.9	45.0	44.1

	48	47	46
2	9.6	9.4	9.2
3	14.4	14.1	13.8
4	19.2	18.8	18.4
5	24.0	23.5	23.0
6	28.8	28.2	27.6
7	33.6	32.9	32.2
8	38.4	37.6	36.8
9	43.2	42.3	41.4

45	44
9.0	8.8
13.5	13.2
18.0	17.6
22.5	22.0
27.0	26.4
31.5	30.8
36.0	35.2
40.5	39.6

0.8	0.6
1.2	0.9
1.6	1.2
2.0	1.5
2.4	1.8
2.8	2.1
3.2	2.4
3.6	2.7

From the top:
For **15°+** or **195°+** read as printed; fo: **105°+** or **285°+**, reac co-function.

From the bottom:
For **74°+** or **254** read as printed; **164°+** or **344°+**, read co-function.

Prop. Pts.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	′
	9.44 034	44	9.45 750	47	10.54 250	9.98 284	60
	9.44 078	44	9.45 797	48	10.54 203	9.98 281	
	.44 122	44	.45 845	47	10.54 155	9.98 277	
	.44 166	44	9.45 892	48	10.54 108	9.98 273	
	9.44 210	43	9.45 940	47	10.54 060	9.98 270	
	9.44 253	44	9.45 987	48	10.54 013	9.98 266	55
	9.44 297	44	9.46 035	47	10.53 965	9.98 262	54
	9.44 341	44	9.46 082	48	10.53 918	9.98 259	53
	9.44 385	43	9.46 130	47	10.53 870	9.98 255	52
	9.44 428	44	9.46 177	47	10.53 823	9.98 251	51
10	9.44 472	44	9.46 224	47	10.53 776	9.98 248	50
11	9.44 516	43	9.46 271	48	10.53 729	9.98 244	49
12	9.44 559	43	9.46 319	47	10.53 681	9.98 240	48
13	9.44 602	44	9.46 366	47	10.53 634	9.98 237	47
14	9.44 646	43	9.46 413	47	10.53 587	9.98 233	46
15	9.44 689	44	9.46 460	47	10.53 540	9.98 229	45
16	9.44 733	43	9.46 507	47	10.53 493	9.98 226	44
17	9.44 776	43	9.46 554	47	10.53 446	9.98 222	43
18	9.44 819	43	9.46 601	47	10.53 399	9.98 218	42
19	9.44 862	43	9.46 648	46	10.53 352	9.98 215	41
20	9.44 905	43	9.46 694	47	10.53 306	9.98 211	40
21	9.44 948	44	9.46 741	47	10.53 259	9.98 207	39
22	9.44 992	43	9.46 788	47	10.53 212	9.98 204	38
23	9.45 035	42	9.46 835	46	10.53 165	9.98 200	37
24	9.45 077	43	9.46 881	47	10.53 119	9.98 196	36
25	9.45 120	43	9.46 928	47	10.53 072	9.98 192	35
26	9.45 163	43	9.46 975	46	10.53 025	9.98 189	34
27	9.45 206	43	9.47 021	47	10.52 979	9.98 185	33
28	9.45 249	43	9.47 068	46	10.52 932	9.98 181	32
29	9.45 292	42	9.47 114	46	10.52 886	9.98 177	31
30	9.45 334	43	9.47 160	47	10.52 840	9.98 174	30
31	9.45 377	42	9.47 207	46	10.52 793	9.98 170	29
32	9.45 419	43	9.47 253	46	10.52 747	9.98 166	28
33	9.45 462	42	9.47 299	47	10.52 701	9.98 162	27
34	9.45 504	43	9.47 346	46	10.52 654	9.98 159	26
35	9.45 547	42	9.47 392	46	10.52 608	9.98 15[illegible]	25
36	9.45 589	43	9.47 438	46	10.52 562	9.98 151	24
37	9.45 632	42	9.47 484	46	10.52 516	9.98 147	23
38	9.45 674	42	9.47 530	46	10.52 470	9.98 144	2[illegible]
39	9.45 716	42	9.47 576	46	10.52 424	9.98 140	21
40	9.45 758	43	9.47 622	46	10.52 378	9.98 136	20
41	9.45 801	42	9.47 668	46	10.52 33[illegible]	9.98 13[illegible]	19
42	9.45 843	42	9.47 714	46	10.52 286	9.98 129	18
43	9.45 885	42	9.47 760	46	10.52 240	9.98 125	17
44	9.45 927	42	9.47 806	46	10.52 194	9.98 121	16
45	9.45 969	42	9.47 852	45	10.52 148	9.98 11[illegible]	15
46	9.46 011	42	9.47 897	46	10.52 103	9.98 113	14
47	9.46 053	42	9.47 943	46	10.52 05[illegible]	9.98 110	1[illegible]
48	9.46 095	41	9.47 989	46	10.52 011	9.98 106	12
49	9.46 136	42	9.48 035	45	10.51 965	9.98 102	11
50	9.46 178		9.48 080	46	10.51 920	9.98 098	10
51	9.46 220		9.48 126	45	10.51 874	9.98 094	
52	9.46 262		9.48 171	46	10.51 829	9.98 090	
53	9.46 303		9.48 217	45	10.51 783	9.98 087	
54	9.46 345		9.48 26[illegible]	45	10.51 738	9.98 083	
55	9.46 386		9.48 30[illegible]	46	10.51 693	9.98 079	
56	9.46 428		9.48 353	45	10.51 647	9.98 075	
57	9.46 469		9.48 398	45	10.51 60[illegible]	9.98 071	
58	9.46 511		9.48 443	46	10.51 557	9.98 067	
59	9.46 55[illegible]		9.48 489	45	10.51 511	9.98 063	
60	9.46 594		9.48 534		10.51 466	9.98 060	
	L Cos	d	L Ctn	c d	L Tan	L Sin	d

Prop. Pts.

48	47	46
9.6	9.4	9.2
14.4	14.1	13.8
19.2	18.8	18.4
24.0	23.5	23.0
28.8	28.2	27.6
33.6	32.9	32.2
38.4	37.6	36.8
43.2	42.3	41.4

45	44	43
9.0	8.8	8.6
13.5	13.2	12.9
18.0	17.6	17.2
22.5	22.0	21.5
27.0	26.4	25.8
31.5	30.8	30.1
36.0	35.2	34.4
40.5	39.6	38.7

	42	41
2	8.4	8.2
3	12.6	12.3
4	16.8	16.4
5	21.0	20.5
6	25.2	24.6
7	29.4	28.7
8	33.6	32.8
9	37.8	36.9

	4	
2	0.8	0.6
3	1.2	0.9
4	1.6	1.2
5	2.0	1.5
6	2.4	1.8
7	2.8	2.1
8	3.2	2.4
9	3.6	2.7

From the top:
For **16°+** or **196°+**, read as printed; for **106°+** or **286°+**, read co-function.

From the bottom:
For **73°+** or **253°+**, read as printed; for **163°+** or **343°+**, read co-function.

Prop. Pts.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.46 594	41	9.48 534	45	10.51 466	9.98 060	4	60
1	9.46 635	41	9.48 579	45	10.51 421	9.98 056	4	59
2	9.46 676	41	9.48 624	45	10.51 376	9.98 052	4	58
3	9.46 717	41	9.48 669	45	10.51 331	9.98 048	4	57
4	9.46 758	42	9.48 714	45	10.51 286	9.98 044	4	56
5	9.46 800	41	9.48 759	45	10.51 241	9.98 040	4	55
6	9.46 841	41	9.48 804	45	10.51 196	9.98 036	4	54
7	9.46 882	41	9.48 849	45	10.51 151	9.98 032	3	53
8	9.46 923	41	9.48 894	45	10.51 106	9.98 029	4	52
9	9.46 964	41	9.48 939	45	10.51 061	9.98 025	4	51
10	9.47 005	40	9.48 984	45	10.51 016	9.98 021	4	50
11	9.47 045	41	9.49 029	44	10.50 971	9.98 017	4	49
12	9.47 086	41	9.49 073	45	10.50 927	9.98 013	4	48
13	9.47 127	41	9.49 118	45	10.50 882	9.98 009	4	47
14	9.47 168	41	9.49 163	44	10.50 837	9.98 005	4	46
15	9.47 209	40	9.49 207	45	10.50 793	9.98 001	4	45
16	9.47 249	41	9.49 252	44	10.50 748	9.97 997	4	44
17	9.47 290	40	9.49 296	45	10.50 704	9.97 993	4	43
18	9.47 330	41	9.49 341	44	10.50 659	9.97 989	3	42
19	9.47 371	40	9.49 385	45	10.50 615	9.97 986	4	41
20	9.47 411	41	9.49 430	44	10.50 570	9.97 982	4	40
21	9.47 452	40	9.49 474	45	10.50 526	9.97 978	4	39
22	9.47 492	41	9.49 519	44	10.50 481	9.97 974	4	38
23	9.47 533	40	9.49 563	44	10.50 437	9.97 970	4	37
24	9.47 573	40	9.49 607	45	10.50 393	9.97 966	4	36
25	9.47 613	41	9.49 652	44	10.50 348	9.97 962	4	35
26	9.47 654	40	9.49 696	44	10.50 304	9.97 958	4	34
27	9.47 694	40	9.49 740	44	10.50 260	9.97 954	4	33
28	9.47 734	40	9.49 784	44	10.50 216	9.97 950	4	32
29	9.47 774	40	9.49 828	44	10.50 172	9.97 946	4	31
30	9.47 814	40	9.49 872	44	10.50 128	9.97 942	4	30
31	9.47 854	40	9.49 916	44	10.50 084	9.97 938	4	29
32	9.47 894	40	9.49 960	44	10.50 040	9.97 934	4	28
33	9.47 934	40	9.50 004	44	10.49 996	9.97 930	4	27
34	9.47 974	40	9.50 048	44	10.49 952	9.97 926	4	26
35	9.48 014	40	9.50 092	44	10.49 908	9.97 922	4	25
36	9.48 054	40	9.50 136	44	10.49 864	9.97 918	4	24
37	9.48 094	39	9.50 180	43	10.49 820	9.97 914	4	23
38	9.48 133	40	9.50 223	44	10.49 777	9.97 910	4	22
39	9.48 173	40	9.50 267	44	10.49 733	9.97 906	4	21
40	9.48 213	39	9.50 311	44	10.49 689	9.97 902	4	20
41	9.48 252	40	9.50 355	43	10.49 645	9.97 898	4	19
42	9.48 292	40	9.50 398	44	10.49 602	9.97 894	4	18
43	9.48 332	39	9.50 442	43	10.49 558	9.97 890	4	17
44	9.48 371	40	9.50 485	44	10.49 515	9.97 886	4	16
45	9.48 411	39	9.50 529	43	10.49 471	9.97 882	4	15
46	9.48 450	40	9.50 572	44	10.49 428	9.97 878	4	14
47	9.48 490	39	9.50 616	43	10.49 384	9.97 874	4	13
48	9.48 529	39	9.50 659	44	10.49 341	9.97 870	4	12
49	9.48 568	39	9.50 703	43	10.49 297	9.97 866	5	11
50	9.48 607	40	9.50 746	43	10.49 254	9.97 861	4	10
51	9.48 647	39	9.50 789	44	10.49 211	9.97 857	4	9
52	9.48 686	39	9.50 833	43	10.49 167	9.97 853	4	8
53	9.48 725	39	9.50 876	43	10.49 124	9.97 849	4	7
54	9.48 764	39	9.50 919	43	10.49 081	9.97 845	4	6
55	9.48 803	39	9.50 962	43	10.49 038	9.97 841	4	5
56	9.48 842	39	9.51 005	43	10.48 995	9.97 837	4	4
57	9.48 881	39	9.51 048	44	10.48 952	9.97 833	4	3
58	9.48 920	39	9.51 092	43	10.48 908	9.97 829	4	2
59	9.48 959	39	9.51 135	43	10.48 865	9.97 825	4	1
60	9.48 998		9.51 178		10.48 822	9.97 821		0
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	45	44	43
2	9.0	8.8	8.6
3	13.5	13.2	12.9
4	18.0	17.6	17.2
5	22.5	22.0	21.5
6	27.0	26.4	25.8
7	31.5	30.8	30.1
8	36.0	35.2	34.4
9	40.5	39.6	38.7

	42	41	40
2	8.4	8.2	8.0
3	12.6	12.3	12.0
4	16.8	16.4	16.0
5	21.0	20.5	20.0
6	25.2	24.6	24.0
7	29.4	28.7	28.0
8	33.6	32.8	32.0
9	37.8	36.9	36.0

	39	5
2	7.8	1.0
3	11.7	1.5
4	15.6	2.0
5	19.5	2.5
6	23.4	3.0
7	27.3	3.5
8	31.2	4.0
9	35.1	4.5

	4	3
2	0.8	0.6
3	1.2	0.9
4	1.6	1.2
5	2.0	1.5
6	2.4	1.8
7	2.8	2.1
8	3.2	2.4
9	3.6	2.7

From the top:

For **17°+** or **197°+**, read as printed; for **107°+** or **287°+**, read co-function.

From the bottom:

For **72°+** or **252°+** read as printed; fo **162°+** or **342°+**, rea co-function.

Prop. Pts.

72° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	cd	L Ctn	L Cos	d	′
0	9.48 998		9.51 178		10.48 822	9.97 821		
1	9.49 037		9.51 221		10.48 779	9.97 817		
2	9.49 076		9.51 264		10.48 736	9.97 812		
3	9.49 115		9.51 306		10.48 694	9.97 808		
4	9.49 153		9.51 349		10.48 651	9.97 804		
5	9.49 192		9.51 392		10.48 608	9.97 800		
6	9.49 231		9.51 435		10.48 565	9.97 796		
7	9.49 269		9.51 478		10.48 522	9.97 792		
8	9.49 308		9.51 520		10.48 480	9.97 788		
9	9.49 347		9.51 563		10.48 437	9.97 784		
10	9.49 385		9.51 606		10.48 394	9.97 779		
11	9.49 424		9.51 648		10.48 352	9.97 775		
12	9.49 462		9.51 691		10.48 309	9.97 771		
13	9.49 500		9.51 734		10.48 266	9.97 767		
14	9.49 539		9.51 776		10.48 224	9.97 763		
15	9.49 577		9.51 819		10.48 181	9.97 759		
16	9.49 615		9.51 861		10.48 139	9.97 754		
17	9.49 654		9.51 903		10.48 097	9.97 750		
18	9.49 692		9.51 946		10.48 054	9.97 746		
19	9.49 730		9.51 988		10.48 012	9.97 742		
20	9.49 768		9.52 031		10.47 969	9.97 738		
21	9.49 806		9.52 073		10.47 927	9.97 734		
22	9.49 844		9.52 115		10.47 885	9.97 729		
23	9.49 882		9.52 157		10.47 843	9.97 725		
24	9.49 920		9.52 200		10.47 800	9.97 721		
25	9.49 958		9.52 242		10.47 758	9.97 717		
26	9.49 996		9.52 284		10.47 716	9.97 713		
27	9.50 034		9.52 326		10.47 674	9.97 708		
28	9.50 072		9.52 368		10.47 632	9.97 704		
29	9.50 110		9.52 410		10.47 590	9.97 700		
30	9.50 148		9.52 452		10.47 548	9.97 696		
31	9.50 185		9.52 494		10.47 506	9.97 691		
32	9.50 223		9.52 536		10.47 464	9.97 687		
33	9.50 261		9.52 578		10.47 422	9.97 683		
34	9.50 298		9.52 620		10.47 380	9.97 679		
35	9.50 336		9.52 661		10.47 339	9.97 674		
36	9.50 374		9.52 703		10.47 297	9.97 670		
37	9.50 411		9.52 745		10.47 255	9.97 666		
38	9.50 449		9.52 787		10.47 213	9.97 662		
39	9.50 486		9.52 829		10.47 171	9.97 657		
40	9.50 523		9.52 870		10.47 130	9.97 653		
41	9.50 561		9.52 912		10.47 088	9.97 649		
42	9.50 598		9.52 953		10.47 047	9.97 645		
43	9.50 635		9.52 995		10.47 005	9.97 640		
44	9.50 673		9.53 037		10.46 963	9.97 636		
45	9.50 710		9.53 078		10.46 922	9.97 632		
46	9.50 747		9.53 120		10.46 880	9.97 628		
47	9.50 784		9.53 161		10.46 839	9.97 623		
48	9.50 821		9.53 202		10.46 798	9.97 619		
49	9.50 858		9.53 244		10.46 756	9.97 615		
50	9.50 896		9.53 285		10.46 715	9.97 610		
51	9.50 933		9.53 327		10.46 673	9.97 606		
52	9.50 970		9.53 368		10.46 632	9.97 602		
53	9.51 007		9.53 409		10.46 591	9.97 597		
54	9.51 043		9.53 450		10.46 550	9.97 593		
55	9.51 080		9.53 492		10.46 508	9.97 589		
56	9.51 117		9.53 533		10.46 467	9.97 584		
57	9.51 154		9.53 574		10.46 426	9.97 580		
58	9.51 191		9.53 615		10.46 385	9.97 576		
59	9.51 227		9.53 656		10.46 344	9.97 571		
60	9.51 264		9.53 697		10.46 303	9.97 567		
	L Cos	d	L Ctn	cd	L Tan	L Sin	d	

Prop. Pts.

43	42	41
8.6	8.4	8.2
12.9	12.6	12.3
17.2	16.8	16.4
21.5	21.0	20.5
25.8	25.2	24.6
30.1	29.4	28.7
34.4	33.6	32.8
38.7	37.8	36.9

39	38	37
7.8	7.6	7.4
11.7	11.4	11.1
15.6	15.2	14.8
19.5	19.0	18.5
23.4	22.8	22.2
27.3	26.6	25.9
31.2	30.4	29.6
35.1	34.2	33.3

36	5	
7.2	1.0	0.8
10.8	1.5	1.2
14.4	2.0	1.6
18.0	2.5	2.0
21.6	3.0	2.4
25.2	3.5	2.8
28.8	4.0	3.2
32.4	4.5	3.6

From the top:

For **18°+** or **198°+**, read as printed; for **108°+** or **288°+**, read co-function.

From the bottom:

For **71°+** or **251°+**, read as printed; for **161°+** or **341°+**, read co-function.

Prop. Pts.

71° — Logarithms of Trigonometric Functions

L Sin	d	L Tan	cd	L Ctn	L Cos	d
9.51 264		9.53 697		10.46 303	9.97 567	
9.51 301		9.53 738		10.46 262	9.97 563	
9.51 338		9.53 779		10.46 221	9.97 558	
9.51 374	37	9.53 820		10.46 180	9.97 554	
9.51 411		9.53 861		10.46 139	9.97 550	
9.51 447		9.53 902		10.46 098	9.97 545	
9.51 484		9.53 943		10.46 057	9.97 541	
9.51 520		9.53 984		10.46 016	9.97 536	
9.51 557		9.54 025		10.45 975	9.97 532	
9.51 593		9.54 065		10.45 935	9.97 528	
9.51 629		9.54 106		10.45 894	9.97 523	
9.51 666		9.54 147		10.45 853	9.97 519	
9.51 702	36	9.54 187		10.45 813	9.97 515	
9.51 738		9.54 228		10.45 772	9.97 510	
9.51 774	37	9.54 269		10.45 731	9.97 506	
9.51 811		9.54 309		10.45 691	9.97 501	
9.51 847		9.54 350		10.45 650	9.97 497	
9.51 883		9.54 390		10.45 610	9.97 492	
9.51 919		9.54 431		10.45 569	9.97 488	
9.51 955		9.54 471		10.45 529	9.97 484	
9.51 991		9.54 512		10.45 488	9.97 479	
9.52 027		9.54 552		10.45 448	9.97 475	
9.52 063		9.54 593		10.45 407	9.97 470	
9.52 099		9.54 633		10.45 367	9.97 466	
9.52 135		9.54 673		10.45 327	9.97 461	
9.52 171		9.54 714		10.45 286	9.97 457	
9.52 207		9.54 754		10.45 246	9.97 453	
9.52 242		9.54 794		10.45 206	9.97 448	
9.52 278		9.54 835		10.45 165	9.97 444	
9.52 314		9.54 875		10.45 125	9.97 439	
9.52 350		9.54 915		10.45 085	9.97 435	
9.52 385		9.54 955		10.45 045	9.97 430	
9.52 421		9.54 995		10.45 005	9.97 426	
9.52 456		9.55 035		10.44 965	9.97 421	
9.52 492		9.55 075		10.44 925	9.97 417	
9.52 527		9.55 115		10.44 885	9.97 412	
9.52 563		9.55 155		10.44 845	9.97 408	
9.52 598		9.55 195		10.44 805	9.97 403	
9.52 634		9.55 235		10.44 765	9.97 399	
9.52 669		9.55 275		10.44 725	9.97 394	
9.52 705		9.55 315		10.44 685	9.97 390	
9.52 740		9.55 355		10.44 645	9.97 385	
9.52 775		9.55 395		10.44 605	9.97 381	
9.52 811		9.55 434		10.44 566	9.97 376	
9.52 846		9.55 474		10.44 526	9.97 372	
9.52 881		9.55 514		10.44 486	9.97 367	
9.52 916		9.55 554		10.44 446	9.97 363	
9.52 951		9.55 593		10.44 407	9.97 358	
9.52 986		9.55 633		10.44 367	9.97 353	
9.53 021		9.55 673		10.44 327	9.97 349	
9.53 056		9.55 712		10.44 288	9.97 344	
9.53 092		9.55 752		10.44 248	9.97 340	
9.53 126		9.55 791		10.44 209	9.97 335	
9.53 161		9.55 831		10.44 169	9.97 331	
9.53 196		9.55 870		10.44 130	9.97 326	
9.53 231		9.55 910		10.44 090	9.97 322	
9.53 266		9.55 949		10.44 051	9.97 317	
9.53 301		9.55 989		10.44 011	9.97 312	
9.53 336		9.56 028		10.43 972	9.97 308	
9.53 370		9.56 067		10.43 933	9.97 303	
9.53 405		9.56 107		10.43 893	9.97 299	
L Cos		L Ctn	cd	L Tan	L Sin	d

Prop. Pts.

41	40	39
8.2	8.0	7.8
12.3	12.0	11.7
16.4	16.0	15.6
20.5	20.0	19.5
24.6	24.0	23.4
28.7	28.0	27.3
32.8	32.0	31.2
36.9	36.0	35.1

37	36	35
7.4	7.2	7.0
11.1	10.8	10.5
14.8	14.4	14.0
18.5	18.0	17.5
22.2	21.6	21.0
25.9	25.2	24.5
29.6	28.8	28.0
33.3	32.4	31.5

34	5	
6.8	1.0	0.8
10.2	1.5	1.2
13.6	2.0	1.6
17.0	2.5	2.0
20.4	3.0	2.4
23.8	3.5	2.8
27.2	4.0	3.2
30.6	4.5	3.6

From the top:

For **19°+** or **199°+**, read as printed; for **109°+** or **289°+**, read co-function.

From the bottom:

For **70°+** or **250°+**, read as printed; for **160°+** or **340°+**, read co-function.

Prop. Pts.

	L Sin	d	L Tan	cd	L Ctn	L Cos	d
	9.53 405		9.56 107		10.43 893	9.97 299	
	9.53 440		9.56 146		10.43 854	9.97 294	
	9.53 475		9.56 185		10.43 815	9.97 289	
	9.53 509		9.56 224		10.43 776	9.97 285	
	9.53 544		9.56 264		10.43 736	9.97 280	
	9.53 578		9.56 303		10.43 697	9.97 276	
	9.53 613		9.56 342		10.43 658	9.97 271	
	9.53 647		9.56 381		10.43 619	9.97 266	
	9.53 682		9.56 420		10.43 580	9.97 262	
	9.53 716		9.56 459		10.43 541	9.97 257	
10	9.53 751		9.56 498		10.43 502	9.97 252	
11	9.53 785		9.56 537		10.43 463	9.97 248	
12	9.53 819		9.56 576		10.43 424	9.97 243	
13	9.53 854		9.56 615		10.43 385	9.97 238	
14	9.53 888		9.56 654		10.43 346	9.97 234	
15	9.53 922		9.56 693		10.43 307	9.97 229	
16	9.53 957		9.56 732		10.43 268	9.97 224	
17	9.53 991		9.56 771		10.43 229	9.97 220	
18	9.54 025		9.56 810		10.43 190	9.97 215	
19	9.54 059		9.56 849		10.43 151	9.97 210	
20	9.54 093		9.56 887		10.43 113	9.97 206	
21	9.54 127		9.56 926		10.43 074	9.97 201	
22	9.54 161		9.56 965		10.43 035	9.97 196	
23	9.54 195		9.57 004		10.42 996	9.97 192	
24	9.54 229		9.57 042		10.42 958	9.97 187	
25	9.54 263		9.57 081		10.42 919	9.97 182	
26	9.54 297		9.57 120		10.42 880	9.97 178	
27	9.54 331		9.57 158		10.42 842	9.97 173	
28	9.54 365		9.57 197		10.42 803	9.97 168	
29	9.54 399		9.57 235		10.42 765	9.97 163	
30	9.54 433		9.57 274		10.42 726	9.97 159	
31	9.54 466		9.57 312		10.42 688	9.97 154	
32	9.54 500		9.57 351		10.42 649	9.97 149	
33	9.54 534		9.57 389		10.42 611	9.97 145	
34	9.54 567		9.57 428		10.42 572	9.97 140	
35	9.54 601		9.57 466		10.42 534	9.97 135	
36	9.54 635		9.57 504		10.42 496	9.97 130	
37	9.54 668		9.57 543		10.42 457	9.97 126	
38	9.54 702		9.57 581		10.42 419	9.97 121	
39	9.54 735		9.57 619		10.42 381	9.97 116	
40	9.54 769		9.57 658		10.42 342	9.97 111	
41	9.54 802		9.57 696		10.42 304	9.97 107	
42	9.54 836		9.57 734		10.42 266	9.97 102	
43	9.54 869		9.57 772		10.42 228	9.97 097	
44	9.54 903		9.57 810		10.42 190	9.97 092	
45	9.54 936		9.57 849		10.42 151	9.97 087	
46	9.54 969		9.57 887		10.42 113	9.97 083	
47	9.55 003		9.57 925		10.42 075	9.97 078	
48	9.55 036		9.57 963		10.42 037	9.97 073	
49	9.55 069		9.58 001		10.41 999	9.97 068	
50	9.55 102		9.58 039		10.41 961	9.97 063	
51	9.55 136		9.58 077		10.41 923	9.97 059	
52	9.55 169		9.58 115		10.41 885	9.97 054	
53	9.55 202		9.58 153		10.41 847	9.97 049	
54	9.55 235		9.58 191		10.41 809	9.97 044	
55	9.55 268		9.58 229		10.41 771	9.97 039	
56	9.55 301		9.58 267		10.41 733	9.97 035	
57	9.55 334		9.58 304		10.41 696	9.97 030	
58	9.55 367		9.58 342		10.41 658	9.97 025	
59	9.55 400		9.58 380		10.41 620	9.97 020	
60	9.55 433		9.58 418		10.41 582	9.97 015	
	L Cos	d	L Ctn	cd	L Tan	L Sin	d

Prop. Pts.

40	39	38
8.0	7.8	7.6
12.0	11.7	11.4
16.0	15.6	15.2
20.0	19.5	19.0
24.0	23.4	22.8
28.0	27.3	26.6
32.0	31.2	30.4
36.0	35.1	34.2

37	35	34
7.4	7.0	6.8
11.1	10.5	10.2
14.8	14.0	13.6
18.5	17.5	17.0
22.2	21.0	20.4
25.9	24.5	23.8
29.6	28.0	27.2
33.3	31.5	30.6

33	5	4
6.6	1.0	0.8
9.9	1.5	1.2
13.2	2.0	1.6
16.5	2.5	2.0
19.8	3.0	2.4
23.1	3.5	2.8
26.4	4.0	3.2
29.7	4.5	3.6

From the top:

For **20°+** or **200°+**, read as printed; for **110°+** or **290°+**, read co-function.

From the bottom:

For **69°+** or **249°+**, read as printed; for **159°+** or **339°+**, read co-function.

Prop. Pts.

′	L Sin	d	L Tan	cd	L Ctn	L Cos	d
0	9.55 433		9.58 418		10.41 582	9.97 015	
1	9.55 466		9.58 455		10.41 545	9.97 010	
2	9.55 499		9.58 493		10.41 507	9.97 005	
3	9.55 532		9.58 531		10.41 469	9.97 001	
4	9.55 564		9.58 569		10.41 431		
5	9.55 597		9.58 606		10.41 394	9.96 991	
6	9.55 630		9.58 644		10.41 356	9.96 986	
7	9.55 663		9.58 681		10.41 319	9.96 981	
	9.55 695		9.58 719		10.41 281	9.96 976	
	9.55 728		9.58 757		10.41 243	9.96 971	
	9.55 761		9.58 794		10.41 206	9.96 966	
	9.55 793		9.58 832		10.41 168	9.96 962	
	9.55 826		9.58 869		10.41 131	9.96 957	
	9.55 858		9.58 907		10.41 093	9.96 952	
	9.55 891		9.58 944		10.41 056	9.96 947	
	9.55 923		9.58 981		10.41 019	9.96 942	
	9.55 956		9.59 019		10.40 981	9.96 937	
	9.55 988		9.59 056		10.40 944	9.96 932	
	9.56 021		9.59 094		10.40 906	9.96 927	
	9.56 053		9.59 131		10.40 869	9.96 922	
	9.56 085		9.59 168		10.40 832	9.96 917	
	9.56 118		9.59 205		10.40 795	9.96 912	
	9.56 150		9.59 243		10.40 757	9.96 907	
	9.56 182		9.59 280		10.40 720	9.96 903	
	9.56 215		9.59 317		10.40 683	9.96 898	
	9.56 247		9.59 354		10.40 646	9.96 893	
	9.56 279		9.59 391		10.40 609		
	9.56 311		9.59 429		10.40 571	9.96 883	
	9.56 343		9.59 466		10.40 534	9.96 878	
	9.56 375		9.59 503		10.40 497	9.96 873	
	9.56 408		9.59 540		10.40 460		
	9.56 440		9.59 577		10.40 423	9.96 863	
	9.56 472		9.59 614		10.40 386	9.96 858	
	9.56 504		9.59 651		10.40 349	9.96 853	
	9.56 536		9.59 688		10.40 312	9.96 848	
	9.56 568		9.59 725		10.40 275	9.96 843	
	9.56 599		9.59 762		10.40 238	9.96 838	
	9.56 631		9.59 799		10.40 201	9.96 833	
	9.56 663		9.59 835		10.40 165	9.96 828	
	9.56 695		9.59 872		10.40 128	9.96 823	
	9.56 727		9.59 909		10.40 091	9.96 818	
	9.56 759		9.59 946		10.40 054	9.96 813	
	9.56 790		9.59 983		10.40 017	9.96 808	
	9.56 822		9.60 019		10.39 981	9.96 803	
	9.56 854		9.60 056		10.39 944	9.96 798	
	9.56 886		9.60 093		10.39 907	9.96 793	
	9.56 917		9.60 130		10.39 870	9.96 788	
	9.56 949		9.60 166		10.39 834	9.96 783	
	9.56 980		9.60 203		10.39 797	9.96 778	
	9.57 012		9.60 240		10.39 760	9.96 772	
	9.57 044		9.60 276		10.39 724	9.96 767	
	9.57 075		9.60 313		10.39 687	9.96 762	
	9.57 107		9.60 349		10.39 651	9.96 757	
	9.57 138		9.60 386		10.39 614	9.96 752	
	9.57 169		9.60 422		10.39 578	9.96 747	
	9.57 201		9.60 459		10.39 541	9.96 742	
	9.57 232		9.60 495		10.39 505	9.96 737	
	9.57 264		9.60 532		10.39 468	9.96 732	
	9.57 295		9.60 568		10.39 432	9.96 727	
	9.57 326		9.60 605		10.39 395	9.96 722	
	9.57 358		9.60 641		10.39 359	9.96 717	
	L Cos	d	L Ctn	cd	L Tan	L Sin	

Prop. Pts.

38	37	36
7.6	7.4	7.2
11.4	11.1	10.8
15.2	14.8	14.4
19.0	18.5	18.0
22.8	22.2	21.6
26.6	25.9	25.2
30.4	29.6	28.8
34.2	33.3	32.4

33	32	31
6.6	6.4	6.2
9.9	9.6	9.3
13.2	12.8	12.4
16.5	16.0	15.5
19.8	19.2	18.6
23.1	22.4	21.7
26.4	25.6	24.8
29.7	28.8	27.9

6	5	
1.2	1.0	0.8
1.8	1.5	1.2
2.4	2.0	1.6
3.0	2.5	2.0
3.6	3.0	2.4
4.2	3.5	2.8
4.8	4.0	3.2
5.4	4.5	3.6

From the top:

For **21°+** or **201°+**, read as printed; for **111°+** or **291°+**, read co-function.

From the bottom:

For **68°+** or **248°+**, read as printed; for **158°+** or **338°+**, read co-function.

Prop. Pts.

′	L Sin	d	L Tan	c d	L Ctn	L Cos
0	9.57 358		9.60 641		10.39 359	9.96 717
1	9.57 389		9.60 677		10.39 323	9.96 711
2	9.57 420		9.60 714		10.39 286	9.96 706
3	9.57 451		9.60 750		10.39 250	9.96 701
4	9.57 482		9.60 786		10.39 214	9.96 696
5	9.57 514		9.60 823		10.39 177	9.96 691
6	9.57 545		9.60 859		10.39 141	9.96 686
7	9.57 576		9.60 895		10.39 105	9.96 681
8	9.57 607		9.60 931		10.39 069	9.96 676
9	9.57 638		9.60 967		10.39 033	9.96 670
10	9.57 669		9.61 004		10.38 996	9.96 665
11	9.57 700		9.61 040		10.38 960	9.96 660
12	9.57 731		9.61 076		10.38 924	9.96 655
13	9.57 762		9.61 112		10.38 888	9.96 650
14	9.57 793		9.61 148		10.38 852	9.96 645
15	9.57 824		9.61 184		10.38 816	9.96 640
16	9.57 855		9.61 220		10.38 780	9.96 634
17	9.57 885		9.61 256		10.38 744	9.96 629
18	9.57 916		9.61 292		10.38 708	9.96 624
19	9.57 947		9.61 328		10.38 672	9.96 619
20	9.57 978		9.61 364		10.38 636	9.96 614
21	9.58 008		9.61 400		10.38 600	9.96 608
22	9.58 039		9.61 436		10.38 564	9.96 603
23	9.58 070		9.61 472		10.38 528	9.96 598
24	9.58 101		9.61 508		10.38 492	9.96 593
25	9.58 131		9.61 544		10.38 456	9.96 588
26	9.58 162		9.61 579		10.38 421	9.96 582
27	9.58 192		9.61 615		10.38 385	9.96 577
28	9.58 223		9.61 651		10.38 349	9.96 572
29	9.58 253		9.61 687		10.38 313	9.96 567
30	9.58 284		9.61 722		10.38 278	9.96 562
31	9.58 314		9.61 758		10.38 242	9.96 556
32	9.58 345		9.61 794		10.38 206	9.96 551
33	9.58 375		9.61 830		10.38 170	9.96 546
34	9.58 406		9.61 865		10.38 135	9.96 541
35	9.58 436		9.61 901		10.38 099	9.96 535
36	9.58 467		9.61 936		10.38 064	9.96 530
37	9.58 497		9.61 972		10.38 028	9.96 525
38	9.58 527		9.62 008		10.37 992	9.96 520
39	9.58 557		9.62 043		10.37 957	9.96 514
40	9.58 588		9.62 079		10.37 921	9.96 509
41	9.58 618		9.62 114		10.37 886	9.96 504
42	9.58 648		9.62 150		10.37 850	9.96 498
43	9.58 678		9.62 185		10.37 815	9.96 493
44	9.58 709		9.62 221		10.37 779	9.96 488
45	9.58 739		9.62 256		10.37 744	9.96 483
46	9.58 769		9.62 292		10.37 708	9.96 477
47	9.58 799		9.62 327		10.37 673	9.96 472
48	9.58 829		9.62 362		10.37 638	9.96 467
49	9.58 859		9.62 398		10.37 602	9.96 461
50	9.58 889		9.62 433		10.37 567	9.96 456
51	9.58 919		9.62 468		10.37 532	9.96 451
52	9.58 949		9.62 504		10.37 496	9.96 445
53	9.58 979		9.62 539		10.37 461	9.96 440
54	9.59 009		9.62 574		10.37 426	9.96 435
55	9.59 039		9.62 609		10.37 391	9.96 429
56	9.59 069		9.62 645		10.37 355	9.96 424
57	9.59 098		9.62 680		10.37 320	9.96 419
58	9.59 128		9.62 715		10.37 285	9.96 413
59	9.59 158		9.62 750		10.37 250	9.96 408
60	9.59 188		9.62 785		10.37 215	9.96 403
	L Cos	d	L Ctn	c d	L Tan	L Sin

Prop. Pts.

37	36	35
7.4	7.2	7.0
11.1	10.8	10.5
14.8	14.4	14.0
18.5	18.0	17.5
22.2	21.6	21.0
25.9	25.2	24.5
29.6	28.8	28.0
33.3	32.4	31.5

32	31	30
6.4	6.2	6.0
9.6	9.3	9.0
12.8	12.4	12.0
16.0	15.5	15.0
19.2	18.6	18.0
22.4	21.7	21.0
25.6	24.8	24.0
28.8	27.9	27.0

29	6	
5.8	1.2	1.0
8.7	1.8	1.5
11.6	2.4	2.0
14.5	3.0	2.5
17.4	3.6	3.0
20.3	4.2	3.5
23.2	4.8	4.0
26.1	5.4	4.5

From the top:

For **22°+** or **202°+**, read as printed; for **112°+** or **292°+**, read co-function.

From the bottom:

For **67°+** or **247°+**, read as printed; for **157°+** or **337°+**, read co-function.

Prop. Pts.

L Sin	L Tan		L Ctn	L Cos	
9.59 188	9.62 785		10.37 215	9.96 403	**60**
9.59 218	9.62 820		10.37 180	9.96 397	59
9.59 247	9.62 855		10.37 145	9.96 392	58
9.59 277	9.62 890		10.37 110	9.96 387	57
9.59 307	9.62 926		10.37 074	9.96 381	56
9.59 336	9.62 961		10.37 039	9.96 376	**55**
9.59 366	9.62 996		10.37 004	9.96 370	54
9.59 396	9.63 031		10.36 969	9.96 365	53
9.59 425	9.63 066		10.36 934	9.96 360	52
9.59 455	9.63 101		10.36 899	9.96 354	51
9.59 484	9.63 135		10.36 865	9.96 349	**50**
9.59 514	9.63 170		10.36 830	9.96 343	49
9.59 543	9.63 205		10.36 795	9.96 338	48
9.59 573	9.63 240		10.36 760	9.96 333	47
9.59 602	9.63 275		10.36 725	9.96 327	46
9.59 632	9.63 310		10.36 690	9.96 322	**45**
9.59 661	9.63 345		10.36 655	9.96 316	44
9.59 690	9.63 379		10.36 621	9.96 311	43
9.59 720	9.63 414		10.36 586	9.96 305	42
9.59 749	9.63 449		10.36 551	9.96 300	41
9.59 778	9.63 484		10.36 516	9.96 294	**40**
9.59 808	9.63 519		10.36 481	9.96 289	39
9.59 837	9.63 553		10.36 447	9.96 284	38
9.59 866	9.63 588		10.36 412	9.96 278	37
9.59 895	9.63 623		10.36 377	9.96 273	36
9.59 924	9.63 657		10.36 343	9.96 267	**35**
9.59 954	9.63 692		10.36 308	9.96 262	34
9.59 983	9.63 726		10.36 274	9.96 256	33
9.60 012	9.63 761		10.36 239	9.96 251	32
9.60 041	9.63 796		10.36 204	9.96 245	31
9.60 070	9.63 830		10.36 170	9.96 240	**30**
9.60 099	9.63 865		10.36 135	9.96 234	29
9.60 128	9.63 899		10.36 101	9.96 229	28
9.60 157	9.63 934		10.36 066	9.96 223	27
9.60 186	9.63 968		10.36 032	9.96 218	26
9.60 215	9.64 003		10.35 997	9.96 212	**25**
9.60 244	9.64 037		10.35 963	9.96 207	24
9.60 273	9.64 072		10.35 928	9.96 201	23
9.60 302	9.64 106		10.35 894	9.96 196	22
9.60 331	9.64 140		10.35 860	9.96 190	21
9.60 359	9.64 175		10.35 825	9.96 185	**20**
9.60 388	9.64 209		10.35 791	9.96 179	19
9.60 417	9.64 243		10.35 757	9.96 174	18
9.60 446	9.64 278		10.35 722	9.96 168	17
9.60 474	9.64 312		10.35 688	9.96 162	16
9.60 503	9.64 346		10.35 654	9.96 157	**15**
9.60 532	9.64 381		10.35 619	9.96 151	14
9.60 561	9.64 415		10.35 585	9.96 146	13
9.60 589	9.64 449		10.35 551	9.96 140	12
9.60 618	9.64 483		10.35 517	9.96 135	11
9.60 646	9.64 517		10.35 483	9.96 129	**10**
9.60 675	9.64 552		10.35 448	9.96 123	9
9.60 704	9.64 586		10.35 414	9.96 118	
9.60 732	9.64 620		10.35 380	9.96 112	
9.60 761	9.64 654		10.35 346	9.96 107	
9.60 789	9.64 688		10.35 312	9.96 101	
9.60 818	9.64 722		10.35 278	9.96 095	
9.60 846	9.64 756		10.35 244	9.96 090	
9.60 875	9.64 790		10.35 210	9.96 084	
9.60 903	9.64 824		10.35 176	9.96 079	
9.60 931	9.64 858		10.35 142	9.96 073	
L Cos	L Ctn	c d	L Tan	L Sin	d

Prop. Pts.

36	35
7.2	7.0
10.8	10.5
14.4	14.0
18.0	17.5
21.6	21.0
25.2	24.5
28.8	28.0
32.4	31.5

34	30
6.8	6.0
10.2	9.0
13.6	12.0
17.0	15.0
20.4	18.0
23.8	21.0
27.2	24.0
30.6	27.0

	29	28
2	5.8	5.6
3	8.7	8.4
4	11.6	11.2
5	14.5	14.0
6	17.4	16.8
7	20.3	19.6
8	23.2	22.4
9	26.1	25.2

	6	5
2	1.2	1.0
3	1.8	1.5
4	2.4	2.0
5	3.0	2.5
6	3.6	3.0
7	4.2	3.5
8	4.8	4.0
9	5.4	4.5

From the top:
For **23°+** or **203°+** read as printed; for **113°+** or **293°+**, read co-function.

From the bottom:
For **66°+** or **246°+**, read as printed; for **156°+** or **336°+**, read co-function.

Prop. Pts.

L Sin	d	L Tan	c d	L Ctn	L Cos	d
9.60 931		9.64 858		10.35 142	9.96 073	
9.60 960		9.64 892		10.35 108	9.96 067	
9.60 988		9.64 926		10.35 074	9.96 062	
9.61 016		9.64 960		10.35 040	9.96 056	
9.61 045		9.64 994		10.35 006	9.96 050	
9.61 073		9.65 028		10.34 972	9.96 045	
9.61 101		9.65 062		10.34 938	9.96 039	
9.61 129		9.65 096		10.34 904	9.96 034	
9.61 158		9.65 130		10.34 870	9.96 028	
9.61 186		9.65 164		10.34 836	9.96 022	
9.61 214		9.65 197		10.34 803	9.96 017	
9.61 242		9.65 231		10.34 769	9.96 011	
9.61 270		9.65 265		10.34 735	9.96 005	
9.61 298		9.65 299		10.34 701	9.96 000	
9.61 326		9.65 333		10.34 667	9.95 994	
9.61 354		9.65 366		10.34 634	9.95 988	
9.61 382		9.65 400		10.34 600	9.95 982	
9.61 411		9.65 434		10.34 566	9.95 977	
9.61 438		9.65 467		10.34 533	9.95 971	
9.61 466		9.65 501		10.34 499	9.95 965	
9.61 494		9.65 535		10.34 465	9.95 960	
9.61 522		9.65 568		10.34 432	9.95 954	
9.61 550		9.65 602		10.34 398	9.95 948	
9.61 578		9.65 636		10.34 364	9.95 942	
9.61 606		9.65 669		10.34 331	9.95 937	
9.61 634		9.65 703		10.34 297	9.95 931	
9.61 662		9.65 736		10.34 264	9.95 925	
9.61 689		9.65 770		10.34 230	9.95 920	
9.61 717		9.65 803		10.34 197	9.95 914	
9.61 745		9.65 837		10.34 163	9.95 908	
9.61 773		9.65 870		10.34 130	9.95 902	
9.61 800		9.65 904		10.34 096	9.95 897	
9.61 828		9.65 937		10.34 063	9.95 891	
9.61 856		9.65 971		10.34 029	9.95 885	
9.61 883		9.66 004		10.33 996	9.95 879	
9.61 911		9.66 038		10.33 962	9.95 873	
9.61 939		9.66 071		10.33 929	9.95 868	
9.61 966		9.66 104		10.33 896	9.95 862	
9.61 994		9.66 138		10.33 862	9.95 856	
9.62 021		9.66 171		10.33 829	9.95 850	
9.62 049		9.66 204		10.33 796	9.95 844	
9.62 076		9.66 238		10.33 762	9.95 839	
9.62 104		9.66 271		10.33 729	9.95 833	
9.62 131		9.66 304		10.33 696	9.95 827	
9.62 159		9.66 337		10.33 663	9.95 821	
9.62 186		9.66 371		10.33 629	9.95 815	
9.62 214		9.66 404		10.33 596	9.95 810	
9.62 241		9.66 437		10.33 563	9.95 804	
9.62 268		9.66 470		10.33 530	9.95 798	
9.62 296		9.66 503		10.33 497	9.95 792	
9.62 323		9.66 537		10.33 463	9.95 786	
9.62 350		9.66 570		10.33 430	9.95 780	
9.62 377		9.66 603		10.33 397	9.95 775	
9.62 405		9.66 636		10.33 364	9.95 769	
9.62 432		9.66 669		10.33 331	9.95 763	
9.62 459		9.66 702		10.33 298	9.95 757	
9.62 486		9.66 735		10.33 265	9.95 751	
9.62 513		9.66 768		10.33 232	9.95 745	
9.62 541		9.66 801		10.33 199	9.95 739	
9.62 568		9.66 834		10.33 166	9.95 733	
9.62 595		9.66 867		10.33 133	9.95 728	
L Cos	d	L Ctn	c d	L Tan	L Sin	d ′

Prop. Pts.

	34	33
2	6.8	6.6
3	10.2	9.9
4	13.6	13.2
5	17.0	16.5
6	20.4	19.8
7	23.8	23.1
8	27.2	26.4
9	30.6	29.7

	29	28
2	5.8	5.6
3	8.7	8.4
4	11.6	11.2
5	14.5	14.0
6	17.4	16.8
7	20.3	19.6
8	23.2	22.4
9	26.1	25.2

	27	6
2	5.4	1.2
3	8.1	1.8
4	10.8	2.4
5	13.5	3.0
6	16.2	3.6
7	18.9	4.2
8	21.6	4.8
9	24.3	5.4

	5
2	1.0
3	1.5
4	2.0
5	2.5
6	3.0
7	3.5
8	4.0
9	4.5

From the top:

For **24°+** or **204°+**, read as printed; for **114°+** or **294°+**, read co-function.

From the bottom:

For **65°+** or **245°+** read as printed; for **155°+** or **335°+**, read co-function.

Prop. Pts.

L Sin	d	L Tan	cd	L Ctn	L Cos	
9.62 595		9.66 867		10.33 133	9.95 728	60
9.62 622		9.66 900		10.33 100	9.95 722	59
9.62 649		9.66 933		10.33 067	9.95 716	58
9.62 676		9.66 966		10.33 034	9.95 710	57
9.62 703		9.66 999		10.33 001	9.95 704	56
9.62 730		9.67 032		10.32 968	9.95 698	55
9.62 757		9.67 065		10.32 935	9.95 692	54
9.62 784		9.67 098		10.32 902	9.95 686	53
9.62 811		9.67 131		10.32 869	9.95 680	52
9.62 838		9.67 163		10.32 837	9.95 674	51
9.62 865		9.67 196		10.32 804	9.95 668	50
9.62 892		9.67 229		10.32 771	9.95 663	49
9.62 918		9.67 262		10.32 738	9.95 657	48
9.62 945		9.67 295		10.32 705	9.95 651	47
9.62 972		9.67 327		10.32 673	9.95 645	46
9.62 999		9.67 360		10.32 640	9.95 639	45
9.63 026		9.67 393		10.32 607	9.95 633	44
9.63 052		9.67 426		10.32 574	9.95 627	43
9.63 079		9.67 458		10.32 542	9.95 621	42
9.63 106		9.67 491		10.32 509	9.95 615	41
9.63 133		9.67 524		10.32 476	9.95 609	40
9.63 159		9.67 556		10.32 444	9.95 603	39
9.63 186		9.67 589		10.32 411	9.95 597	38
9.63 213		9.67 622		10.32 378	9.95 591	37
9.63 239		9.67 654		10.32 346	9.95 585	36
9.63 266		9.67 687		10.32 313	9.95 579	35
9.63 292		9.67 719		10.32 281	9.95 573	34
9.63 319		9.67 752		10.32 248	9.95 567	33
9.63 345		9.67 785		10.32 215	9.95 561	32
9.63 372		9.67 817		10.32 183	9.95 555	31
9.63 398		9.67 850		10.32 150	9.95 549	30
9.63 425		9.67 882		10.32 118	9.95 543	29
9.63 451		9.67 915		10.32 085	9.95 537	28
9.63 478		9.67 947		10.32 053	9.95 531	27
9.63 504		9.67 980		10.32 020	9.95 525	26
9.63 531		9.68 012		10.31 988	9.95 519	25
9.63 557		9.68 044		10.31 956	9.95 513	24
9.63 583		9.68 077		10.31 923	9.95 507	23
9.63 610		9.68 109		10.31 891	9.95 500	22
9.63 636		9.68 142		10.31 858	9.95 494	21
9.63 662		9.68 174		10.31 826	9.95 488	20
9.63 689		9.68 206		10.31 794	9.95 482	19
9.63 715		9.68 239		10.31 761	9.95 476	18
9.63 741		9.68 271		10.31 729	9.95 470	17
9.63 767		9.68 303		10.31 697	9.95 464	16
9.63 794		9.68 336		10.31 664	9.95 458	15
9.63 820		9.68 368		10.31 632	9.95 452	14
9.63 846		9.68 400		10.31 600	9.95 446	13
9.63 872		9.68 432		10.31 568	9.95 440	12
9.63 898		9.68 465		10.31 535	9.95 434	11
9.63 924		9.68 497		10.31 503	9.95 427	10
9.63 950		9.68 529		10.31 471	9.95 421	9
9.63 976		9.68 561		10.31 439	9.95 415	8
9.64 002		9.68 593		10.31 407	9.95 409	7
9.64 028		9.68 626		10.31 374	9.95 403	6
9.64 054		9.68 658		10.31 342	9.95 397	5
9.64 080		9.68 690		10.31 310	9.95 391	4
9.64 106		9.68 722		10.31 278	9.95 384	3
9.64 132		9.68 754		10.31 246	9.95 378	2
9.64 158		9.68 786		10.31 214	9.95 372	1
9.64 184		9.68 818		10.31 182	9.95 366	0
L Cos		L Ctn	cd	L Tan	L Sin	d ′

Prop. Pts.

	33	32
2	6.6	6.4
3	9.9	9.6
4	13.2	12.8
5	16.5	16.0
6	19.8	19.2
7	23.1	22.4
8	26.4	25.6
9	29.7	28.8

27	26
5.4	5.2
8.1	7.8
10.8	10.4
13.5	13.0
16.2	15.6
18.9	18.2
21.6	20.8
24.3	23.4

	7	
2	1.4	1.2
3	2.1	1.8
4	2.8	2.4
5	3.5	3.0
6	4.2	3.6
7	4.9	4.2
8	5.6	4.8
9	6.3	5.4

1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5

From the top:

For **25°+** or **205°+**, read as printed; for **115°+** or **295°+**, read co-function.

From the bottom:

For **64°+** or **244°+**, read as printed; for **154°+** or **334°+**, read co-function.

Prop. Pts.

	L Sin	d	L Tan	c d	L Ctn	L Cos	d
	9.64 184		9.68 818		10.31 182	9.95 366	
	9.64 210		9.68 850		10.31 150	9.95 360	
	9.64 236		9.68 882		10.31 118	9.95 354	
	9.64 262		9.68 914		10.31 086	9.95 348	
	9.64 288		9.68 946		10.31 054	9.95 341	
	9.64 313		9.68 978		10.31 022	9.95 335	
	9.64 339		9.69 010		10.30 990	9.95 329	
	9.64 365		9.69 042		10.30 958	9.95 323	
	9.64 391		9.69 074		10.30 926	9.95 317	
	9.64 417		9.69 106		10.30 894	9.95 310	
10	9.64 442		9.69 138		10.30 862	9.95 304	
11	9.64 468		9.69 170		10.30 830	9.95 298	
12	9.64 494		9.69 202		10.30 798	9.95 292	
13	9.64 519		9.69 234		10.30 766	9.95 286	
14	9.64 545		9.69 266		10.30 734	9.95 279	
15	9.64 571		9.69 298		10.30 702	9.95 273	
16	9.64 596		9.69 329		10.30 671	9.95 267	
17	9.64 622		9.69 361		10.30 639	9.95 261	
18	9.64 647		9.69 393		10.30 607	9.95 254	
19	9.64 673		9.69 425		10.30 575	9.95 248	
20	9.64 698		9.69 457		10.30 543	9.95 242	
21	9.64 724		9.69 488		10.30 512	9.95 236	
22	9.64 749		9.69 520		10.30 480	9.95 229	
23	9.64 775		9.69 552		10.30 448	9.95 223	
24	9.64 800		9.69 584		10.30 416	9.95 217	
25	9.64 826		9.69 615		10.30 385	9.95 211	
26	9.64 851		9.69 647		10.30 353	9.95 204	
27	9.64 877		9.69 679		10.30 321	9.95 198	
28	9.64 902		9.69 710		10.30 290	9.95 192	
29	9.64 927		9.69 742		10.30 258	9.95 185	
30	9.64 953		9.69 774		10.30 226	9.95 179	
31	9.64 978		9.69 805		10.30 195	9.95 173	
32	9.65 003		9.69 837		10.30 163	9.95 167	
33	9.65 029		9.69 868		10.30 132	9.95 160	
34	9.65 054		9.69 900		10.30 100	9.95 154	
35	9.65 079		9.69 932		10.30 068	9.95 148	
36	9.65 104		9.69 963		10.30 037	9.95 141	
37	9.65 130		9.69 995		10.30 005	9.95 135	
38	9.65 155		9.70 026		10.29 974	9.95 129	
39	9.65 180		9.70 058		10.29 942	9.95 122	
40	9.65 205		9.70 089		10.29 911	9.95 116	
41	9.65 230		9.70 121		10.29 879	9.95 110	
42	9.65 255		9.70 152		10.29 848	9.95 103	
43	9.65 281		9.70 184		10.29 816	9.95 097	
44	9.65 306		9.70 215		10.29 785	9.95 090	
45	9.65 331		9.70 247		10.29 753	9.95 084	
46	9.65 356		9.70 278		10.29 722	9.95 078	
47	9.65 381		9.70 309		10.29 691	9.95 071	
48	9.65 406		9.70 341		10.29 659	9.95 065	
49	9.65 431		9.70 372		10.29 628	9.95 059	
50	9.65 456		9.70 404		10.29 596	9.95 052	
51	9.65 481		9.70 435		10.29 565	9.95 046	
52	9.65 506		9.70 466		10.29 534	9.95 039	
53	9.65 531		9.70 498		10.29 502	9.95 033	
54	9.65 556		9.70 529		10.29 471	9.95 027	
55	9.65 580		9.70 560		10.29 440	9.95 020	
56	9.65 605		9.70 592		10.29 408	9.95 014	
57	9.65 630		9.70 623		10.29 377	9.95 007	
58	9.65 655		9.70 654		10.29 346	9.95 001	
59	9.65 680		9.70 685		10.29 315	9.94 995	
60	9.65 705		9.70 717		10.29 283	9.94 988	
	L Cos	d	L Ctn	c d	L Tan	L Sin	d

Prop. Pts.

	32	31
2	6.4	6.2
3	9.6	9.3
4	12.8	12.4
5	16.0	15.5
6	19.2	18.6
7	22.4	21.7
8	25.6	24.8
9	28.8	27.9

	26	25
2	5.2	5.0
3	7.8	7.5
4	10.4	10.0
5	13.0	12.5
6	15.6	15.0
7	18.2	17.5
8	20.8	20.0
9	23.4	22.5

	24	7
2	4.8	1.4
3	7.2	2.1
4	9.6	2.8
5	12.0	3.5
6	14.4	4.2
7	16.8	4.9
8	19.2	5.6
9	21.6	6.3

6
1.2
1.8
2.4
3.0
3.6
4.2
4.8
5.4

From the top:

For **26°+** or **206°+**, read as printed; for **116°+** or **296°+**, read co-function.

From the bottom:

For **63°+** or **243°+**, read as printed; for **153°+** or **333°+**, read co-function.

Prop. Pts.

L Sin	d	L Tan	cd	L Ctn	L Cos	d	
9.65 705		9.70 717		10.29 283	9.94 988		60
9.65 729		9.70 748		10.29 252	9.94 982		59
9.65 754		9.70 779		10.29 221	9.94 975		58
9.65 779		9.70 810		10.29 190	9.94 969		57
9.65 804		9.70 841		10.29 159	9.94 962		56
9.65 828		9.70 873		10.29 127	9.94 956		55
9.65 853		9.70 904		10.29 096	9.94 949		54
9.65 878		9.70 935		10.29 065	9.94 943		53
9.65 902		9.70 966		10.29 034	9.94 936		52
9.65 927		9.70 997		10.29 003	9.94 930		51
9.65 952		9.71 028		10.28 972	9.94 923		50
9.65 976		9.71 059		10.28 941	9.94 917		49
9.66 001		9.71 090		10.28 910	9.94 911		48
9.66 025		9.71 121		10.28 879	9.94 904		47
9.66 050		9.71 153		10.28 847	9.94 898		46
9.66 075		9.71 184		10.28 816	9.94 891		45
9.66 099		9.71 215		10.28 785	9.94 885		44
9.66 124		9.71 246		10.28 754	9.94 878		43
9.66 148		9.71 277		10.28 723	9.94 871		42
9.66 173		9.71 308		10.28 692	9.94 865		41
9.66 197		9.71 339		10.28 661	9.94 858		40
9.66 221		9.71 370		10.28 630	9.94 852		39
9.66 246		9.71 401		10.28 599	9.94 845		38
9.66 270		9.71 431		10.28 569	9.94 839		37
9.66 295		9.71 462		10.28 538	9.94 832		36
9.66 319		9.71 493		10.28 507	9.94 826		35
9.66 343		9.71 524		10.28 476	9.94 819		34
9.66 368		9.71 555		10.28 445	9.94 813		33
9.66 392		9.71 586		10.28 414	9.94 806		32
9.66 416		9.71 617		10.28 383	9.94 799		31
9.66 441		9.71 648		10.28 352	9.94 793		30
9.66 465		9.71 679		10.28 321	9.94 786		29
9.66 489		9.71 709		10.28 291	9.94 780		28
9.66 513		9.71 740		10.28 260	9.94 773		27
9.66 537		9.71 771		10.28 229	9.94 767		26
9.66 562		9.71 802		10.28 198	9.94 760		25
9.66 586		9.71 833		10.28 167	9.94 753		24
9.66 610		9.71 863		10.28 137	9.94 747		23
9.66 634		9.71 894		10.28 106	9.94 740		22
9.66 658		9.71 925		10.28 075	9.94 734		21
9.66 682		9.71 955		10.28 045	9.94 727		20
9.66 706		9.71 986		10.28 014	9.94 720		19
9.66 731		9.72 017		10.27 983	9.94 714		18
9.66 755		9.72 048		10.27 952	9.94 707		17
9.66 779		9.72 078		10.27 922	9.94 700		16
9.66 803		9.72 109		10.27 891	9.94 694		15
9.66 827		9.72 140		10.27 860	9.94 687		14
9.66 851		9.72 170		10.27 830	9.94 680		13
9.66 875		9.72 201		10.27 799	9.94 674		12
9.66 899		9.72 231		10.27 769	9.94 667		11
9.66 922		9.72 262		10.27 738	9.94 660		10
9.66 946		9.72 293		10.27 707	9.94 654		9
9.66 970		9.72 323		10.27 677	9.94 647		8
9.66 994		9.72 354		10.27 646	9.94 640		7
9.67 018		9.72 384		10.27 616	9.94 634		6
9.67 042		9.72 415		10.27 585	9.94 627		5
9.67 066		9.72 445		10.27 555	9.94 620		4
9.67 090		9.72 476		10.27 524	9.94 614		3
9.67 113		9.72 506		10.27 494	9.94 607		2
9.67 137		9.72 537		10.27 463	9.94 600		1
9.67 161		9.72 567		10.27 433	9.94 593		
L Cos	d	L Ctn	cd	L Tan	L Sin	d	′

Prop. Pts.

	32	31
2	6.4	6.2
3	9.6	9.3
4	12.8	12.4
5	16.0	15.5
6	19.2	18.6
7	22.4	21.7
8	25.6	24.8
9	28.8	27.9

	30	25
2	6.0	5.0
3	9.0	7.5
4	12.0	10.0
5	15.0	12.5
6	18.0	15.0
7	21.0	17.5
8	24.0	20.0
9	27.0	22.5

	24	23
2	4.8	4.6
3	7.2	6.9
4	9.6	9.2
5	12.0	11.5
6	14.4	13.8
7	16.8	16.1
8	19.2	18.4
9	21.6	20.7

7	6
1.4	1.2
2.1	1.8
2.8	2.4
3.5	3.0
4.2	3.6
4.9	4.2
5.6	4.8
6.3	5.4

From the top: For **27°+** or **207°+** read as printed; for **117°+** or **297°+**, read co-function.

From the bottom: For **62°+** or **242°+**, read as printed; for **152°+** or **332°+**, read co-function.

Prop. Pts.

'	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
	9.67 161	24	9.72 567	31	10.27 433	9.94 593		60
	9.67 185	23	9.72 598	30	10.27 402	9.94 587		59
	9.67 208	24	9.72 628	31	10.27 372	9.94 580		58
	9.67 232	24	9.72 659	30	10.27 341	9.94 573		57
	9.67 256	24	9.72 689	31	10.27 311	9.94 567		56
	9.67 280	23	9.72 720	30	10.27 280	9.94 560		**55**
	9.67 303	24	9.72 750	30	10.27 250	9.94 553		54
	9.67 327	23	9.72 780	31	10.27 220	9.94 546		53
8	9.67 350	24	9.72 811	30	10.27 189	9.94 540		52
9	9.67 374	24	9.72 841	31	10.27 159	9.94 533		51
10	9.67 398	23	9.72 872	30	10.27 128	9.94 526		**50**
11	9.67 421	24	9.72 902	30	10.27 098	9.94 519		49
12	9.67 445	23	9.72 932	31	10.27 068	9.94 513		48
13	9.67 468	24	9.72 963	30	10.27 037	9.94 506		47
14	9.67 492	23	9.72 993	30	10.27 007	9.94 499		46
	9.67 515	24	9.73 023	31	10.26 977	9.94 492		**45**
16	9.67 539	23	9.73 054	30	10.26 946	9.94 485		44
17	9.67 562	24	9.73 084	30	10.26 916	9.94 479		43
18	9.67 586	23	9.73 114	30	10.26 886	9.94 472		42
19	9.67 609	24	9.73 144	31	10.26 856	9.94 465		41
20	9.67 633	23	9.73 175	30	10.26 825	9.94 458		**40**
21	9.67 656	24	9.73 205	30	10.26 795	9.94 451		39
22	9.67 680	23	9.73 235	30	10.26 765	9.94 445		38
23	9.67 703	23	9.73 265	30	10.26 735	9.94 438		37
24	9.67 726	24	9.73 295	31	10.26 705	9.94 431		36
25	9.67 750	23	9.73 326	30	10.26 674	9.94 424		**35**
26	9.67 773	23	9.73 356	30	10.26 644	9.94 417		34
27	9.67 796	24	9.73 386	30	10.26 614	9.94 410		33
28	9.67 820	23	9.73 416	30	10.26 584	9.94 404		32
29	9.67 843	23	9.73 446	30	10.26 554	9.94 397		31
30	9.67 866	24	9.73 476	31	10.26 524	9.94 390		**30**
31	9.67 890	23	9.73 507	30	10.26 493	9.94 383		29
32	9.67 913	23	9.73 537	30	10.26 463	9.94 376		28
33	9.67 936	23	9.73 567	30	10.26 433	9.94 369		27
34	9.67 959	23	9.73 597	30	10.26 403	9.94 362		26
35	9.67 982	24	9.73 627	30	10.26 373	9.94 355		**25**
36	9.68 006	23	9.73 657	30	10.26 343	9.94 349		24
37	9.68 029	23	9.73 687	30	10.26 313	9.94 342		23
38	9.68 052	23	9.73 717	30	10.26 283	9.94 335		22
39	9.68 075	23	9.73 747	30	10.26 253	9.94 328		21
40	9.68 098	23	9.73 777	30	10.26 223	9.94 321		**20**
41	9.68 121	23	9.73 807	30	10.26 193	9.94 314		19
42	9.68 144	23	9.73 837	30	10.26 163	9.94 307		18
43	9.68 167	23	9.73 867	30	10.26 133	9.94 300		17
44	9.68 190	23	9.73 897	30	10.26 103	9.94 293		16
45	9.68 213	24	9.73 927	30	10.26 073	9.94 286		**15**
46	9.68 237	23	9.73 957	30	10.26 043	9.94 279		14
47	9.68 260	23	9.73 987	30	10.26 013	9.94 273		13
48	9.68 283	22	9.74 017	30	10.25 983	9.94 266		12
49	9.68 305	23	9.74 047	30	10.25 953	9.94 259		11
50	9.68 328	23	9.74 077	30	10.25 923	9.94 252		**10**
51	9.68 351	23	9.74 107	30	10.25 893	9.94 245		9
52	9.68 374	23	9.74 137	29	10.25 863	9.94 238		8
53	9.68 397	23	9.74 166	30	10.25 834	9.94 231		7
54	9.68 420	23	9.74 196	30	10.25 804	9.94 224		6
55	9.68 443	23	9.74 226	30	10.25 774	9.94 217		**5**
	9.68 466	23	9.74 256	30	10.25 744	9.94 210		4
	9.68 489	23	9.74 286	30	10.25 714	9.94 203		3
	9.68 512	22	9.74 316	29	10.25 684	9.94 196		2
	9.68 534	23	9.74 345	30	10.25 655	9.94 189		1
	9.68 557		9.74 375		10.25 625	9.94 182		0
	L Cos		L Ctn		L Tan	L Sin		

Prop. Pts.

	31	30
2	6.2	6.0
3	9.3	9.0
4	12.4	12.0
5	15.5	15.0
6	18.6	18.0
7	21.7	21.0
8	24.8	24.0
9	27.9	27.0

	29	24
2	5.8	4.8
3	8.7	7.2
4	11.6	9.6
5	14.5	12.0
6	17.4	14.4
7	20.3	16.8
8	23.2	19.2
9	26.1	21.6

	23	22
2	4.6	4.4
3	6.9	6.6
4	9.2	8.8
5	11.5	11.0
6	13.8	13.2
7	16.1	15.4
8	18.4	17.6
9	20.7	19.8

1.4	1.2
2.1	1.8
2.8	2.4
3.5	3.0
4.2	3.6
4.9	4.2
5.6	4.8
6.3	5.4

From the top:

For **28°+** or **208°+**, read as printed; for **118°+** or **298°+**, read co-function.

From the bottom:

For **61°+** or **241°+**, read as printed; for **151°+** or **331°+**, read co-function.

Prop. Pts.

L Sin	L Tan	c d	L Ctn	L Cos	d
9.68 557	9.74 375		10.25 625	9.94 182	
9.68 580	9.74 405		10.25 595	9.94 175	
9.68 603	9.74 435		10.25 565	9.94 168	
9.68 625	9.74 465		10.25 535	9.94 161	
9.68 648	9.74 494		10.25 506	9.94 154	
9.68 671	9.74 524		10.25 476	9.94 147	
9.68 694	9.74 554		10.25 446	9.94 140	
9.68 716	9.74 583		10.25 417	9.94 133	
9.68 739	9.74 613		10.25 387	9.94 126	
9.68 762	9.74 643		10.25 357	9.94 119	
9.68 784	9.74 673		10.25 327	9.94 112	
9.68 807	9.74 702		10.25 298	9.94 105	
9.68 829	9.74 732		10.25 268	9.94 098	
9.68 852	9.74 762		10.25 238	9.94 090	
9.68 875	9.74 791		10.25 209	9.94 083	
9.68 897	9.74 821		10.25 179	9.94 076	
9.68 920	9.74 851		10.25 149	9.94 069	
9.68 942	9.74 880		10.25 120	9.94 062	
9.68 965	9.74 910		10.25 090	9.94 055	
9.68 987	9.74 939		10.25 061	9.94 048	
9.69 010	9.74 969		10.25 031	9.94 041	
9.69 032	9.74 998		10.25 002	9.94 034	
9.69 055	9.75 028		10.24 972	9.94 027	
9.69 077	9.75 058		10.24 942	9.94 020	
9.69 100	9.75 087		10.24 913	9.94 012	
9.69 122	9.75 117		10.24 883	9.94 005	
9.69 144	9.75 146		10.24 854	9.93 998	
9.69 167	9.75 176		10.24 824	9.93 991	
9.69 189	9.75 205		10.24 795	9.93 984	
9.69 212	9.75 235		10.24 765	9.93 977	
9.69 234	9.75 264		10.24 736	9.93 970	
9.69 256	9.75 294		10.24 706	9.93 963	
9.69 279	9.75 323		10.24 677	9.93 955	
9.69 301	9.75 353		10.24 647	9.93 948	
9.69 323	9.75 382		10.24 618	9.93 941	
9.69 345	9.75 411		10.24 589	9.93 934	
9.69 368	9.75 441		10.24 559	9.93 927	
9.69 390	9.75 470		10.24 530	9.93 920	
9.69 412	9.75 500		10.24 500	9.93 912	
9.69 434	9.75 529		10.24 471	9.93 905	
9.69 456	9.75 558		10.24 442	9.93 898	
9.69 479	9.75 588		10.24 412	9.93 891	
9.69 501	9.75 617		10.24 383	9.93 884	
9.69 523	9.75 647		10.24 353	9.93 876	
9.69 545	9.75 676		10.24 324	9.93 869	
9.69 567	9.75 705		10.24 295	9.93 862	
9.69 589	9.75 735		10.24 265	9.93 855	
9.69 611	9.75 764		10.24 236	9.93 847	
9.69 633	9.75 793		10.24 207	9.93 840	
9.69 655	9.75 822		10.24 178	9.93 833	
9.69 677	9.75 852		10.24 148	9.93 826	
9.69 699	9.75 881		10.24 119	9.93 819	
9.69 721	9.75 910		10.24 090	9.93 811	
9.69 743	9.75 939		10.24 061	9.93 804	
9.69 765	9.75 969		10.24 031	9.93 797	
9.69 787	9.75 998		10.24 002	9.93 789	
9.69 809	9.76 027		10.23 973	9.93 782	
9.69 831	9.76 056		10.23 944	9.93 775	
9.69 853	9.76 086		10.23 914	9.93 768	
9.69 875	9.76 115		10.23 885	9.93 760	
9.69 897	9.76 144		10.23 856	9.93 753	
L Cos	L Ctn	c d	L Tan	L Sin	

Prop. Pts.

30	29
6.0	5.8
9.0	8.7
12.0	11.6
15.0	14.5
18.0	17.4
21.0	20.3
24.0	23.2
27.0	26.1

	23	22
2	4.6	4.4
3	6.9	6.6
4	9.2	8.8
5	11.5	11.0
6	13.8	13.2
7	16.1	15.4
8	18.4	17.6
9	20.7	19.8

	8	
2	1.6	1.4
3	2.4	2.1
4	3.2	2.8
5	4.0	3.5
6	4.8	4.2
7	5.6	4.9
8	6.4	5.6
9	7.2	6.3

From the top:

For **29°+** or **209°+**, read as printed; for **119°+** or **299°+**, read co-function.

From the bottom:

For **60°+** or **240°+**, read as printed; for **150°+** or **330°+**, read co-function.

Prop. Pts.

′	L Sin	d	L Tan	c	L Ctn	L Cos	
	9.69 89	22	9.76 144		10.23 85	9.93 75	60
	9.69 91	22	9.76 17		10.23 8.	9.93 74	59
	9.69 94	22	9.76 20		10.23 79	9.93 73	58
	9.69 96	21	9.76 23		10.23 76	9.93 73	57
	9.69 98	22	9.76 26		10.23 73	9.93 7	56
	9.70 00	22	9.76 29		10.23 71	9.93 71	55
	9.70 02	22	9.76 31		10.23 68	9.93 70	54
	9.70 05	22	9.76 34		10.23 65	9.93 70	53
	9.70 07	21	9.76 37		10.23 62	9.93 69	52
	9.70 09	22	9.76 40		10.23 59	9.93 68	51
10	9.70 11	22	9.76 43		10.23 56	9.93 68	50
11	9.70 13	22	9.76 464		10.23 53	9.93 67	49
12	9.70 15	21	9.76 49		10.23 50	9.93 66	48
13	9.70 18	22	9.76 52		10.23 47	9.93 65	47
14	9.70 20	22	9.76 55		10.23 44	9.93 65	46
15	9.70 224	21	9.76 58		10.23 42	9.93 64	45
16	9.70 24	22	9.76 60		10.23 39	9.93 63	44
17	9.70 26	21	9.76 63		10.23 36	9.93 62	43
18	9.70 288	22	9.76 66		10.23 332	9.93 62	42
19	9.70 31	22	9.76 697		10.23 30	9.93 614	41
20	9.70 332	21	9.76 725		10.23 275	9.93 60	40
21	9.70 353	22	9.76 754		10.23 24	9.93 599	39
22	9.70 375	21	9.76 783		10.23 217	9.93 591	38
23	9.70 396	22	9.76 81		10.23 18	9.93 584	37
24	9.70 418	21	9.76 841		10.23 159	9.93 577	36
25	9.70 439	22	9.76 870		10.23 130	9.93 569	35
26	9.70 461	21	9.76 899		10.23 101	9.93 562	34
27	9.70 482	22	9.76 928		10.23 07	9.93 554	33
28	9.70 504	21	9.76 957		10.23 043	9.93 547	32
29	9.70 525	22	9.76 986		10.23 014	9.93 539	31
30	9.70 547	21	9.77 015		10.22 985	9.93 53	30
31	9.70 568	22	9.77 044		10.22 956	9.93 525	29
32	9.70 590	21	9.77 073		10.22 927	9.93 517	28
33	9.70 611	22	9.77 101		10.22 899	9.93 510	27
34	9.70 633	21	9.77 130		10.22 870	9.93 502	26
35	9.70 654	21	9.77 159		10.22 841	9.93 495	25
36	9.70 675	22	9.77 188		10.22 812	.93 487	24
37	9.70 697	21	9.77 217		10.22 783	9.93 480	23
38	9.70 718	21	9.77 246		10.22 754	.93 47	22
39	9.70 739	22	9.77 274		10.22 726	9.93 465	21
40	9.70 761	21	9.77 303		0.22 697	.93 457	20
41	9.70 782	21	9.77 332		0.22 668	.93 450	19
42	9.70 803	21	9.77 361		0.22 639	.93 442	18
43	9.70 824	22	9.77 390		0.22 610	.93 435	17
44	9.70 846	1	9.77 418		0.22 582	.93 427	16
45	9.70 867	1	9.77 447		0.22 553	.93 420	15
46	9.70 888	1	9.77 476		0.22 524	.93 412	14
47	9.70 909	2	9.77 505		0.22 495	.93 405	13
48	9.70 931	1	9.77 533		0.22 467	.93 397	12
49	9.70 952	1	9.77 562		0.22 438	.93 390	11
50	9.70 973	1	9.77 591		0.22 409	.93 382	10
51	9.70 994	1	9.77 619		0.22 381	.93 375	9
52	9.71 015	1	9.77 648		0.22 352	.93 367	8
53	9.71 036	2	9.77 677		0.22 323	.93 360	
54	9.71 058	1	9.77 706		0.22 294	.93 352	
55	9.71 079	1	9.77 734		0.22 266	.93 344	5
56	9.71 100	1	9.77 763		0.22 237	.93 337	4
57	9.71 121	1	9.77 791		0.22 209	.93 329	3
58	9.71 142	1	9.77 820		0.22 180	.93 322	2
59	9.71 163	1	9.77 849		0.22 151	93 314	1
60	9.71 184		9.77 877		0.22 123	93 307	0
	L Cos	d	L Ctn		Tan	L Sin	′

Prop. Pts.

30	29
6.0	5.8
9.0	8.7
12.0	11.6
15.0	14.5
18.0	17.4
21.0	20.3
24.0	23.2
27.0	26.1

28	22
5.6	4.4
8.4	6.6
11.2	8.8
14.0	11.0
16.8	13.2
19.6	15.4
22.4	17.6
25.2	19.8

21	8
4.2	1.6
6.3	2.4
8.4	3.2
10.5	4.0
12.6	4.8
14.7	5.6
16.8	6.4
18.9	7.2

7
1.4
2.1
2.8
3.5
4.2
4.9
5.6
6.3

From the top:

For **30°+** or **210°**, read as printed; for **120°+** or **300°+**, read co-function.

From the bottom:

For **59°+** or **239°+**, read as printed; for **149°+** or **329°+**, read co-function.

Prop. Pts

L Sin	L Tan	c d	L Ctn	L Cos	
9.71 184	9.77 877		10.22 123	9.93 307	**60**
9.71 205	9.77 906		10.22 094	9.93 299	59
9.71 226	9.77 935		10.22 065	9.93 291	58
9.71 247	9.77 963		10.22 037	9.93 284	57
9.71 268	9.77 992		10.22 008	9.93 276	56
9.71 289	9.78 020		10.21 980	9.93 269	**55**
9.71 310	9.78 049		10.21 951	9.93 261	54
9.71 331	9.78 077		10.21 923	9.93 253	53
9.71 352	9.78 106		10.21 894	9.93 246	52
9.71 373	9.78 135		10.21 865	9.93 238	51
9.71 393	9.78 163		10.21 837	9.93 230	**50**
9.71 414	9.78 192		10.21 808	9.93 223	49
9.71 435	9.78 220		10.21 780	9.93 215	48
9.71 456	9.78 249		10.21 751	9.93 207	47
9.71 477	9.78 277		10.21 723	9.93 200	46
9.71 498	9.78 306		10.21 694	9.93 192	**45**
9.71 519	9.78 334		10.21 666	9.93 184	44
9.71 539	9.78 363		10.21 637	9.93 177	43
9.71 560	9.78 391		10.21 609	9.93 169	42
9.71 581	9.78 419		10.21 581	9.93 161	41
9.71 602	9.78 448		10.21 552	9.93 154	**40**
9.71 622	9.78 476		10.21 524	9.93 146	39
9.71 643	9.78 505		10.21 495	9.93 138	38
9.71 664	9.78 533		10.21 467	9.93 131	37
9.71 685	9.78 562		10.21 438	9.93 123	36
9.71 705	9.78 590		10.21 410	9.93 115	**35**
9.71 726	9.78 618		10.21 382	9.93 108	34
9.71 747	9.78 647		10.21 353	9.93 100	33
9.71 767	9.78 675		10.21 325	9.93 092	32
9.71 788	9.78 704		10.21 296	9.93 084	31
9.71 809	9.78 732		10.21 268	9.93 077	**30**
9.71 829	9.78 760		10.21 240	9.93 069	29
9.71 850	9.78 789		10.21 211	9.93 061	28
9.71 870	9.78 817		10.21 183	9.93 053	27
9.71 891	9.78 845		10.21 155	9.93 046	26
9.71 911	9.78 874		10.21 126	9.93 038	**25**
9.71 932	9.78 902		10.21 098	9.93 030	24
9.71 952	9.78 930		10.21 070	9.93 022	23
9.71 973	9.78 959		10.21 041	9.93 014	22
9.71 994	9.78 987		10.21 013	9.93 007	21
9.72 014	9.79 015		10.20 985	9.92 999	**20**
9.72 034	9.79 043		10.20 957	9.92 991	19
9.72 055	9.79 072		10.20 928	9.92 983	18
9.72 075	9.79 100		10.20 900	9.92 976	17
9.72 096	9.79 128		10.20 872	9.92 968	16
9.72 116	9.79 156		10.20 844	9.92 960	**15**
9.72 137	9.79 185		10.20 815	9.92 952	14
9.72 157	9.79 213		10.20 787	9.92 944	13
9.72 177	9.79 241		10.20 759	9.92 936	12
9.72 198	9.79 269		10.20 731	9.92 929	11
9.72 218	9.79 297		10.20 703	9.92 921	**10**
9.72 238	9.79 326		10.20 674	9.92 913	9
9.72 259	9.79 354		10.20 646	9.92 905	8
9.72 279	9.79 382		10.20 618	9.92 897	7
9.72 299	9.79 410		10.20 590	9.92 889	6
9.72 320	9.79 438		10.20 562	9.92 881	**5**
9.72 340	9.79 466		10.20 534	9.92 874	
9.72 360	9.79 495		10.20 505	9.92 866	
9.72 381	9.79 523		10.20 477	9.92 858	
9.72 401	9.79 551		10.20 449	9.92 850	
9.72 421	9.79 579		10.20 421	9.92 842	
L Cos	L Ctn	c d	L Tan	L Sin	

Prop. Pts.

	29	28
2	5.8	5.6
3	8.7	8.4
4	11.6	11.2
5	14.5	14.0
6	17.4	16.8
7	20.3	19.6
8	23.2	22.4
9	26.1	25.2

	21	20
2	4.2	4.0
3	6.3	6.0
4	8.4	8.0
5	10.5	10.0
6	12.6	12.0
7	14.7	14.0
8	16.8	16.0
9	18.9	18.0

	8	
2	1.6	1.4
3	2.4	2.1
4	3.2	2.8
5	4.0	3.5
6	4.8	4.2
7	5.6	4.9
8	6.4	5.6
9	7.2	6.3

From the top:

For **31°+** or **211°+**, read as printed; for **121°+** or **301°+**, read co-function.

From the bottom:

For **58°+** or **238°+**, read as printed; for **148°+** or **328°+**, read co-function.

Prop. Pts.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	′
	9.72 421	20	9.79 579	28	10.20 421	9.92 842	60
	9.72 441	20	9.79 607	28	10.20 393	9.92 83[illegible]	5[illegible]
	9.72 461	21	9.79 635	28	10.20 365	9.92 826	5[illegible]
	9.72 482	20	9.79 663	28	10.20 337	9.92 818	5[illegible]
	9.72 502	20	9.79 691	28	10.20 309	9.92 810	56
	9.72 52[illegible]	20	9.79 719	28	10.20 281	9.92 803	5[illegible]
	9.72 542	20	9.79 747	29	10.20 253	9.92 795	5[illegible]
	9.72 56[illegible]	20	9.79 776	28	10.20 224	9.92 787	53
	9.72 58[illegible]	20	9.79 804	28	10.20 196	9.92 779	52
	9.72 60[illegible]	20	9.79 83[illegible]	28	10.20 168	9.92 771	51
10	9.72 62[illegible]	21	9.79 860	28	10.20 140	9.92 763	50
11	9.72 643	20	9.79 888	28	10.20 112	9.92 755	49
1[illegible]	9.72 663	20	9.79 916	28	10.20 084	9.92 747	48
13	9.72 683	20	9.79 944	28	10.20 056	9.92 739	47
14	9.72 703	20	9.79 97[illegible]	28	10.20 028	9.92 731	46
15	9.72 723	20	9.80 000	28	10.20 000	9.92 723	45
16	9.72 743	20	9.80 028	28	10.19 97[illegible]	9.92 715	44
17	9.72 763	20	9.80 056	28	10.19 944	9.92 707	43
18	9.72 783	20	9.80 084	28	10.19 916	9.92 699	4[illegible]
19	9.72 803	20	9.80 112	28	10.19 888	9.92 691	41
20	9.72 823	20	9.80 140	28	10.19 860	9.92 683	40
21	9.72 843	20	9.80 168	27	10.19 832	9.92 675	39
22	9.72 863	20	9.80 195	28	10.19 805	9.92 667	38
23	9.72 883	19	9.80 223	28	10.19 777	9.92 659	37
24	9.72 902	20	9.80 251	28	10.19 749	9.92 651	36
25	9.72 922	20	9.80 279	28	10.19 721	9.92 643	35
26	9.72 942	20	9.80 307	28	10.19 693	9.92 635	34
27	9.72 962	20	9.80 335	28	10.19 665	9.92 627	33
28	9.72 982	20	9.80 363	28	10.19 637	9.92 619	32
29	9.73 002	20	9.80 391	28	10.19 609	9.92 611	31
30	9.73 022	19	9.80 419	28	10.19 581	9.92 603	30
31	9.73 041	20	9.80 447	27	10.19 553	9.92 595	29
32	9.73 061	20	9.80 474	28	10.19 526	9.92 587	28
33	9.73 081	20	9.80 502	28	10.19 498	9.92 579	27
34	9.73 101	20	9.80 530	28	10.19 470	9.92 571	26
35	9.73 121	19	9.80 558	28	10.19 442	9.92 563	25
36	9.73 140	20	9.80 586	28	10.19 414	9.92 555	24
37	9.73 160	20	9.80 614	28	10.19 386	9.92 546	23
38	9.73 180	20	9.80 642	27	10.19 358	9.92 538	22
39	9.73 200	19	9.80 669	28	10.19 331	9.92 530	21
40	9.73 219	20	9.80 697	28	10.19 303	9.92 522	20
41	9.73 239	20	9.80 725	28	10.19 275	9.92 514	19
42	9.73 259	19	9.80 753	28	10.19 247	9.92 506	18
43	9.73 278	20	9.80 781	27	10.19 219	9.92 498	17
44	9.73 298	20	9.80 808	28	10.19 192	9.92 490	16
45	9.73 318	19	9.80 836	28	10.19 164	9.92 482	15
46	9.73 337	20	9.80 864	28	10.19 136	9.92 473	14
47	9.73 357	20	9.80 892	27	10.19 108	9.92 465	13
48	9.73 377	19	9.80 919	28	10.19 081	9.92 457	12
49	9.73 396	20	9.80 947	28	10.19 053	9.92 449	11
50	9.73 416	19	9.80 975	28	10.19 025	9.92 441	10
51	9.73 435	20	9.81 003	27	10.18 997	9.92 433	
52	9.73 455	19	9.81 030	28	10.18 970	9.92 425	
53	9.73 474	20	[illegible].81 058	28	10.18 942	9.92 416	
54	9.73 494	19	9.81 086	27	10.18 914	[illegible].92 408	
55	9.73 513	20	9.81 113	28	10.18 887	[illegible].92 400	
56	9.73 533	19	9.81 141	28	[illegible]0.18 859	[illegible].92 392	
57	9.73 552	20	9.81 169	27	10.18 831	9.92 384	
[illegible]8	9.73 572	19	9.81 196	28	10.18 804	9.92 376	
[illegible]9	[illegible].73 591	20	[illegible].81 224	28	10.18 776	9.92 367	
60	[illegible].73 611		[illegible].81 252		10.18 748	[illegible]92 359	
	L Cos	d	L Ctn	c d	L Tan	L Sin	′

Prop. Pts.

	29	28
2	5.8	5.6
3	8.7	8.4
4	11.6	11.2
5	14.5	14.0
6	17.4	16.8
7	20.3	19.6
8	23.2	22.4
9	26.1	25.2

27	21
5.4	4.2
8.1	6.3
10.8	8.4
13.5	10.5
16.2	12.6
18.9	14.7
21.6	16.8
24.3	18.9

	20	19
2	4.0	3.8
3	6.0	5.7
4	8.0	7.6
5	10.0	9.5
6	12.0	11.4
7	14.0	13.3
8	16.0	15.2
9	18.0	17.1

	9	8	
	1.8	1.6	1.4
	2.7	2.4	2.1
	3.6	3.2	2.8
	4.5	4.0	3.5
	5.4	4.8	4.2
	6.3	5.6	4.9
	7.2	6.4	5.6
9	8.1	7.2	6.3

From the top:

For **32°+** or **212°+**, read as printed; for **122°+** or **302°+**, read co-function.

From the bottom:

For **57°+** or **237°+**, read as printed; for **147°+** or **327°+**, read [illegible]o-function.

Prop. Pts.

L Sin	d	L Tan	c d	L Ctn	L Cos
9.73 611	19	9.81 252	27	10.18 748	9.92 359
9.73 630	20	9.81 279	28	10.18 721	9.92 351
9.73 650	19	9.81 307	28	10.18 693	9.92 343
9.73 669	20	9.81 335	27	10.18 665	9.92 335
9.73 689	19	9.81 362	28	10.18 638	9.92 326
9.73 708	19	9.81 390	28	10.18 610	9.92 318
9.73 727	20	9.81 418	27	10.18 582	9.92 310
9.73 747	19	9.81 445	28	10.18 555	9.92 302
9.73 766	19	9.81 473	27	10.18 527	9.92 293
9.73 785	20	9.81 500	28	10.18 500	9.92 285
9.73 805	19	9.81 528	28	10.18 472	9.92 277
9.73 824	19	9.81 556	27	10.18 444	9.92 269
9.73 843	20	9.81 583	28	10.18 417	9.92 260
9.73 863	19	9.81 611	27	10.18 389	9.92 252
9.73 882	19	9.81 638	28	10.18 362	9.92 244
9.73 901	20	9.81 666	27	10.18 334	9.92 235
9.73 921	19	9.81 693	28	10.18 307	9.92 227
9.73 940	19	9.81 721	27	10.18 279	9.92 219
9.73 959	19	9.81 748	28	10.18 252	9.92 211
9.73 978	19	9.81 776	27	10.18 224	9.92 202
9.73 997	20	9.81 803	28	10.18 197	9.92 194
9.74 017	19	9.81 831	27	10.18 169	9.92 186
9.74 036	19	9.81 858	28	10.18 142	9.92 177
9.74 055	19	9.81 886	27	10.18 114	9.92 169
9.74 074	19	9.81 913	28	10.18 087	9.92 161
9.74 093	20	9.81 941	27	10.18 059	9.92 152
9.74 113	19	9.81 968	28	10.18 032	9.92 144
9.74 132	19	9.81 996	27	10.18 004	9.92 136
9.74 151	19	9.82 023	28	10.17 977	9.92 127
9.74 170	19	9.82 051	27	10.17 949	9.92 119
9.74 189	19	9.82 078	28	10.17 922	9.92 111
9.74 208	19	9.82 106	27	10.17 894	9.92 102
9.74 227	19	9.82 133	28	10.17 867	9.92 094
9.74 246	19	9.82 161	27	10.17 839	9.92 086
9.74 265	19	9.82 188	27	10.17 812	9.92 077
9.74 284	19	9.82 215	28	10.17 785	9.92 069
9.74 303	19	9.82 243	27	10.17 757	9.92 060
9.74 322	19	9.82 270	28	10.17 730	9.92 052
9.74 341	19	9.82 298	27	10.17 702	9.92 044
9.74 360	19	9.82 325	27	10.17 675	9.92 035
9.74 379	19	9.82 352	28	10.17 648	9.92 027
9.74 398	19	9.82 380	27	10.17 620	9.92 018
9.74 417	19	9.82 407	28	10.17 593	9.92 010
9.74 436	19	9.82 435	27	10.17 565	9.92 002
9.74 455	19	9.82 462	27	10.17 538	9.91 993
9.74 474	19	9.82 489	28	10.17 511	9.91 985
9.74 493	19	9.82 517	27	10.17 483	9.91 976
9.74 512	19	9.82 544	27	10.17 456	9.91 968
9.74 531	18	9.82 571	28	10.17 429	9.91 959
9.74 549	19	9.82 599	27	10.17 401	9.91 951
9.74 568	19	9.82 626	27	10.17 374	9.91 942
9.74 587	19	9.82 653	28	10.17 347	9.91 934
9.74 606	19	9.82 681	27	10.17 319	9.91 925
9.74 625	19	9.82 708	27	10.17 292	9.91 917
9.74 644	18	9.82 735	27	10.17 265	9.91 908
9.74 662	19	9.82 762	28	10.17 238	9.91 900
9.74 681	19	9.82 790	27	10.17 210	9.91 891
9.74 700	19	9.82 817	27	10.17 183	9.91 883
9.74 719	18	9.82 844	27	10.17 156	9.91 874
9.74 73'	19	9.82 871	28	10.17 129	9.91 866
9.74 756		9.82 899		10.17 101	9.91 857
L Cos	d	L Ctn	c d	L Tan	L Sin | d | '

Prop. Pts.

	28	27
2	5.6	5.4
3	8.4	8.1
4	11.2	10.8
5	14.0	13.5
6	16.8	16.2
7	19.6	18.9
8	22.4	21.6
9	25.2	24.3

	20	19
2	4.0	3.8
3	6.0	5.7
4	8.0	7.6
5	10.0	9.5
6	12.0	11.4
7	14.0	13.3
8	16.0	15.2
9	18.0	17.1

	18	9
2	3.6	1.8
3	5.4	2.7
4	7.2	3.6
5	9.0	4.5
6	10.8	5.4
7	12.6	6.3
8	14.4	7.2
9	16.2	8.1

1.6
2.4
3.2
4.0
4.8
5.6
6.4
7.2

From the top:
For **33°+** or **213°+**, read as printed; for **123°+** or **303°+**, read co-function.

From the bottom:
For **56°+** or **236°+** read as printed; for **146°+** or **326°+**, read co-function.

Prop. Pts.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.74 756	19	9.82 899	27	10.17 101	9.91 857	8	60
1	9.74 775	19	9.82 926	27	10.17 074	9.91 849	9	59
2	9.74 794	18	9.82 953	27	10.17 047	9.91 840	8	58
3	9.74 812	19	9.82 980	28	10.17 020	9.91 832	9	57
4	9.74 831	19	9.83 008	27	10.16 992	9.91 823	8	56
5	9.74 850	18	9.83 035	27	10.16 965	9.91 815	9	55
6	9.74 868	19	9.83 062	27	10.16 938	9.91 806	8	54
7	9.74 887	19	9.83 089	28	10.16 911	9.91 798	9	53
8	9.74 906	18	9.83 117	27	10.16 883	9.91 789	8	52
9	9.74 924	19	9.83 144	27	10.16 856	9.91 781	9	51
10	9.74 943	18	9.83 171	27	10.16 829	9.91 772	9	50
11	9.74 961	19	9.83 198	27	10.16 802	9.91 763	8	49
12	9.74 980	19	9.83 225	27	10.16 775	9.91 755	9	48
13	9.74 999	18	9.83 252	28	10.16 748	9.91 746	8	47
14	9.75 017	19	9.83 280	27	10.16 720	9.91 738	9	46
15	9.75 036	18	9.83 307	27	10.16 693	9.91 729	9	45
16	9.75 054	19	9.83 334	27	10.16 666	9.91 720	8	44
17	9.75 073	18	9.83 361	27	10.16 639	9.91 712	9	43
18	9.75 091	19	9.83 388	27	10.16 612	9.91 703	8	42
19	9.75 110	18	9.83 415	27	10.16 585	9.91 695	9	41
20	9.75 128	19	9.83 442	28	10.16 558	9.91 686	9	40
21	9.75 147	18	9.83 470	27	10.16 530	9.91 677	8	39
22	9.75 165	19	9.83 497	27	10.16 503	9.91 669	9	38
23	9.75 184	18	9.83 524	27	10.16 476	9.91 660	9	37
24	9.75 202	19	9.83 551	27	10.16 449	9.91 651	8	36
25	9.75 221	18	9.83 578	27	10.16 422	9.91 643	9	35
26	9.75 239	19	9.83 605	27	10.16 395	9.91 634	9	34
27	9.75 258	18	9.83 632	27	10.16 368	9.91 625	8	33
28	9.75 276	18	9.83 659	27	10.16 341	9.91 617	9	32
29	9.75 294	19	9.83 686	27	10.16 314	9.91 608	9	31
30	9.75 313	18	9.83 713	27	10.16 287	9.91 599	8	30
31	9.75 331	19	9.83 740	28	10.16 260	9.91 591	9	29
32	9.75 350	18	9.83 768	27	10.16 232	9.91 582	9	28
33	9.75 368	18	9.83 795	27	10.16 205	9.91 573	8	27
34	9.75 386	19	9.83 822	27	10.16 178	9.91 565	9	26
35	9.75 405	18	9.83 849	27	10.16 151	9.91 556	9	25
36	9.75 423	18	9.83 876	27	10.16 124	9.91 547	9	24
37	9.75 441	18	9.83 903	27	10.16 097	9.91 538	8	23
38	9.75 459	19	9.83 930	27	10.16 070	9.91 530	9	22
39	9.75 478	18	9.83 957	27	10.16 043	9.91 521	9	21
40	9.75 496	18	9.83 984	27	10.16 016	9.91 512	8	20
41	9.75 514	19	9.84 011	27	10.15 989	9.91 504	9	19
42	9.75 533	18	9.84 038	27	10.15 962	9.91 495	9	18
43	9.75 551	18	9.84 065	27	10.15 935	9.91 486	9	17
44	9.75 569	18	9.84 092	27	10.15 908	9.91 477	8	16
45	9.75 587	18	9.84 119	27	10.15 881	9.91 469	9	15
46	9.75 605	19	9.84 146	27	10.15 854	9.91 460	9	14
47	9.75 624	18	9.84 173	27	10.15 827	9.91 451	9	13
48	9.75 642	18	9.84 200	27	10.15 800	9.91 442	9	12
49	9.75 660	18	9.84 227	27	10.15 773	9.91 433	8	11
50	9.75 678	18	9.84 254	26	10.15 746	9.91 425	9	10
51	9.75 696	18	9.84 280	27	10.15 720	9.91 416	9	9
52	9.75 714	19	9.84 307	27	10.15 693	9.91 407	9	8
53	9.75 733	18	9.84 334	27	10.15 666	9.91 398	9	7
54	9.75 751	18	9.84 361	27	10.15 639	9.91 389	8	6
55	9.75 769	18	9.84 388	27	10.15 612	9.91 381	9	5
56	9.75 787	18	9.84 415	27	10.15 585	9.91 372	9	4
57	9.75 805	18	9.84 442	27	10.15 558	9.91 363	9	3
58	9.75 823	18	9.84 469	27	10.15 531	9.91 354	9	2
59	9.75 841	18	9.84 496	27	10.15 504	9.91 345	9	1
60	9.75 859		9.84 523		10.15 477	9.91 336		0
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	′

Prop. Pts.

	28	27
2	5.6	5.4
3	8.4	8.1
4	11.2	10.8
5	14.0	13.5
6	16.8	16.2
7	19.6	18.9
8	22.4	21.6
9	25.2	24.3

	26	19
2	5.2	3.8
3	7.8	5.7
4	10.4	7.6
5	13.0	9.5
6	15.6	11.4
7	18.2	13.3
8	20.8	15.2
9	23.4	17.1

	18	9
2	3.6	1.8
3	5.4	2.7
4	7.2	3.6
5	9.0	4.5
6	10.8	5.4
7	12.6	6.3
8	14.4	7.2
9	16.2	8.1

	8
2	1.6
3	2.4
4	3.2
5	4.0
6	4.8
7	5.6
8	6.4
9	7.2

From the top:

For **34°+** or **214°+**, read as printed; for **124°+** or **304°+**, read co-function.

From the bottom:

For **55°+** or **235°+**, read as printed; for **145°+** or **325°+**, read co-function.

Prop. Pts.

L Sin	d	L Tan	cd	L Ctn	L Cos
9.75 859		9.84 523		10.15 477	9.91 336
9.75 877		9.84 550		10.15 450	9.91 328
9.75 895		9.84 576		10.15 424	9.91 319
9.75 913		9.84 603		10.15 397	9.91 310
9.75 931		9.84 630		10.15 370	9.91 301
9.75 949		9.84 657		10.15 343	9.91 292
9.75 967		9.84 684		10.15 316	9.91 283
9.75 985		9.84 711		10.15 289	9.91 274
9.76 003		9.84 738		10.15 262	9.91 266
9.76 021		9.84 764		10.15 236	9.91 257
9.76 039		9.84 791		10.15 209	9.91 248
9.76 057		9.84 818		10.15 182	9.91 239
9.76 075		9.84 845		10.15 155	9.91 230
9.76 093		9.84 872		10.15 128	9.91 221
9.76 111		9.84 899		10.15 101	9.91 212
9.76 129		9.84 925		10.15 075	9.91 203
9.76 146		9.84 952		10.15 048	9.91 194
9.76 164		9.84 979		10.15 021	9.91 185
9.76 182		9.85 006		10.14 994	9.91 176
9.76 200		9.85 033		10.14 967	9.91 167
9.76 218		9.85 059		10.14 941	9.91 158
9.76 236		9.85 086		10.14 914	9.91 149
9.76 253		9.85 113		10.14 887	9.91 141
9.76 271		9.85 140		10.14 860	9.91 132
9.76 289		9.85 166		10.14 834	9.91 123
9.76 307		9.85 193		10.14 807	9.91 114
9.76 324		9.85 220		10.14 780	9.91 105
9.76 342		9.85 247		10.14 753	9.91 096
9.76 360		9.85 273		10.14 727	9.91 087
9.76 378		9.85 300		10.14 700	9.91 078
9.76 395		9.85 327		10.14 673	9.91 069
9.76 413		9.85 354		10.14 646	9.91 060
9.76 431		9.85 380		10.14 620	9.91 051
9.76 448		9.85 407		10.14 593	9.91 042
9.76 466		9.85 434		10.14 566	9.91 033
9.76 484		9.85 460		10.14 540	9.91 023
9.76 501		9.85 487		10.14 513	9.91 014
9.76 519		9.85 514		10.14 486	9.91 005
9.76 537		9.85 540		10.14 460	9.90 996
9.76 554		9.85 567		10.14 433	9.90 987
9.76 572		9.85 594		10.14 406	9.90 978
9.76 590		9.85 620		10.14 380	9.90 969
9.76 607		9.85 647		10.14 353	9.90 960
9.76 625		9.85 674		10.14 326	9.90 951
9.76 642		9.85 700		10.14 300	9.90 942
9.76 660		9.85 727		10.14 273	9.90 933
9.76 677		9.85 754		10.14 246	9.90 924
9.76 695		9.85 780		10.14 220	9.90 915
9.76 712		9.85 807		10.14 193	9.90 906
9.76 730		9.85 834		10.14 166	9.90 896
9.76 747		9.85 860		10.14 140	9.90 887
9.76 765		9.85 887		10.14 113	9.90 878
9.76 782		9.85 913		10.14 087	9.90 869
9.76 800		9.85 940		10.14 060	9.90 860
9.76 817		9.85 967		10.14 033	9.90 851
9.76 835		9.85 993		10.14 007	9.90 842
9.76 852		9.86 020		10.13 980	9.90 832
9.76 870		9.86 046		10.13 954	9.90 823
9.76 887		9.86 073		10.13 927	9.90 814
9.76 904		9.86 100		10.13 900	9.90 805
9.76 922		9.86 126		10.13 874	9.90 796
L Cos	d	L Ctn	cd	L Tan	L Sin

Prop. Pts.

	27	26
2	5.4	5.2
3	8.1	7.8
4	10.8	10.4
5	13.5	13.0
6	16.2	15.6
7	18.9	18.2
8	21.6	20.8
9	24.3	23.4

	18	17
2	3.6	3.4
3	5.4	5.1
4	7.2	6.8
5	9.0	8.5
6	10.8	10.2
7	12.6	11.9
8	14.4	13.6
9	16.2	15.3

	10	9	8
2	2.0	1.8	1.6
3	3.0	2.7	2.4
4	4.0	3.6	3.2
5	5.0	4.5	4.0
6	6.0	5.4	4.8
7	7.0	6.3	5.6
8	8.0	7.2	6.4
9	9.0	8.1	7.2

From the top:

For **35°+** or **215°+**, read as printed; for **125°+** or **305°+**, read co-function.

From the bottom:

For **54°+** or **234°+**, read as printed; for **144°+** or **324°+**, read co-function.

Prop. Pts.

	L Sin	d	L Tan	cd	L Ctn	L Cos	
	9.76 922		9.86 126		10.13 874	9.90 796	
	9.76 939		9.86 153		10.13 847	9.90 787	
	9.76 957		9.86 179		10.13 821	9.90 777	
	9.76 974		9.86 206		10.13 794	9.90 768	
	9.76 991		9.86 232		10.13 768	9.90 759	
5	9.77 009		9.86 259		10.13 741	9.90 750	
6	9.77 026		9.86 285		10.13 715	9.90 741	
7	9.77 043		9.86 312		10.13 688	9.90 731	10
8	9.77 061		9.86 338		10.13 662	9.90 722	
9	9.77 078		9.86 365		10.13 635	9.90 713	
10	9.77 095		9.86 392		10.13 608	9.90 704	
11	9.77 112		9.86 418		10.13 582	9.90 694	10
12	9.77 130		9.86 445		10.13 555	9.90 685	
13	9.77 147		9.86 471		10.13 529	9.90 676	
14	9.77 164		9.86 498		10.13 502	9.90 667	10
15	9.77 181		9.86 524		10.13 476	9.90 657	
16	9.77 199		9.86 551		10.13 449	9.90 648	
17	9.77 216		9.86 577		10.13 423	9.90 639	
18	9.77 233		9.86 603		10.13 397	9.90 630	
19	9.77 250		9.86 630		10.13 370	9.90 620	
20	9.77 268		9.86 656		10.13 344	9.90 611	
21	9.77 285		9.86 683		10.13 317	9.90 602	
22	9.77 302		9.86 709		10.13 291	9.90 592	
23	9.77 319		9.86 736		10.13 264	9.90 583	
24	9.77 336		9.86 762		10.13 238	9.90 574	
25	9.77 353		9.86 789		10.13 211	9.90 565	
26	9.77 370		9.86 815		10.13 185	9.90 555	
27	9.77 387		9.86 842		10.13 158	9.90 546	
28	9.77 405		9.86 868		10.13 132	9.90 537	
29	9.77 422		9.86 894		10.13 106	9.90 527	
30	9.77 439		9.86 921		10.13 079	9.90 518	
31	9.77 456		9.86 947		10.13 053	9.90 509	
32	9.77 473		9.86 974		10.13 026	9.90 499	10
33	9.77 490		9.87 000		10.13 000	9.90 490	
34	9.77 507		9.87 027		10.12 973	9.90 480	
35	9.77 524		9.87 053		10.12 947	9.90 471	
36	9.77 541		9.87 079		10.12 921	9.90 462	
37	9.77 558		9.87 106		10.12 894	9.90 452	
38	9.77 575		9.87 132		10.12 868	9.90 443	
39	9.77 592		9.87 158		10.12 842	9.90 434	
40	9.77 609		9.87 185		10.12 815	9.90 424	
41	9.77 626		9.87 211		10.12 789	9.90 415	
	9.77 643		9.87 238		10.12 762	9.90 405	
	9.77 660		9.87 264		10.12 736	9.90 396	
44	9.77 677		9.87 290		10.12 710	9.90 386	
45	9.77 694		9.87 317		10.12 683	9.90 377	
46	9.77 711		9.87 343		10.12 657	9.90 368	
47	9.77 728		9.87 369		10.12 631	9.90 358	
48	9.77 744		9.87 396		10.12 604	9.90 349	
49	9.77 761		9.87 422		10.12 578	9.90 339	
50	9.77 778		9.87 448		10.12 552	9.90 330	
51	9.77 795		9.87 475		10.12 525	9.90 320	
52	9.77 812		9.87 501		10.12 499	9.90 311	
53	9.77 829		9.87 527		10.12 473	9.90 301	
54	9.77 846		9.87 554		10.12 446	9.90 292	
55	9.77 862		9.87 580		10.12 420	9.90 282	
56	9.77 879		9.87 606		10.12 394	9.90 273	
57	9.77 896		9.87 633		10.12 367	9.90 263	
58	9.77 913		9.87 659		10.12 341	9.90 254	
59	9.77 930		9.87 685		10.12 315	9.90 244	
60	9.77 946		9.87 711		10.12 289	9.90 235	
	L Cos	d	L Ctn	cd	L Tan	L Sin	d

Prop. Pts.

	27	26
2	5.4	5.2
3	8.1	7.8
4	10.8	10.4
5	13.5	13.0
6	16.2	15.6
7	18.9	18.2
8	21.6	20.8
9	24.3	23.4

	18	17
2	3.6	3.4
3	5.4	5.1
4	7.2	6.8
5	9.0	8.5
6	10.8	10.2
7	12.6	11.9
8	14.4	13.6
9	16.2	15.3

	16	10
2	3.2	2.0
3	4.8	3.0
4	6.4	4.0
5	8.0	5.0
6	9.6	6.0
7	11.2	7.0
8	12.8	8.0
9	14.4	9.0

1.8
2.7
3.6
4.5
5.4
6.3
7.2
8.1

From the top:

For **36°+** or **216°+**, read as printed; for **126°+** or **306°+**, read co-function.

From the bottom:

For **53°+** or **233°+**, read as printed; for **143°+** or **323°+**, read co-function.

Prop. Pts.

L Sin		cd	L Ctn	L Cos	d
9.77 946	9.87 711		10.12 289	9.90 235	
9.77 963	9.87 738		10.12 262	9.90 225	10
9.77 980	9.87 764		10.12 236	9.90 216	
9.77 997	9.87 790		10.12 210	9.90 206	
9.78 013	9.87 817		10.12 183	9.90 197	
9.78 030	9.87 843		10.12 157	9.90 187	
9.78 047	9.87 869		10.12 131	9.90 178	
9.78 063	9.87 895		10.12 105	9.90 168	
9.78 080	9.87 922		10.12 078	9.90 159	
9.78 097	9.87 948		10.12 052	9.90 149	
9.78 113	9.87 974		10.12 026	9.90 139	
9.78 130	9.88 000		10.12 000	9.90 130	
9.78 147	9.88 027		10.11 973	9.90 120	
9.78 163	9.88 053		10.11 947	9.90 111	
9.78 180	9.88 079		10.11 921	9.90 101	
9.78 197	9.88 105		10.11 895	9.90 091	
9.78 213	9.88 131		10.11 869	9.90 082	
9.78 230	9.88 158		10.11 842	9.90 072	
9.78 246	9.88 184		10.11 816	9.90 063	
9.78 263	9.88 210		10.11 790	9.90 053	
9.78 280	9.88 236		10.11 764	9.90 043	
9.78 296	9.88 262		10.11 738	9.90 034	
9.78 313	9.88 289		10.11 711	9.90 024	
9.78 329	9.88 315		10.11 685	9.90 014	
9.78 346	9.88 341		10.11 659	9.90 005	
9.78 362	9.88 367		10.11 633	9.89 995	
9.78 379	9.88 393		10.11 607	9.89 985	
9.78 395	9.88 420		10.11 580	9.89 976	
9.78 412	9.88 446		10.11 554	9.89 966	
9.78 428	9.88 472		10.11 528	9.89 956	
9.78 445	9.88 498		10.11 502	9.89 947	
9.78 461	9.88 524		10.11 476	9.89 937	
9.78 478	9.88 550		10.11 450	9.89 927	
9.78 494	9.88 577		10.11 423	9.89 918	
9.78 510	9.88 603		10.11 397	9.89 908	
9.78 527	9.88 629		10.11 371	9.89 898	
9.78 543	9.88 655		10.11 345	9.89 888	
9.78 560	9.88 681		10.11 319	9.89 879	
9.78 576	9.88 707		10.11 293	9.89 869	
9.78 592	9.88 733		10.11 267	9.89 859	
9.78 609	9.88 759		10.11 241	9.89 849	
9.78 625	9.88 786		10.11 214	9.89 840	
9.78 642	9.88 812		10.11 188	9.89 830	
9.78 658	9.88 838		10.11 162	9.89 820	
9.78 674	9.88 864		10.11 136	9.89 810	
9.78 691	9.88 890		10.11 110	9.89 801	
9.78 707	9.88 916		10.11 084	9.89 791	
9.78 723	9.88 942		10.11 058	9.89 781	
9.78 739	9.88 968		10.11 032	9.89 771	
9.78 756	9.88 994		10.11 006	9.89 761	
9.78 772	9.89 020		10.10 980	9.89 752	
9.78 788	9.89 046		10.10 954	9.89 742	
9.78 805	9.89 073		10.10 927	9.89 732	
9.78 821	9.89 099		10.10 901	9.89 722	
9.78 837	9.89 125		10.10 875	9.89 712	
9.78 853	9.89 151		10.10 849	9.89 702	
9.78 869	9.89 177		10.10 823	9.89 693	
9.78 886	9.89 203		10.10 797	9.89 683	
9.78 902	9.89 229		10.10 771	9.89 673	
9.78 918	9.89 255		10.10 745	9.89 663	
9.78 934	9.89 281		10.10 719	9.89 653	
L Cos	L Ctn	cd	L Tan	L Sin	d

Prop. Pts.

	27	26
2	5.4	5.2
3	8.1	7.8
4	10.8	10.4
5	13.5	13.0
6	16.2	15.6
7	18.9	18.2
8	21.6	20.8
9	24.3	23.4

	17	16
2	3.4	3.2
3	5.1	4.8
4	6.8	6.4
5	8.5	8.0
6	10.2	9.6
7	11.9	11.2
8	13.6	12.8
9	15.3	14.4

9
1.8
2.7
3.6
4.5
5.4
6.3
7.2
8.1

From the top:

For **37°+** or **217°+**, read as printed; for **127°+** or **307°+**, read co-function.

From the bottom:

For **52°+** or **232°+**, read as printed; for **142°+** or **322°+**, read co-function.

Prop. Pts.

	L Sin	d					
	9.78 934		9.89 281	26	10.10 719	9.89 653	10
			9.89 307	26	10.10 693	9.89 643	10
			9.89 333	26	10.10 667	9.89 633	
			9.89 359	26	10.10 641	9.89 624	10
			9.89 385	26	10.10 615	9.89 614	10
			9.89 411	26	10.10 589	9.89 604	10
			9.89 437	26	10.10 563	9.89 594	10
			9.89 463	26	10.10 537	9.89 584	10
			9.89 489	26	10.10 511	9.89 574	10
			9.89 515	26	10.10 485	9.89 564	10
			9.89 541	26	10.10 459	9.89 554	10
			9.89 567	26	10.10 433	9.89 544	10
			9.89 593	26	10.10 407	9.89 534	10
			9.89 619	26	10.10 381	9.89 524	10
			9.89 645	26	10.10 355	9.89 514	10
			9.89 671	26	10.10 329	9.89 504	
			9.89 697	26	10.10 303	9.89 495	10
			9.89 723	26	10.10 277	9.89 485	10
			9.89 749	26	10.10 251	9.89 475	10
			9.89 775	26	10.10 225	9.89 465	10
20			9.89 801	26	10.10 199	9.89 455	10
			9.89 827	26	10.10 173	9.89 445	10
			9.89 853	26	10.10 147	9.89 435	10
			9.89 879	26	10.10 121	9.89 425	10
			9.89 905	26	10.10 095	9.89 415	10
25			9.89 931	26	10.10 069	9.89 405	10
			9.89 957	26	10.10 043	9.89 395	10
			9.89 983	26	10.10 017	9.89 385	10
			9.90 009	26	10.09 991	9.89 375	11
29			9.90 035	26	10.09 965	9.89 364	10
30			9.90 061	25	10.09 939	9.89 354	10
31			9.90 086	26	10.09 914	9.89 344	10
32			9.90 112	26	10.09 888	9.89 334	10
33			9.90 138	26	10.09 862	9.89 324	10
34			9.90 164	26	10.09 836	9.89 314	10
35			9.90 190	26	10.09 810	9.89 304	10
			9.90 216	26	10.09 784	9.89 294	10
			9.90 242	26	10.09 758	9.89 284	10
			9.90 268	26	10.09 732	9.89 274	10
			9.90 294	26	10.09 706	9.89 264	10
			9.90 320	26	10.09 680	9.89 254	10
			9.90 346	25	10.09 654	9.89 244	11
42			9.90 371	26	10.09 629	9.89 233	10
			9.90 397	26	10.09 603	9.89 223	10
			9.90 423	26	10.09 577	9.89 213	10
			9.90 449	26	10.09 551	9.89 203	10
			9.90 475	26	10.09 525	9.89 193	
			9.90 501	26	10.09 499	9.89 183	
			9.90 527	26	10.09 473	9.89 173	
49			9.90 553	25	10.09 447	9.89 162	
			9.90 578	26	10.09 422	9.89 152	
			9.90 604	26	10.09 396	9.89 142	
			9.90 630	26	10.09 370	9.89 132	
53			9.90 656	26	10.09 344	9.89 122	
			9.90 682	26	10.09 318	9.89 112	
						9.89 101	
56						9.89 091	
57						9.89 081	
						9.89 071	
						9.89 060	
						9.89 050	
					L Tan	L Sin	

Prop. Pts.

26	25
5.2	5.0
7.8	7.5
10.4	10.0
13.0	12.5
15.6	15.0
18.2	17.5
20.8	20.0
23.4	22.5

17	16
3.4	3.2
5.1	4.8
6.8	6.4
8.5	8.0
10.2	9.6
11.9	11.2
13.6	12.8
15.3	14.4

15	11
3.0	2.2
4.5	3.3
6.0	4.4
7.5	5.5
9.0	6.6
10.5	7.7
12.0	8.8
13.5	9.9

10	
2.0	1.8
3.0	2.7
4.0	3.6
5.0	4.5
6.0	5.4
7.0	6.3
8.0	7.2
9.0	

From the top:

For **38°+** or **218°+**, read as printed; for **128°+** or **308°+**, read co-function.

From the bottom:

For **51°+** or **231°+**, read as printed; for **141°+** or **321°+**, read co-function.

L Sin	d	L Tan	cd	L Ctn	L Cos	d	
9.79 887		9.90 837		10.09 163	9.89 050		**60**
9.79 903		9.90 863		10.09 137	9.89 040		59
9.79 918		9.90 889		10.09 111	9.89 030		58
9.79 934		9.90 914		10.09 086	9.89 020		57
9.79 950		9.90 940		10.09 060	9.89 009		56
9.79 965		9.90 966		10.09 034	9.88 999		**55**
9.79 981		9.90 992		10.09 008			54
9.79 996		9.91 018		10.08 982	9.88 978		53
9.80 012		9.91 043		10.08 957	9.88 968		52
9.80 027		9.91 069		10.08 931	9.88 958		51
9.80 043		9.91 095		10.08 905	9.88 948		**50**
9.80 058		9.91 121		10.08 879	9.88 937		49
9.80 074		9.91 147		10.08 853	9.88 927		48
9.80 089		9.91 172		10.08 828	9.88 917		47
9.80 105		9.91 198		10.08 802	9.88 906		46
9.80 120		9.91 224		10.08 776	9.88 896		**45**
9.80 136		9.91 250		10.08 750	9.88 886		44
9.80 151		9.91 276		10.08 724	9.88 875		43
9.80 166		9.91 301		10.08 699	9.88 865		42
9.80 182		9.91 327		10.08 673	9.88 855		41
9.80 197		9.91 353		10.08 647	9.88 844		**40**
9.80 213		9.91 379		10.08 621	9.88 834		39
9.80 228		9.91 404		10.08 596	9.88 824		38
9.80 244		9.91 430		10.08 570	9.88 813		37
9.80 259		9.91 456		10.08 544	9.88 803		36
9.80 274		9.91 482		10.08 518	9.88 793		**35**
9.80 290		9.91 507		10.08 493	9.88 782		34
9.80 305		9.91 533		10.08 467	9.88 772		33
9.80 320		9.91 559		10.08 441	9.88 761		32
9.80 336		9.91 585		10.08 415	9.88 751		31
9.80 351		9.91 610		10.08 390	9.88 741		**30**
9.80 366		9.91 636		10.08 364	9.88 730		29
9.80 382		9.91 662		10.08 338	9.88 720		28
9.80 397		9.91 688		10.08 312	9.88 709		27
9.80 412		9.91 713		10.08 287	9.88 699		26
9.80 428		9.91 739		10.08 261	9.88 688		**25**
9.80 443		9.91 765		10.08 235	9.88 678		24
9.80 458		9.91 791		10.08 209	9.88 668		23
9.80 473		9.91 816		10.08 184	9.88 657		22
9.80 489		9.91 842		10.08 158	9.88 647		21
9.80 504		9.91 868		10.08 132	9.88 636		**20**
9.80 519		9.91 893		10.08 107	9.88 626		19
9.80 534		9.91 919		10.08 081	9.88 615		18
9.80 550		9.91 945		10.08 055	9.88 605		17
9.80 565		9.91 971		10.08 029	9.88 594		16
9.80 580		9.91 996		10.08 004	9.88 584		**15**
9.80 595		9.92 022		10.07 978	9.88 573		14
9.80 610		9.92 048		10.07 952	9.88 563		13
9.80 625		9.92 073		10.07 927	9.88 552		12
9.80 641		9.92 099		10.07 901	9.88 542		11
9.80 656		9.92 125		10.07 875	9.88 531		**10**
9.80 671		9.92 150		10.07 850	9.88 521		9
9.80 686		9.92 176		10.07 824	9.88 510		8
9.80 701		9.92 202		10.07 798	9.88 499		7
9.80 716		9.92 227		10.07 773	9.88 489		6
9.80 731		9.92 253		10.07 747	9.88 478		**5**
9.80 746		9.92 279		10.07 721	9.88 468		4
9.80 762		9.92 304		10.07 696	9.88 457		3
9.80 777		9.92 330		10.07 670	9.88 447		2
9.80 792		9.92 356		10.07 644	9.88 436		1
9.80 807		9.92 381		10.07 619	9.88 425		**0**
L Cos		**L Ctn**		**L Tan**	**L Sin**		

Prop. Pts.

26	
5.2	5.0
7.8	7.5
10.4	10.0
13.0	12.5
15.6	15.0
18.2	17.5
20.8	20.0
23.4	22.5

	16	**15**
2	3.2	3.0
3	4.8	4.5
4	6.4	6.0
5	8.0	7.5
6	9.6	9.0
7	11.2	10.5
8	12.8	12.0
9	14.4	13.5

	11	**10**
2	2.2	2.0
3	3.3	3.0
4	4.4	4.0
5	5.5	5.0
6	6.6	6.0
7	7.7	7.0
8	8.8	8.0
9	9.9	9.0

From the top:

For **39°+** or **219°+**, read as printed; for **129°+** or **309°+**, read co-function.

From the bottom:

For **50°+** or **230°+**, read as printed; for **140°+** or **320°+**, read co-function.

Prop. Pts.

50° — Logarithms of Trigonometric Functions

	L Sin							
	9.80 807		9.92 381		10.07 619	9.88 425		
	9.80 822		9.92 407		10.07 593	9.88 415		
	9.80 837		9.92 433		10.07 567	9.88 404		
	9.80 852	15	9.92 458		10.07 542	9.88 394		
	9.80 867		9.92 484		10.07 516	9.88 383		
	9.80 882		9.92 510		10.07 490	9.88 372		
	9.80 897		9.92 535		10.07 465	9.88 362		
	9.80 912	15	9.92 561		10.07 439	9.88 351		
	9.80 927	15	9.92 587		10.07 413	9.88 340		
	9.80 942	15	9.92 612		10.07 388	9.88 330		51
	9.80 957	15	9.92 638		10.07 362	9.88 319		
	9.80 972	15	9.92 663		10.07 337	9.88 308		
	9.80 987	15	9.92 689		10.07 311	9.88 298		48
	9.81 002	15	9.92 715		10.07 285	9.88 287		
	9.81 017	15	9.92 740		10.07 260	9.88 276		
	9.81 032	15	9.92 766		10.07 234	9.88 266	11	45
	9.81 047	14	9.92 792		10.07 208	9.88 255	11	44
	9.81 061	15	9.92 817		10.07 183	9.88 244	10	43
	9.81 076	15	9.92 843		10.07 157	9.88 234	11	42
	9.81 091	15	9.92 868		10.07 132	9.88 223	11	41
	9.81 106	15	9.92 894		10.07 106	9.88 212	11	40
	9.81 121	15	9.92 920		10.07 080	9.88 201	10	39
	9.81 136	15	9.92 945		10.07 055	9.88 191	11	38
	9.81 151	15	9.92 971		10.07 029	9.88 180	11	37
	9.81 166	14	9.92 996		10.07 004	9.88 169	11	36
	9.81 180	15	9.93 022		10.06 978	9.88 158	10	35
	9.81 195	15	9.93 048		10.06 952	9.88 148	11	34
	9.81 210	15	9.93 073		10.06 927	9.88 137	11	33
	9.81 225	15	9.93 099		10.06 901	9.88 126	11	32
	9.81 240	14	9.93 124		10.06 876	9.88 115	10	31
	9.81 254	15	9.93 150		10.06 850	9.88 105	11	30
	9.81 269	15	9.93 175		10.06 825	9.88 094	11	29
	9.81 284	15	9.93 201		10.06 799	9.88 083	11	28
	9.81 299	15	9.93 227		10.06 773	9.88 072	11	27
	9.81 314	14	9.93 252		10.06 748	9.88 061	10	26
	9.81 328	15	9.93 278		10.06 722	9.88 051	11	25
	9.81 343	15	9.93 303		10.06 697	9.88 040	11	24
	9.81 358	14	9.93 329		10.06 671	9.88 029	11	23
	9.81 372	15	9.93 354		10.06 646	9.88 018	11	22
	9.81 387	15	9.93 380	26	10.06 620	9.88 007	11	21
10	9.81 402	15	9.93 406	25	10.06 594	9.87 996	11	20
	9.81 417	14	9.93 431	26	10.06 569	9.87 985	10	19
	9.81 431	15	9.93 457	25	10.06 543	9.87 975	11	18
	.81 446	15	9.93 482	26	10.06 518	9.87 964	11	17
	.81 461	14	9.93 508	25	10.06 492	9.87 953	11	16
	.1 475	15	9.93 533	26	10.06 467	9.87 942	11	15
	81 490	15	9.93 559	25	10.06 441	9.87 931	11	14
	81 505	14	9.93 584	26	10.06 416	9.87 920	11	13
	.81 519	15	9.93 610	26	10.06 390	9.87 909	11	12
	.81 534	15	9.93 636	25	10.06 364	9.87 898	11	11
	9.81 549	14	9.93 661	26	10.06 339	9.87 887	10	10
	9.81 563	15	9.93 687	25	10.06 313	9.87 877	11	9
	9.81 578		9.93 712	26	10.06 288	9.87 866	11	8
53	9.81 592		9.93 738	25	10.06 262	9.87 855	11	7
54	9.81 607		9.93 763	26	10.06 237	9.87 844	11	6
55	9.81 622		9.93 789	25	10.06 211	9.87 833	11	5
56	9.81 636		9.93 814	26	10.06 186	9.87 822	11	4
57	9.81 651		9.93 840	25	10.06 160	9.87 811	11	3
58	9.81 665		9.93 865	26	10.06 135	9.87 800	11	2
59	9.81 680		9.93 891	25	10.06 109	9.87 789	11	1
60	9.81 694		9.93 916		10.06 084	9.87 778		
	L Cos	d	L Ctn	cd	L Tan	L Sin		

Prop. Pts.

26	25
5.2	5.0
7.8	7.5
10.4	10.0
13.0	12.5
15.6	15.0
18.2	17.5
20.8	20.0
23.4	22.5

15	14
3.0	2.8
4.5	4.2
6.0	5.6
7.5	7.0
9.0	8.4
10.5	9.8
12.0	11.2
13.5	12.6

11	10
2.2	2.0
3.3	3.0
4.4	4.0
5.5	5.0
6.6	6.0
7.7	7.0
8.8	8.0
9.9	9.0

From the top:

For **40°+** or **220°+**, read as printed; for **130°+** or **310°+**, read co-function.

From the bottom:

For **49°+** or **229°+**, read as printed; for **139°+** or **319°+**, read co-function.

Prop. Pts

′	L Sin	d	L Tan	cd	L Ctn	L Cos	d	
0	9.81 694	15	9.93 916	26	10.06 084	9.87 778	11	60
1	9.81 709	14	9.93 942	25	10.06 058	9.87 767	11	59
2	9.81 723	15	9.93 967	26	10.06 033	9.87 756	11	58
3	9.81 738	14	9.93 993	25	10.06 007	9.87 745	11	57
4	9.81 752	15	9.94 018	26	10.05 982	9.87 734	11	56
5	9.81 767	14	9.94 044	25	10.05 956	9.87 723	11	55
6	9.81 781	15	9.94 069	26	10.05 931	9.87 712	11	54
7	9.81 796	14	9.94 095	25	10.05 905	9.87 701	11	53
8	9.81 810	15	9.94 120	26	10.05 880	9.87 690	11	52
9	9.81 825	14	9.94 146	25	10.05 854	9.87 679	11	51
10	9.81 839	15	9.94 171	26	10.05 829	9.87 668	11	50
11	9.81 854	14	9.94 197	25	10.05 803	9.87 657	11	49
12	9.81 868	14	9.94 222	26	10.05 778	9.87 646	11	48
13	9.81 882	15	9.94 248	25	10.05 752	9.87 635	11	47
14	9.81 897	14	9.94 273	26	10.05 727	9.87 624	11	46
15	9.81 911	15	9.94 299	25	10.05 701	9.87 613	12	45
16	9.81 926	14	9.94 324	26	10.05 676	9.87 601	11	44
17	9.81 940	15	9.94 350	25	10.05 650	9.87 590	11	43
18	9.81 955	14	9.94 375	26	10.05 625	9.87 579	11	42
19	9.81 969	14	9.94 401	25	10.05 599	9.87 568	11	41
20	9.81 983	15	9.94 426	26	10.05 574	9.87 557	11	40
21	9.81 998	14	9.94 452	25	10.05 548	9.87 546	11	39
22	9.82 012	14	9.94 477	26	10.05 523	9.87 535	11	38
23	9.82 026	15	9.94 503	25	10.05 497	9.87 524	11	37
24	9.82 041	14	9.94 528	26	10.05 472	9.87 513	12	36
25	9.82 055	14	9.94 554	25	10.05 446	9.87 501	11	35
26	9.82 069	15	9.94 579	25	10.05 421	9.87 490	11	34
27	9.82 084	14	9.94 604	26	10.05 396	9.87 479	11	33
28	9.82 098	14	9.94 630	25	10.05 370	9.87 468	11	32
29	9.82 112	14	9.94 655	26	10.05 345	9.87 457	11	31
30	9.82 126	15	9.94 681	25	10.05 319	9.87 446	12	30
31	9.82 141	14	9.94 706	26	10.05 294	9.87 434	11	29
32	9.82 155	14	9.94 732	25	10.05 268	9.87 423	11	28
33	9.82 169	15	9.94 757	26	10.05 243	9.87 412	11	27
34	9.82 184	14	9.94 783	25	10.05 217	9.87 401	11	26
35	9.82 198	14	9.94 808	26	10.05 192	9.87 390	12	25
36	9.82 212	14	9.94 834	25	10.05 166	9.87 378	11	24
37	9.82 226	14	9.94 859	25	10.05 141	9.87 367	11	23
38	9.82 240	15	9.94 884	26	10.05 116	9.87 356	11	22
39	9.82 255	14	9.94 910	25	10.05 090	9.87 345	11	21
40	9.82 269	14	9.94 935	26	10.05 065	9.87 334	12	20
41	9.82 283	14	9.94 961	25	10.05 039	9.87 322	11	19
42	9.82 297	14	9.94 986	26	10.05 014	9.87 311	11	18
43	9.82 311	15	9.95 012	25	10.04 988	9.87 300	12	17
44	9.82 326	14	9.95 037	25	10.04 963	9.87 288	11	16
45	9.82 340	14	9.95 062	26	10.04 938	9.87 277	11	15
46	9.82 354	14	9.95 088	25	10.04 912	9.87 266	11	14
47	9.82 368	14	9.95 113	26	10.04 887	9.87 255	12	13
48	9.82 382	14	9.95 139	25	10.04 861	9.87 243	11	12
49	9.82 396	14	9.95 164	26	10.04 836	9.87 232	11	11
50	9.82 410	14	9.95 190	25	10.04 810	9.87 221	12	10
51	9.82 424	15	9.95 215	25	10.04 785	9.87 209	11	9
52	9.82 439	14	9.95 240	26	10.04 760	9.87 198	11	8
53	9.82 453	14	9.95 266	25	10.04 734	9.87 187	12	7
54	9.82 467	14	9.95 291	26	10.04 709	9.87 175	11	6
55	9.82 481	14	9.95 317	25	10.04 683	9.87 164	11	5
56	9.82 495	14	9.95 342	26	10.04 658	9.87 153	12	4
57	9.82 509	14	9.95 368	25	10.04 632	9.87 141	11	3
58	9.82 523	14	9.95 393	25	10.04 607	9.87 130	11	2
59	9.82 537	14	9.95 418	26	10.04 582	9.87 119	12	1
60	9.82 551		9.95 444		10.04 556	9.87 107		0
	L Cos	d	L Ctn	cd	L Tan	L Sin	d	′

Prop. Pts.

	26	25
2	5.2	5.0
3	7.8	7.5
4	10.4	10.0
5	13.0	12.5
6	15.6	15.0
7	18.2	17.5
8	20.8	20.0
9	23.4	22.5

	15	14
2	3.0	2.8
3	4.5	4.2
4	6.0	5.6
5	7.5	7.0
6	9.0	8.4
7	10.5	9.8
8	12.0	11.2
9	13.5	12.6

	12	11
2	2.4	2.2
3	3.6	3.3
4	4.8	4.4
5	6.0	5.5
6	7.2	6.6
7	8.4	7.7
8	9.6	8.8
9	10.8	9.9

From the top: For **41°+** or **221°+**, read as printed; for **131°+** or **311°+**, read co-function.

From the bottom: For **48°+** or **228°+**, read as printed; for **138°+** or **318°+**, read co-function.

Prop. Pts.

48° — Logarithms of Trigonometric Functions

′	L Sin	d	L Tan	cd	L Ctn	L Cos	d	
0	9.82 551		9.95 444		10.04 556	9.87 107		60
1	9.82 565	14	9.95 469	25	10.04 531	9.87 096	11	59
2	9.82 579	14	9.95 495	26	10.04 505	9.87 085	11	58
3	9.82 593	14	9.95 520	25	10.04 480	9.87 073	12	57
4	9.82 607	14	9.95 545	25	10.04 455	9.87 062	11	56
5	9.82 621	14	9.95 571	26	10.04 429	9.87 050	12	55
6	9.82 635	14	9.95 596	25	10.04 404	9.87 039	11	54
7	9.82 649	14	9.95 622	26	10.04 378	9.87 028	11	53
8	9.82 663	14	9.95 647	25	10.04 353	9.87 016	12	52
9	9.82 677	14	9.95 672	25	10.04 328	9.87 005	11	51
10	9.82 691	14	9.95 698	26	10.04 302	9.86 993	12	50
11	9.82 705	14	9.95 723	25	10.04 277	9.86 982	11	49
12	9.82 719	14	9.95 748	25	10.04 252	9.86 970	12	48
13	9.82 733	14	9.95 774	26	10.04 226	9.86 959	11	47
14	9.82 747	14	9.95 799	25	10.04 201	9.86 947	12	46
15	9.82 761	14	9.95 825	26	10.04 175	9.86 936	11	45
16	9.82 775	14	9.95 850	25	10.04 150	9.86 924	12	44
17	9.82 788	13	9.95 875	25	10.04 125	9.86 913	11	43
18	9.82 802	14	9.95 901	26	10.04 099	9.86 902	11	42
19	9.82 816	14	9.95 926	25	10.04 074	9.86 890	12	41
20	9.82 830	14	9.95 952	26	10.04 048	9.86 879	11	40
21	9.82 844	14	9.95 977	25	10.04 023	9.86 867	12	39
22	9.82 858	14	9.96 002	25	10.03 998	9.86 855	12	38
23	9.82 872	14	9.96 028	26	10.03 972	9.86 844	11	37
24	9.82 885	13	9.96 053	25	10.03 947	9.86 832	12	36
25	9.82 899	14	9.96 078	25	10.03 922	9.86 821	11	35
26	9.82 913	14	9.96 104	26	10.03 896	9.86 809	12	34
27	9.82 927	14	9.96 129	25	10.03 871	9.86 798	11	33
28	9.82 941	14	9.96 155	26	10.03 845	9.86 786	12	32
29	9.82 955	14	9.96 180	25	10.03 820	9.86 775	11	31
30	9.82 968	13	9.96 205	25	10.03 795	9.86 763	12	30
31	9.82 982	14	9.96 231	26	10.03 769	9.86 752	11	29
32	9.82 996	14	9.96 256	25	10.03 744	9.86 740	12	28
33	9.83 010	14	9.96 281	25	10.03 719	9.86 728	12	27
34	9.83 023	13	9.96 307	26	10.03 693	9.86 717	11	26
35	9.83 037	14	9.96 332	25	10.03 668	9.86 705	12	25
36	9.83 051	14	9.96 357	25	10.03 643	9.86 694	11	24
37	9.83 065	14	9.96 383	26	10.03 617	9.86 682	12	23
38	9.83 078	13	9.96 408	25	10.03 592	9.86 670	12	22
39	9.83 092	14	9.96 433	25	10.03 567	9.86 659	11	21
40	9.83 106	14	9.96 459	26	10.03 541	9.86 647	12	20
41	9.83 120	14	9.96 484	25	10.03 516	9.86 635	12	19
42	9.83 133	13	9.96 510	26	10.03 490	9.86 624	11	18
43	9.83 147	14	9.96 535	25	10.03 465	9.86 612	12	17
44	9.83 161	14	9.96 560	25	10.03 440	9.86 600	12	16
45	9.83 174	13	9.96 586		10.03 414	9.86 589	11	15
46	9.83 188	14	9.96 611		10.03 389	9.86 577	12	14
47	9.83 202	14	9.96 636		10.03 364	9.86 565	12	13
48	9.83 215	13	9.96 662		10.03 338	9.86 554	11	12
49	9.83 229	14	9.96 687		10.03 313	9.86 542	12	11
50	9.83 242		9.96 712		10.03 288	9.86 530	12	10
51	9.83 256		9.96 738		10.03 262	9.86 518	12	9
52	9.83 270		9.96 763		10.03 237	9.86 507	11	8
53	9.83 283		9.96 788		10.03 212	9.86 495	12	7
54	9.83 297		9.96 814		10.03 186	9.86 483	12	6
55	9.83 310		9.96 839		10.03 161	9.86 472	11	5
56	9.83 324	14	9.96 864		10.03 136	9.86 460	12	4
57	9.83 338	14	9.96 890		10.03 110	9.86 448	12	3
58	9.83 351	13	9.96 915		10.03 085	9.86 436	12	2
59	9.83 365	14	9.96 940		10.03 060	9.86 425	11	1
60	9.83 378	13	9.96 966		10.03 034	9.86 413	12	0
	L Cos	d	L Ctn	cd	L Tan	L Sin		′

Prop. Pts.

	26	25
2	5.2	5.0
3	7.8	7.5
4	10.4	10.0
5	13.0	12.5
6	15.6	15.0
7	18.2	17.5
8	20.8	20.0
9	23.4	22.5

	14	13
2	2.8	2.6
3	4.2	3.9
4	5.6	5.2
5	7.0	6.5
6	8.4	7.8
7	9.8	9.1
8	11.2	10.4
9	12.6	11.7

	12	11
2	2.4	2.2
3	3.6	3.3
4	4.8	4.4
5	6.0	5.5
6	7.2	6.6
7	8.4	7.7
8	9.6	8.8
9	10.8	

From the top:

For **42°+** or **222°+**, read as printed; for **132°+** or **312°+**, read co-function.

From the bottom:

For **47°+** or **227°+**, read as printed; for **137°+** or **317°+**, read co-function.

. Pts

L Sin	d	L Tan	cd	L Ctn	L Cos	d
9.83 378		9.96 966		10.03 034	9.86 413	
9.83 392		9.96 991		10.03 009	9.86 401	
9.83 405		9.97 016		10.02 984	9.86 389	
9.83 419		9.97 042		10.02 958	9.86 377	
9.83 432		9.97 067		10.02 933	9.86 366	
9.83 446		9.97 092		10.02 908	9.86 354	
9.83 459		9.97 118		10.02 882	9.86 342	
9.83 473		9.97 143		10.02 857	9.86 330	
9.83 486		9.97 168		10.02 832	9.86 318	
9.83 500		9.97 193		10.02 807	9.86 306	
9.83 513		9.97 219		10.02 781	9.86 295	
9.83 527		9.97 244		10.02 756	9.86 283	
9.83 540		9.97 269		10.02 731	9.86 271	
9.83 554		9.97 295		10.02 705	9.86 259	
9.83 567		9.97 320		10.02 680	9.86 247	
9.83 581		9.97 345		10.02 655	9.86 235	
9.83 594		9.97 371		10.02 629	9.86 223	
9.83 608		9.97 396		10.02 604	9.86 211	
9.83 621		9.97 421		10.02 579	9.86 200	
9.83 634		9.97 447		10.02 553	9.86 188	
9.83 648		9.97 472		10.02 528	9.86 176	
9.83 661		9.97 497		10.02 503	9.86 164	
9.83 674		9.97 523		10.02 477	9.86 152	
9.83 688		9.97 548		10.02 452	9.86 140	
9.83 701		9.97 573		10.02 427	9.86 128	
9.83 715		9.97 598		10.02 402	9.86 116	
9.83 728		9.97 624		10.02 376	9.86 104	
9.83 741		9.97 649		10.02 351	9.86 092	
9.83 755		9.97 674		10.02 326	9.86 080	
9.83 768		9.97 700		10.02 300	9.86 068	
9.83 781		9.97 725		10.02 275	9.86 056	
9.83 795		9.97 750		10.02 250	9.86 044	
9.83 808		9.97 776		10.02 224	9.86 032	
9.83 821		9.97 801		10.02 199	9.86 020	
9.83 834		9.97 826		10.02 174	9.86 008	
9.83 848		9.97 851		10.02 149	9.85 996	
9.83 861		9.97 877		10.02 123	9.85 984	
9.83 874		9.97 902		10.02 098	9.85 972	
9.83 887		9.97 927		10.02 073	9.85 960	
9.83 901		9.97 953		10.02 047	9.85 948	
9.83 914		9.97 978		10.02 022	9.85 936	
9.83 927		9.98 003		10.01 997	9.85 924	
9.83 940		9.98 029		10.01 971	9.85 912	
9.83 954		9.98 054		10.01 946	9.85 900	
9.83 967		9.98 079		10.01 921	9.85 888	
9.83 980		9.98 104		10.01 896	9.85 876	
9.83 993		9.98 130		10.01 870	9.85 864	
9.84 006		9.98 155		10.01 845	9.85 851	
9.84 020		9.98 180		10.01 820	9.85 839	
9.84 033		9.98 206		10.01 794	9.85 827	
9.84 046		9.98 231		10.01 769	9.85 815	
9.84 059		9.98 256		10.01 744	9.85 803	
9.84 072		9.98 281		10.01 719	9.85 791	
9.84 085		9.98 307		10.01 693	9.85 779	
9.84 098		9.98 332		10.01 668	9.85 766	
9.84 112		9.98 357		10.01 643	9.85 754	
9.84 125		9.98 383		10.01 617	9.85 742	
9.84 138		9.98 408		10.01 592	9.85 730	
9.84 151		9.98 433		10.01 567	9.85 718	
9.84 164		9.98 458		10.01 542	9.85 706	
9.84 177		9.98 484		10.01 516	9.85 693	
L Cos		L Ctn		L Tan	L Sin	d

Prop. Pts.

	26	25
2	5.2	5.0
3	7.8	7.5
4	10.4	10.0
5	13.0	12.5
6	15.6	15.0
7	18.2	17.5
8	20.8	20.0
9	23.4	22.5

14	13
2.8	2.6
4.2	3.9
5.6	5.2
7.0	6.5
8.4	7.8
9.8	9.1
11.2	10.4
12.6	11.7

	12	11
2	2.4	2.2
3	3.6	3.3
4	4.8	4.4
5	6.0	5.5
6	7.2	6.6
7	8.4	7.7
8	9.6	8.8
9	10.8	9.9

From the top:

For **43°+** or **223°+**, read as printed; for **133°+** or **313°+**, read co-function.

From the bottom:

For **46°+** or **226°+**, read as printed; for **136°+** or **316°+**, read co-function.

Prop. Pts.

′	L Sin	d	L Tan	c d	L Ctn	L Cos	
	9.84 177	1	9.98 484	2	10.01 51	9.85 69	
	9.84 19	1	9.98 509	25	10.01 49	9.85 68	5
	9.84 20	13	9.98 534	26	10.01 46	9.85 66	5
	9.84 216	1	9.98 56	25	10.01 44	9.85 65	5
	9.84 229	1	9.98 585	25	10.01 41	9.85 64	5
	9.84 242	13	9.98 610	25	10.01 390	9.85 632	5
	9.84 255	1	9.98 63	26	10.01 365	9.85 62	5
	9.84 269	13	9.98 66	25	10.01 339	9.85 60	5
	9.84 282	13	9.98 686	25	10.01 314	9.85 596	5
	9.84 29	1	9.98 711	26	10.01 289	9.85 583	5
10	9.84 308	1	9.98 737	25	10.01 263	9.85 571	5
11	9.84 321	1	9.98 762	25	10.01 238	9.85 55	4
12	9.84 334	13	9.98 78	25	10.01 21	9.85 547	4
	9.84 347	13	9.98 81	26	10.01 188	9.85 534	4
	9.84 360	13	9.98 838	25	10.01 16	9.85 522	4
15	9.84 373	1	9.98 863	25	10.01 137	9.85 510	**45**
16	9.84 385	13	9.98 888	25	10.01 112	9.85 497	44
17	9.84 398	13	9.98 913	26	10.01 087	9.85 485	43
18	9.84 411	13	9.98 939	2	10.01 061	9.85 473	42
19	9.84 424	13	9.98 964	25	10.01 036	9.85 460	41
20	9.84 437	13	9.98 989	26	10.01 011	9.85 448	**40**
21	9.84 450	13	9.99 015	25	10.00 985	9.85 436	39
22	9.84 463	13	9.99 040	25	10.00 960	9.85 423	38
23	9.84 476	13	9.99 065	25	10.00 935	9.85 411	37
24	9.84 489	13	9.99 090	26	10.00 910	9.85 399	36
25	9.84 502	13	9.99 116	25	10.00 884	9.85 386	**35**
26	9.84 515	13	9.99 141	25	10.00 859	9.85 374	34
27	9.84 528	12	9.99 166	25	10.00 834	9.85 361	33
28	9.84 540	13	9.99 191	26	10.00 809	9.85 349	3
29	9.84 553	13	9.99 217	25	10.00 783	9.85 337	31
30	9.84 566	13	9.99 242	25	10.00 758	9.85 324	**30**
31	9.84 579	13	9.99 267	26	10.00 733	9.85 312	29
32	9.84 592	13	9.99 293	25	10.00 707	9.85 299	28
33	9.84 605	13	9.99 318	25	10.00 682	9.85 287	2
34	9.84 618	12	9.99 343	25	10.00 657	9.85 274	26
35	9.84 630	13	9.99 368	26	10.00 632	9.85 262	**25**
36	9.84 643	13	9.99 394	25	10.00 606	9.85 250	24
37	9.84 656	13	9.99 419	25	10.00 581	9.85 237	23
38	9.84 669	13	9.99 444	25	10.00 556	9.85 225	22
39	9.84 682	12	9.99 469	26	10.00 531	9.85 212	21
40	9.84 694	13	9.99 495	25	10.00 505	9.85 200	**20**
41	9.84 707	13	9.99 520	25	10.00 480	9.85 187	
42	9.84 720	13	9.99 545	25	10.00 455	9.85 175	
43	9.84 733	12	9.99 570	26	10.00 430	9.85 162	17
44	9.84 745	13	.99 596	25	10.00 404	9.85 150	
45	9.84 758	13	9.99 621	25	10.00 379	9.85 137	
46	9.84 771	13	9.99 646	26	10.00 354	9.85 125	
47	9.84 784	12	9.99 672	25	10.00 328	9.85 112	
48	9.84 796	13	9.99 697	25	10.00 303	9.85 100	
49	9.84 809	13	9.99 722	25	10.00 278	.85 087	
50	9.84 822	13	9.99 747	26	10.00 253	.85 074	
	9.84 835	12	.99 773	25	10.00 227	9.85 062	
	9.84 847	13	.99 798	25	.0.00 202	9.85 049	
	9.84 860	13	.99 823	25	.0.00 177	.85 037	
	.84 873	12	.99 848	26	.0.00 152	.85 024	
	.84 885	13	9.99 874	25	.0.00 126	.85 012	
		13	9.99 899	25	.0.00 101	.84 999	
	.84 911	12	.99 924	25	.0.00 076	.84 986	
	.84 923	13	.99 949	26	.0.00 051	.84 974	
	.84 936		.99 975	25	.0.00 025	.84 961	
	.84 949		0.0000		.0.00 000	.84 949	
	Cos		L Ctn		L Tan	L Sin	d

Prop. Pts.

	26	25
2	5.2	5.0
3	7.8	7.5
4	10.4	10.0
5	13.0	12.5
6	15.6	15.0
7	18.2	17.5
8	20.8	20.0
9	23.4	22.5

	14	13
2	2.8	2.6
3	4.2	3.9
4	5.6	5.2
5	7.0	6.5
6	8.4	7.8
7	9.8	9.1
8	11.2	10.4
9	12.6	11.7

12
2.4
3.6
4.8
6.0
7.2
8.4
9.6
10.8

From the top:

For **44°+** or **224°+**, read as printed; for **134°+** or **314°+**, read co-function.

From the bottom:

For **45°+** or **225°+**, read as printed; for **135°+** or **315°+**, read co-function.

Prop. Pts.

Degrees						Minutes		Seconds	
0°	0.00000 00	60°	1.04719 76	120°	2.09439 51	0	0.00000 00	0	0.00000 00
1	0.01745 33	61	1.06465 08	121	2.11184 84	1	0.00029 09	1	0.00000 48
2	0.03490 66	62	1.08210 41	122	2.12930 17	2	0.00058 18	2	0.00000 97
3	0.05235 99	63	1.09955 74	123	2.14675 50	3	0.00087 27	3	0.00001 45
4	0.06981 32	64	1.11701 07	124	2.16420 83	4	0.00116 36	4	0.00001 94
5	0.08726 65	65	1.13446 40	125	2.18166 16	5	0.00145 44	5	0.00002 42
6	0.10471 98	66	1.15191 73	126	2.19911 49	6	0.00174 53	6	0.00002 91
7	0.12217 30	67	1.16937 06	127	2.21656 82	7	0.00203 62	7	0.00003 39
8	0.13962 63	68	1.18682 39	128	2.23402 14	8	0.00232 71	8	0.00003 88
9	0.15707 96	69	1.20427 72	129	2.25147 47	9	0.00261 80	9	0.00004 36
10	0.17453 29	70	1.22173 05	130	2.26892 80	10	0.00290 89	10	0.00004 85
11	0.19198 62	71	1.23918 38	131	2.28638 13	11	0.00319 98	11	0.00005 33
12	0.20943 95	72	1.25663 71	132	2.30383 46	12	0.00349 07	12	0.00005 82
13	0.22689 28	73	1.27409 04	133	2.32128 79	13	0.00378 15	13	0.00006 30
14	0.24434 61	74	1.29154 36	134	2.33874 12	14	0.00407 24	14	0.00006 79
15	0.26179 94	75	1.30899 69	135	2.35619 45	15	0.00436 33	15	0.00007 27
16	0.27925 27	76	1.32645 02	136	2.37364 78	16	0.00465 42	16	0.00007 76
17	0.29670 60	77	1.34390 35	137	2.39110 11	17	0.00494 51	17	0.00008 24
18	0.31415 93	78	1.36135 68	138	2.40855 44	18	0.00523 60	18	0.00008 73
19	0.33161 26	79	1.37881 01	139	2.42600 77	19	0.00552 69	19	0.00009 21
20	0.34906 59	80	1.39626 34	140	2.44346 10	20	0.00581 78	20	0.00009 70
21	0.36651 91	81	1.41371 67	141	2.46091 42	21	0.00610 87	21	0.00010 18
22	0.38397 24	82	1.43117 00	142	2.47836 75	22	0.00639 95	22	0.00010 67
23	0.40142 57	83	1.44862 33	143	2.49582 08	23	0.00669 04	23	0.00011 15
24	0.41887 90	84	1.46607 66	144	2.51327 41	24	0.00698 13	24	0.00011 64
25	0.43633 23	85	1.48352 99	145	2.53072 74	25	0.00727 22	25	0.00012 12
26	0.45378 56	86	1.50098 32	146	2.54818 07	26	0.00756 31	26	0.00012 61
27	0.47123 89	87	1.51843 64	147	2.56563 40	27	0.00785 40	27	0.00013 09
28	0.48869 22	88	1.53588 97	148	2.58308 73	28	0.00814 49	28	0.00013 57
29	0.50614 55	89	1.55334 30	149	2.60054 06	29	0.00843 58	29	0.00014 06
30	0.52359 88	90	1.57079 63	150	2.61799 39	30	0.00872 66	30	0.00014 54
31	0.54105 21	91	1.58824 96	151	2.63544 72	31	0.00901 75	31	0.00015 03
32	0.55850 54	92	1.60570 29	152	2.65290 05	32	0.00930 84	32	0.00015 51
33	0.57595 87	93	1.62315 62	153	2.67035 38	33	0.00959 93	33	0.00016 00
34	0.59341 19	94	1.64060 95	154	2.68780 70	34	0.00989 02	34	0.00016 48
35	0.61086 52	95	1.65806 28	155	2.70526 03	35	0.01018 11	35	0.00016 97
36	0.62831 85	96	1.67551 61	156	2.72271 36	36	0.01047 20	36	0.00017 45
37	0.64577 18	97	1.69296 94	157	2.74016 69	37	0.01076 29	37	0.00017 94
38	0.66322 51	98	1.71042 27	158	2.75762 02	38	0.01105 38	38	0.00018 42
39	0.68067 84	99	1.72787 60	159	2.77507 35	39	0.01134 46	39	0.00018 91
40	0.69813 17	100	1.74532 93	160	2.79252 68	40	0.01163 55	40	0.00019 39
41	0.71558 50	101	1.76278 25	161	2.80998 01	41	0.01192 64	41	0.00019 88
42	0.73303 83	102	1.78023 58	162	2.82743 34	42	0.01221 73	42	0.00020 36
43	0.75049 16	103	1.79768 91	163	2.84488 67	43	0.01250 82	43	0.00020 85
44	0.76794 49	104	1.81514 24	164	2.86234 00	44	0.01279 91	44	0.00021 33
45	0.78539 82	105	1.83259 57	165	2.87979 33	45	0.01309 00	45	0.00021 82
46	0.80285 15	106	1.85004 90	166	2.89724 66	46	0.01338 09	46	0.00022 30
47	0.82030 47	107	1.86750 23	167	2.91469 99	47	0.01367 17	47	0.00022 79
48	0.83775 80	108	1.88495 56	168	2.93215 31	48	0.01396 26	48	0.00023 27
49	0.85521 13	109	1.90240 89	169	2.94960 64	49	0.01425 35	49	0.00023 76
50	0.87266 46	110	1.91986 22	170	2.96705 97	50	0.01454 44	50	0.00024 24
51	0.89011 79	111	1.93731 55	171	2.98451 30	51	0.01483 53	51	0.00024 73
52	0.90757 12	112	1.95476 88	172	3.00196 63	52	0.01512 62	52	0.00025 21
53	0.92502 45	113	1.97222 21	173	3.01941 96	53	0.01541 71	53	0.00025 70
54	0.94247 78	114	1.98967 53	174	3.03687 29	54	0.01570 80	54	0.00026 18
55	0.95993 11	115	2.00712 86	175	3.05432 62	55	0.01599 89	55	0.00026 66
56	0.97738 44	116	2.02458 19	176	3.07177 95	56	0.01628 97	56	0.00027 15
57	0.99483 77	117	2.04203 52	177	3.08923 28	57	0.01658 06	57	0.00027 63
58	1.01229 10	118	2.05948 85	178	3.10668 61	58	0.01687 15	58	0.00028 12
59	1.02974 43	119	2.07694 18	179	3.12413 94	59	0.01716 24	59	0.00028 60
60	1.04719 76	120	2.09439 51	180	3.14159 27	60	0.01745 33	60	0.00029 09

x Radians	Sin x	Cos x	Tan x	Equivalent of x
.00	.00000	1.0000	.00000	0° 00′.0
.01	.01000	.99995	.01000	0° 34′.4
.02	.02000	.99980	.02000	1° 08′.8
.03	.03000	.99955	.03001	1° 43′.1
.04	.03999	.99920	.04002	2° 17′.5
.05	.04998	.99875	.05004	2° 51′.9
.06	.05996	.99820	.06007	3° 26′.3
.07	.06994	.99755	.07011	4° 00′.6
.08	.07991	.99680	.08017	4° 35′.0
.09	.08988	.99595	.09024	5° 09′.4
.10	.09983	.99500	.10033	5° 43′.8
.11	.10978	.99396	.11045	6° 18′.2
.12	.11971	.99281	.12058	6° 52′.5
.13	.12963	.99156	.13074	7° 26′.9
.14	.13954	.99022	.14092	8° 01′.3
.15	.14944	.98877	.15114	8° 35′.7
.16	.15932	.98723	.16138	9° 10′.0
.17	.16918	.98558	.17166	9° 44′.4
.18	.17903	.98384	.18197	10° 18′.8
.19	.18886	.98200	.19232	10° 53′.2
.20	.19867	.98007	.20271	11° 27′.5
.21	.20846	.97803	.21314	12° 01′.9
.22	.21823	.97590	.22362	12° 36′.3
.23	.22798	.97367	.23414	13° 10′.7
.24	.23770	.97134	.24472	13° 45′.1
.25	.24740	.96891	.25534	14° 19′.4
.26	.25708	.96639	.26602	14° 53′.8
.27	.26673	.96377	.27676	15° 28′.2
.28	.27636	.96106	.28755	16° 02′.6
.29	.28595	.95824	.29841	16° 36′.9
.30	.29552	.95534	.30934	17° 11′.3
.31	.30506	.95233	.32033	17° 45′.7
.32	.31457	.94924	.33139	18° 20′.1
.33	.32404	.94604	.34252	18° 54′.5
.34	.33349	.94275	.35374	19° 28′.8
.35	.34290	.93937	.36503	20° 03′.2
.36	.35227	.93590	.37640	20° 37′.6
.37	.36162	.93233	.38786	21° 12′.0
.38	.37092	.92866	.39941	21° 46′.3
.39	.38019	.92491	.41105	22° 20′.7
.40	.38942	.92106	.42279	22° 55′.1
.41	.39861	.91712	.43463	23° 29′.5
.42	.40776	.91309	.44657	24° 03′.9
.43	.41687	.90897	.45862	24° 38′.2
.44	.42594	.90475	.47078	25° 12′.6
.45	.43497	.90045	.48306	25° 47′.0
.46	.44395	.89605	.49545	26° 21′.4
.47	.45289	.89157	.50797	26° 55′.7
.48	.46178	.88699	.52061	27° 30′.1
.49	.47063	.88233	.53339	28° 04′.5
.50	.47943	.87758	.54630	28° 38′.9

x Radians	Sin x	Cos x	Tan x	Equivalent of x
.50	.47943	.87758	.54630	28° 38′.9
.51	.48818	.87274	.55936	29° 13′.3
.52	.49688	.86782	.57256	29° 47′.6
.53	.50553	.86281	.58592	30° 22′.0
.54	.51414	.85771	.59943	30° 56′.4
.55	.52269	.85252	.61311	31° 30′.8
.56	.53119	.84726	.62695	32° 05′.1
.57	.53963	.84190	.64097	32° 39′.5
.58	.54802	.83646	.65517	33° 13′.9
.59	.55636	.83094	.66956	33° 48′.3
.60	.56464	.82534	.68414	34° 22′.6
.61	.57287	.81965	.69892	34° 57′.0
.62	.58104	.81388	.71391	35° 31′.4
.63	.58914	.80803	.72911	36° 05′.8
.64	.59720	.80210	.74454	36° 40′.2
.65	.60519	.79608	.76020	37° 14′.5
.66	.61312	.78999	.77610	37° 48′.9
.67	.62099	.78382	.79225	38° 23′.3
.68	.62879	.77757	.80866	38° 57′.7
.69	.63654	.77125	.82534	39° 32′.0
.70	.64422	.76484	.84229	40° 06′.4
.71	.65183	.75836	.85953	40° 40′.8
.72	.65938	.75181	.87707	41° 15′.2
.73	.66687	.74517	.89492	41° 49′.6
.74	.67429	.73847	.91309	42° 23′.9
.75	.68164	.73169	.93160	42° 58′.3
.76	.68892	.72484	.95045	43° 32′.7
.77	.69614	.71791	.96967	44° 07′.1
.78	.70328	.71091	.98926	44° 41′.4
.79	.71035	.70385	1.0092	45° 15′.8
.80	.71736	.69671	1.0296	45° 50′.2
.81	.72429	.68950	1.0505	46° 24′.6
.82	.73115	.68222	1.0717	46° 59′.0
.83	.73793	.67488	1.0934	47° 33′.3
.84	.74464	.66746	1.1156	48° 07′.7
.85	.75128	.65998	1.1383	48° 42′.1
.86	.75784	.65244	1.1616	49° 16′.5
.87	.76433	.64483	1.1853	49° 50′.8
.88	.77074	.63715	1.2097	50° 25′.2
.89	.77707	.62941	1.2346	50° 59′.6
.90	.78333	.62161	1.2602	51° 34′.0
.91	.78950	.61375	1.2864	52° 08′.3
.92	.79560	.60582	1.3133	52° 42′.7
.93	.80162	.59783	1.3409	53° 17′.1
.94	.80756	.58979	1.3692	53° 51′.5
.95	.81342	.58168	1.3984	54° 25′.9
.96	.81919	.57352	1.4284	55° 00′.2
.97	.82489	.56530	1.4592	55° 34′.6
.98	.83050	.55702	1.4910	56° 09′.0
.99	.83603	.54869	1.5237	56° 43′.4
1.00	.84147	.54030	1.5574	57° 17′.7

x Radians	Sin x	Cos x	Tan x	Equivalent of x	x Radians	Sin x	Cos x	Tan x	Equivalent of x
1.00	.84147	.54030	1.5574	57° 17′.7	**1.30**	.96356	.26750	3.6021	74° 29′.1
1.01	.84683	.53186	1.5922	57° 52′.1	1.31	.96618	.25785	3.7471	75° 03′.4
1.02	.85211	.52337	1.6281	58° 26′.5	1.32	.96872	.24818	3.9033	75° 37′.8
1.03	.85730	.51482	1.6652	59° 00′.9	1.33	.97115	.23848	4.0723	76° 12′.2
1.04	.86240	.50622	1.7036	59° 35′.3	1.34	.97348	.22875	4.2556	76° 46′.6
1.05	.86742	.49757	1.7433	60° 09′.6	1.35	.97572	.21901	4.4552	77° 21′.0
1.06	.87236	.48887	1.7844	60° 44′.0	1.36	.97786	.20924	4.6734	77° 55′.3
1.07	.87720	.48012	1.8270	61° 18′.4	1.37	.97991	.19945	4.9131	78° 29′.7
1.08	.88196	.47133	1.8712	61° 52′.8	1.38	.98185	.18964	5.1774	79° 04′.1
1.09	.88663	.46249	1.9171	62° 27′.1	1.39	.98370	.17981	5.4707	79° 38′.5
1.10	.89121	.45360	1.9648	63° 01′.5	**1.40**	.98545	.16997	5.7979	80° 12′.8
1.11	.89570	.44466	2.0143	63° 35′.9	1.41	.98710	.16010	6.1654	80° 47′.2
1.12	.90010	.43568	2.0660	64° 10′.3	1.42	.98865	.15023	6.5811	81° 21′.6
1.13	.90441	.42666	2.1198	64° 44′.7	1.43	.99010	.14033	7.0555	81° 56′.0
1.14	.90863	.41759	2.1759	65° 19′.0	1.44	.99146	.13042	7.6018	82° 30′.4
1.15	.91276	.40849	2.2345	65° 53′.4	1.45	.99271	.12050	8.2381	83° 04′.7
1.16	.91680	.39934	2.2958	66° 27′.8	1.46	.99387	.11057	8.9886	83° 39′.1
1.17	.92075	.39015	2.3600	67° 02′.2	1.47	.99492	.10063	9.8874	84° 13′.5
1.18	.92461	.38092	2.4273	67° 36′.5	1.48	.99588	.09067	10.983	84° 47′.9
1.19	.92837	.37166	2.4979	68° 10′.9	1.49	.99674	.08071	12.350	85° 22′.2
1.20	.93204	.36236	2.5722	68° 45′.3	**1.50**	.99749	.07074	14.101	85° 56′.6
1.21	.93562	.35302	2.6503	69° 19′.7	1.51	.99815	.06076	16.428	86° 31′.0
1.22	.93910	.34365	2.7328	69° 54′.1	1.52	.99871	.05077	19.670	87° 05′.4
1.23	.94249	.33424	2.8198	70° 28′.4	1.53	.99917	.04079	24.498	87° 39′.8
1.24	.94578	.32480	2.9119	71° 02′.8	1.54	.99953	.03079	32.461	88° 14′.1
1.25	.94898	.31532	3.0096	71° 37′.2	1.55	.99978	.02079	48.078	88° 48′.5
1.26	.95209	.30582	3.1133	72° 11′.6	1.56	.99994	.01080	92.621	89° 22′.9
1.27	.95510	.29628	3.2236	72° 45′.9	*1.57	*1.0000	*.00080	*1255.8	89° 57′.3
1.28	.95802	.28672	3.3413	73° 20′.3	1.58	.99996	−.00920	−108.65	90° 31′.6
1.29	.96084	.27712	3.4672	73° 54′.7	1.59	.99982	−.01920	−52.067	91° 06′.0
1.30	.96356	.26750	3.6021	74° 29′.1	**1.60**	.99957	−.02920	−34.233	91° 40′.4

π radians = 180° 1 radian = 57° 17′ 44″.806 = 57.°2957795

π = 3.14159265 3600″ = 60′ = 1° = 0.01745329 radian

*1 right angle = 90° = $\pi/2$ radians = 1.5707963 radians

Table Va — Radians to Degrees

	Radians	Tenths	Hundredths	Thousandths	Ten-thousandths
1	57°17′44″.8	5°43′46″.5	0°34′22″.6	0° 3′26″.3	0° 0′20″.6
2	114°35′29″.6	11°27′33″.0	1° 8′45″.3	0° 6′52″.5	0° 0′41″.3
3	171°53′14″.4	17°11′19″.4	1°43′07″.9	0°10′18″.8	0° 1′01″.9
4	229°10′59″.2	22°55′05″.9	2°17′30″.6	0°13′45″.1	0° 1′22″.5
5	286°28′44″.0	28°38′52″.4	2°51′53″.2	0°17′11″.3	0° 1′43″.1
6	343°46′28″.8	34°22′38″.9	3°26′15″.9	0°20′37″.6	0° 2′03″.8
7	401° 4′13″.6	40° 6′25″.4	4° 0′38″.5	0°24′03″.9	0° 2′24″.4
8	458°21′58″.4	45°50′11″.8	4°35′01″.2	0°27′30″.1	0° 2′45″.0
9	515°39′43″.3	51°33′58″.3	5° 9′23″.8	0°30′56″.4	0° 3′05″.6

n	n^2	$\sqrt{n}$		n^3	$\sqrt[3]{n}$			$1/n$
1.00	1.0000	1.00000	3.16228	1.00000	1.00000	2.15443		1.00000
1.01	1.0201	1.00499	3.17805	1.03030	1.00332	2.16159	4.65701	.990099
1.02	1.0404	1.00995	3.19374	1.06121	1.00662	2.16870	4.67233	.980392
1.03	1.0609	1.01489	3.20936	1.09273	1.00990	2.17577	4.68755	.970874
1.04	1.0816	1.01980	3.22490	1.12486	1.01316	2.18279	4.70267	.961538
1.05	1.1025	1.02470	3.24037	1.15762	1.01640	2.18976	4.71769	.952381
1.06	1.1236	1.02956	3.25576	1.19102	1.01961	2.19669	4.73262	.943396
1.07	1.1449	1.03441	3.27109	1.22504	1.02281	2.20358	4.74746	.934579
1.08	1.1664	1.03923	3.28634	1.25971	1.02599	2.21042	4.76220	.925926
1.09	1.1881	1.04403	3.30151	1.29503	1.02914	2.21722	4.77686	.917431
1.10	1.2100	1.04881	3.31662	1.33100	1.03228	2.22398	4.79142	.909091
1.11	1.2321	1.05357	3.33167	1.36763	1.03540	2.23070	4.80590	.900901
1.12	1.2544	1.05830	3.34664	1.40493	1.03850	2.23738	4.82028	.892857
1.13	1.2769	1.06301	3.36155	1.44290	1.04158	2.24402	4.83459	.884956
1.14	1.2996	1.06771	3.37639	1.48154	1.04464	2.25062	4.84881	.877193
1.15	1.3225	1.07238	3.39116	1.52088	1.04769	2.25718	4.86294	.869565
1.16	1.3456	1.07703	3.40588	1.56090	1.05072	2.26370	4.87700	.862069
1.17	1.3689	1.08167	3.42053	1.60161	1.05373	2.27019	4.89097	.854701
1.18	1.3924	1.08628	3.43511	1.64303	1.05672	2.27664	4.90487	.847458
1.19	1.4161	1.09087	3.44964	1.68516	1.05970	2.28305	4.91868	.840336
1.20	1.4400	1.09545	3.46410	1.72800	1.06266	2.28943	4.93242	.833333
1.21	1.4641	1.10000	3.47851	1.77156	1.06560	2.29577	4.94609	.826446
1.22	1.4884	1.10454	3.49285	1.81585	1.06853	2.30208	4.95968	.819672
1.23	1.5129	1.10905	3.50714	1.86087	1.07144	2.30835	4.97319	.813008
1.24	1.5376	1.11355	3.52136	1.90662	1.07434	2.31459	4.98663	.806452
1.25	1.5625	1.11803	3.53553	1.95312	1.07722	2.32079	5.00000	.800000
1.26	1.5876	1.12250	3.54965	2.00038	1.08008	2.32697	5.01330	.793651
1.27	1.6129	1.12694	3.56371	2.04838	1.08293	2.33311	5.02653	.787402
1.28	1.6384	1.13137	3.57771	2.09715	1.08577	2.33921	5.03968	.781250
1.29	1.6641	1.13578	3.59166	2.14669	1.08859	2.34529	5.05277	.775194
1.30	1.6900	1.14018	3.60555	2.19700	1.09139	2.35133	5.06580	.769231
1.31	1.7161	1.14455	3.61939	2.24809	1.09418	2.35735	5.07875	.763359
1.32	1.7424	1.14891	3.63318	2.29997	1.09696	2.36333	5.09164	.757576
1.33	1.7689	1.15326	3.64692	2.35264	1.09972	2.36928	5.10447	.751880
1.34	1.7956	1.15758	3.66060	2.40610	1.10247	2.37521	5.11723	.746269
1.35	1.8225	1.16190	3.67423	2.46038	1.10521	2.38110	5.12993	.740741
1.36	1.8496	1.16619	3.68782	2.51546	1.10793	2.38697	5.14256	.735294
1.37	1.8769	1.17047	3.70135	2.57135	1.11064	2.39280	5.15514	.729927
1.38	1.9044	1.17473	3.71484	2.62807	1.11334	2.39861	5.16765	.724638
1.39	1.9321	1.17898	3.72827	2.68562	1.11602	2.40439	5.18010	.719424
1.40	1.9600	1.18322	3.74166	2.74400	1.11869	2.41014	5.19249	.714286
1.41	1.9881	1.18743	3.75500	2.80322	1.12135	2.41587	5.20483	.709220
1.42	2.0164	1.19164	3.76829	2.86329	1.12399	2.42156	5.21710	.704225
1.43	2.0449	1.19583	3.78153	2.92421	1.12662	2.42724	5.22932	.699301
1.44	2.0736	1.20000	3.79473	2.98598	1.12924	2.43288	5.24148	.694444
1.45	2.1025	1.20416	3.80789	3.04862	1.13185	2.43850	5.25359	.689655
1.46	2.1316	1.20830	3.82099	3.11214	1.13445	2.44409	5.26564	.684932
1.47	2.1609	1.21244	3.83406	3.17652	1.13703	2.44966	5.27763	.680272
1.48	2.1904	1.21655	3.84708	3.24179	1.13960	2.45520	5.28957	.675676
1.49	2.2201	1.22066	3.86005	3.30795	1.14216	2.46072	5.30146	.671141
1.50	2.2500	1.22474	3.87298	3.37500	1.14471	2.46621	5.31329	.666667
n	n^2	$\sqrt{n}$		n^3	$\sqrt[3]{n}$			$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10\,n}$	$\sqrt[3]{100\,n}$	$1/n$
1.50	2.2500	1.22474	3.87298	3.37500	1.14471	2.46621	5.31329	.666667
1.51	2.2801	1.22882	3.88587	3.44295	1.14725	2.47168	5.32507	.662252
1.52	2.3104	1.23288	3.89872	3.51181	1.14978	2.47712	5.33680	.657895
1.53	2.3409	1.23693	3.91152	3.58158	1.15230	2.48255	5.34848	.653595
1.54	2.3716	1.24097	3.92428	3.65226	1.15480	2.48794	5.36011	.649351
1.55	2.4025	1.24499	3.93700	3.72388	1.15729	2.49332	5.37169	.645161
1.56	2.4336	1.24900	3.94968	3.79642	1.15978	2.49867	5.38321	.641026
1.57	2.4649	1.25300	3.96232	3.86989	1.16225	2.50399	5.39469	.636943
1.58	2.4964	1.25698	3.97492	3.94431	1.16471	2.50930	5.40612	.632911
1.59	2.5281	1.26095	3.98748	4.01968	1.16717	2.51458	5.41750	.628931
1.60	2.5600	1.26491	4.00000	4.09600	1.16961	2.51984	5.42884	.625000
1.61	2.5921	1.26886	4.01248	4.17328	1.17204	2.52508	5.44012	.621118
1.62	2.6244	1.27279	4.02492	4.25153	1.17446	2.53030	5.45136	.617284
1.63	2.6569	1.27671	4.03733	4.33075	1.17687	2.53549	5.46256	.613497
1.64	2.6896	1.28062	4.04969	4.41094	1.17927	2.54067	5.47370	.609756
1.65	2.7225	1.28452	4.06202	4.49212	1.18167	2.54582	5.48481	.606061
1.66	2.7556	1.28841	4.07431	4.57430	1.18405	2.55095	5.49586	.602410
1.67	2.7889	1.29228	4.08656	4.65746	1.18642	2.55607	5.50688	.598802
1.68	2.8224	1.29615	4.09878	4.74163	1.18878	2.56116	5.51785	.595238
1.69	2.8561	1.30000	4.11096	4.82681	1.19114	2.56623	5.52877	.591716
1.70	2.8900	1.30384	4.12311	4.91300	1.19348	2.57128	5.53966	.588235
1.71	2.9241	1.30767	4.13521	5.00021	1.19582	2.57631	5.55050	.584795
1.72	2.9584	1.31149	4.14729	5.08845	1.19815	2.58133	5.56130	.581395
1.73	2.9929	1.31529	4.15933	5.17772	1.20046	2.58632	5.57205	.578035
1.74	3.0276	1.31909	4.17133	5.26802	1.20277	2.59129	5.58277	.574713
1.75	3.0625	1.32288	4.18330	5.35938	1.20507	2.59625	5.59344	.571429
1.76	3.0976	1.32665	4.19524	5.45178	1.20736	2.60118	5.60408	.568182
1.77	3.1329	1.33041	4.20714	5.54523	1.20964	2.60610	5.61467	.564972
1.78	3.1684	1.33417	4.21900	5.63975	1.21192	2.61100	5.62523	.561798
1.79	3.2041	1.33791	4.23084	5.73534	1.21418	2.61588	5.63574	.558659
1.80	3.2400	1.34164	4.24264	5.83200	1.21644	2.62074	5.64622	.555556
1.81	3.2761	1.34536	4.25441	5.92974	1.21869	2.62559	5.65665	.552486
1.82	3.3124	1.34907	4.26615	6.02857	1.22093	2.63041	5.66705	.549451
1.83	3.3489	1.35277	4.27785	6.12849	1.22316	2.63522	5.67741	.546448
1.84	3.3856	1.35647	4.28952	6.22950	1.22539	2.64001	5.68773	.543478
1.85	3.4225	1.36015	4.30116	6.33162	1.22760	2.64479	5.69802	.540541
1.86	3.4596	1.36382	4.31277	6.43486	1.22981	2.64954	5.70827	.537634
1.87	3.4969	1.36748	4.32435	6.53920	1.23201	2.65428	5.71848	.534759
1.88	3.5344	1.37113	4.33590	6.64467	1.23420	2.65901	5.72865	.531915
1.89	3.5721	1.37477	4.34741	6.75127	1.23639	2.66371	5.73879	.529101
1.90	3.6100	1.37840	4.35890	6.85900	1.23856	2.66840	5.74890	.526316
1.91	3.6481	1.38203	4.37035	6.96787	1.24073	2.67307	5.75897	.523560
1.92	3.6864	1.38564	4.38178	7.07789	1.24289	2.67773	5.76900	.520833
1.93	3.7249	1.38924	4.39318	7.18906	1.24505	2.68237	5.77900	.518135
1.94	3.7636	1.39284	4.40454	7.30138	1.24719	2.68700	5.78896	.515464
1.95	3.8025	1.39642	4.41588	7.41488	1.24933	2.69161	5.79889	.512821
1.96	3.8416	1.40000	4.42719	7.52954	1.25146	2.69620	5.80879	.510204
1.97	3.8809	1.40357	4.43847	7.64537	1.25359	2.70078	5.81865	.507614
1.98	3.9204	1.40712	4.44972	7.76239	1.25571	2.70534	5.82848	.505051
1.99	3.9601	1.41067	4.46094	7.88060	1.25782	2.70989	5.83827	.502513
2.00	4.0000	1.41421	4.47214	8.00000	1.25992	2.71442	5.84804	.500000
n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10\,n}$	$\sqrt[3]{100\,n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
2.00	4.0000	1.41421	4.47214	8.00000	1.25992	2.71442	5.84804	.500000
2.01	4.0401	1.41774	4.48330	8.12060	1.26202	2.71893	5.85777	.497512
2.02	4.0804	1.42127	4.49444	8.24241	1.26411	2.72344	5.86746	.495050
2.03	4.1209	1.42478	4.50555	8.36543	1.26619	2.72792	5.87713	.492611
2.04	4.1616	1.42829	4.51664	8.48966	1.26827	2.73239	5.88677	.490196
2.05	4.2025	1.43178	4.52769	8.61512	1.27033	2.73685	5.89637	.487805
2.06	4.2436	1.43527	4.53872	8.74182	1.27240	2.74129	5.90594	.485437
2.07	4.2849	1.43875	4.54973	8.86974	1.27445	2.74572	5.91548	.483092
2.08	4.3264	1.44222	4.56070	8.99891	1.27650	2.75014	5.92499	.480769
2.09	4.3681	1.44568	4.57165	9.12933	1.27854	2.75454	5.93447	.478469
2.10	4.4100	1.44914	4.58258	9.26100	1.28058	2.75892	5.94392	.476190
2.11	4.4521	1.45258	4.59347	9.39393	1.28261	2.76330	5.95334	.473934
2.12	4.4944	1.45602	4.60435	9.52813	1.28463	2.76766	5.96273	.471698
2.13	4.5369	1.45945	4.61519	9.66360	1.28665	2.77200	5.97209	.469434
2.14	4.5796	1.46287	4.62601	9.80034	1.28866	2.77633	5.98142	.467290
2.15	4.6225	1.46629	4.63681	9.93838	1.29066	2.78065	5.99073	.465116
2.16	4.6656	1.46969	4.64758	10.0777	1.29266	2.78495	6.00000	.462963
2.17	4.7089	1.47309	4.65833	10.2183	1.29465	2.78924	6.00925	.460829
2.18	4.7524	1.47648	4.66905	10.3602	1.29664	2.79352	6.01846	.458716
2.19	4.7961	1.47986	4.67974	10.5035	1.29862	2.79779	6.02765	.456621
2.20	4.8400	1.48324	4.69042	10.6480	1.30059	2.80204	6.03681	.454545
2.21	4.8841	1.48661	4.70106	10.7939	1.30256	2.80628	6.04594	.452489
2.22	4.9284	1.48997	4.71169	10.9410	1.30452	2.81050	6.05505	.450450
2.23	4.9729	1.49332	4.72229	11.0896	1.30648	2.81472	6.06413	.448430
2.24	5.0176	1.49666	4.73286	11.2394	1.30843	2.81892	6.07318	.446429
2.25	5.0625	1.50000	4.74342	11.3906	1.31037	2.82311	6.08220	.444444
2.26	5.1076	1.50333	4.75395	11.5432	1.31231	2.82728	6.09120	.442478
2.27	5.1529	1.50665	4.76445	11.6971	1.31424	2.83145	6.10017	.440529
2.28	5.1984	1.50997	4.77493	11.8524	1.31617	2.83560	6.10911	.438596
2.29	5.2441	1.51327	4.78539	12.0090	1.31809	2.83974	6.11803	.436681
2.30	5.2900	1.51658	4.79583	12.1670	1.32001	2.84387	6.12693	.434783
2.31	5.3361	1.51987	4.80625	12.3264	1.32192	2.84798	6.13579	.432900
2.32	5.3824	1.52315	4.81664	12.4872	1.32382	2.85209	6.14463	.431034
2.33	5.4289	1.52643	4.82701	12.6493	1.32572	2.85618	6.15345	.429185
2.34	5.4756	1.52971	4.83735	12.8129	1.32761	2.86026	6.16224	.427350
2.35	5.5225	1.53297	4.84768	12.9779	1.32950	2.86433	6.17101	.425532
2.36	5.5696	1.53623	4.85798	13.1443	1.33139	2.86838	6.17975	.423729
2.37	5.6169	1.53948	4.86826	13.3121	1.33326	2.87243	6.18846	.421941
2.38	5.6644	1.54272	4.87852	13.4813	1.33514	2.87646	6.19715	.420168
2.39	5.7121	1.54596	4.88876	13.6519	1.33700	2.88049	6.20582	.418410
2.40	5.7600	1.54919	4.89898	13.8240	1.33887	2.88450	6.21447	.416667
2.41	5.8081	1.55242	4.90918	13.9975	1.34072	2.88850	6.22308	.414938
2.42	5.8564	1.55563	4.91935	14.1725	1.34257	2.89249	6.23168	.413223
2.43	5.9049	1.55885	4.92950	14.3489	1.34442	2.89647	6.24025	.411523
2.44	5.9536	1.56205	4.93964	14.5268	1.34626	2.90044	6.24880	.409836
2.45	6.0025	1.56525	4.94975	14.7061	1.34810	2.90439	6.25732	.408163
2.46	6.0516	1.56844	4.95984	14.8869	1.34993	2.90834	6.26583	.406504
2.47	6.1009	1.57162	4.96991	15.0692	1.35176	2.91227	6.27431	.404858
2.48	6.1504	1.57480	4.97996	15.2530	1.35358	2.91620	6.28276	.403226
2.49	6.2001	1.57797	4.98999	15.4382	1.35540	2.92011	6.29119	.401606
2.50	6.2500	1.58114	5.00000	15.6250	1.35721	2.92402	6.29961	.400000
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
2.50	6.2500	1.58114	5.00000	15.6250	1.35721	2.92402	6.29961	.400000
2.51	6.3001	1.58430	5.00999	15.8133	1.35902	2.92791	6.30799	.398406
2.52	6.3504	1.58745	5.01996	16.0030	1.36082	2.93179	6.31636	.396825
2.53	6.4009	1.59060	5.02991	16.1943	1.36262	2.93567	6.32470	.395257
2.54	6.4516	1.59374	5.03984	16.3871	1.36441	2.93953	6.33303	.393701
2.55	6.5025	1.59687	5.04975	16.5814	1.36620	2.94338	6.34133	.392157
2.56	6.5536	1.60000	5.05964	16.7772	1.36798	2.94723	6.34960	.390625
2.57	6.6049	1.60312	5.06952	16.9746	1.36976	2.95106	6.35786	.389105
2.58	6.6564	1.60624	5.07937	17.1735	1.37153	2.95488	6.36610	.387597
2.59	6.7081	1.60935	5.08920	17.3740	1.37330	2.95869	6.37431	.386100
2.60	6.7600	1.61245	5.09902	17.5760	1.37507	2.96250	6.38250	.384615
2.61	6.8121	1.61555	5.10882	17.7796	1.37683	2.96629	6.39068	.383142
2.62	6.8644	1.61864	5.11859	17.9847	1.37859	2.97007	6.39883	.381679
2.63	6.9169	1.62173	5.12835	18.1914	1.38034	2.97385	6.40696	.380228
2.64	6.9696	1.62481	5.13809	18.3997	1.38208	2.97761	6.41507	.378788
2.65	7.0225	1.62788	5.14782	18.6096	1.38383	2.98137	6.42316	.377358
2.66	7.0756	1.63095	5.15752	18.8211	1.38557	2.98511	6.43123	.375940
2.67	7.1289	1.63401	5.16720	19.0342	1.38730	2.98885	6.43928	.374532
2.68	7.1824	1.63707	5.17687	19.2488	1.38903	2.99257	6.44731	.373134
2.69	7.2361	1.64012	5.18652	19.4651	1.39076	2.99629	6.45531	.371747
2.70	7.2900	1.64317	5.19615	19.6830	1.39248	3.00000	6.46330	.370370
2.71	7.3441	1.64621	5.20577	19.9025	1.39419	3.00370	6.47127	.369004
2.72	7.3984	1.64924	5.21536	20.1236	1.39591	3.00739	6.47922	.367647
2.73	7.4529	1.65227	5.22494	20.3464	1.39761	3.01107	6.48715	.366300
2.74	7.5076	1.65529	5.23450	20.5708	1.39932	3.01474	6.49507	.364964
2.75	7.5625	1.65831	5.24404	20.7969	1.40102	3.01841	6.50296	.363636
2.76	7.6176	1.66132	5.25357	21.0246	1.40272	3.02206	6.51083	.362319
2.77	7.6729	1.66433	5.26308	21.2539	1.40441	3.02570	6.51868	.361011
2.78	7.7284	1.66733	5.27257	21.4850	1.40610	3.02934	6.52652	.359712
2.79	7.7841	1.67033	5.28205	21.7176	1.40778	3.03297	6.53434	.358423
2.80	7.8400	1.67332	5.29150	21.9520	1.40946	3.03659	6.54213	.357143
2.81	7.8961	1.67631	5.30094	22.1880	1.41114	3.04020	6.54991	.355872
2.82	7.9524	1.67929	5.31037	22.4258	1.41281	3.04380	6.55767	.354610
2.83	8.0089	1.68226	5.31977	22.6652	1.41448	3.04740	6.56541	.353357
2.84	8.0656	1.68523	5.32917	22.9063	1.41614	3.05098	6.57314	.352113
2.85	8.1225	1.68819	5.33854	23.1491	1.41780	3.05456	6.58084	.350877
2.86	8.1796	1.69115	5.34790	23.3937	1.41946	3.05813	6.58853	.349650
2.87	8.2369	1.69411	5.35724	23.6399	1.42111	3.06169	6.59620	.348432
2.88	8.2944	1.69706	5.36656	23.8879	1.42276	3.06524	6.60385	.347222
2.89	8.3521	1.70000	5.37587	24.1376	1.42440	3.06878	6.61149	.346021
2.90	8.4100	1.70294	5.38516	24.3890	1.42604	3.07232	6.61911	.344828
2.91	8.4681	1.70587	5.39444	24.6422	1.42768	3.07584	6.62671	.343643
2.92	8.5264	1.70880	5.40370	24.8971	1.42931	3.07936	6.63429	.342466
2.93	8.5849	1.71172	5.41295	25.1538	1.43094	3.08287	6.64185	.341297
2.94	8.6436	1.71464	5.42218	25.4122	1.43257	3.08638	6.64940	.340136
2.95	8.7025	1.71756	5.43139	25.6724	1.43419	3.08987	6.65693	.338983
2.96	8.7616	1.72047	5.44059	25.9343	1.43581	3.09336	6.66444	.337838
2.97	8.8209	1.72337	5.44977	26.1981	1.43743	3.09684	6.67194	.336700
2.98	8.8804	1.72627	5.45894	26.4636	1.43904	3.10031	6.67942	.335570
2.99	8.9401	1.72916	5.46809	26.7309	1.44065	3.10378	6.68688	.334448
3.00	9.0000	1.73205	5.47723	27.0000	1.44225	3.10723	6.69433	.333333
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
3.00	9.0000	1.73205	5.47723	27.0000	1.44225	3.10723	6.69433	.333333
3.01	9.0601	1.73494	5.48635	27.2709	1.44385	3.11068	6.70176	.332226
3.02	9.1204	1.73781	5.49545	27.5436	1.44545	3.11412	6.70917	.331126
3.03	9.1809	1.74069	5.50454	27.8181	1.44704	3.11756	6.71657	.330033
3.04	9.2416	1.74356	5.51362	28.0945	1.44863	3.12098	6.72395	.328947
3.05	9.3025	1.74642	5.52268	28.3726	1.45022	3.12440	6.73132	.327869
3.06	9.3636	1.74929	5.53173	28.6526	1.45180	3.12781	6.73866	.326797
3.07	9.4249	1.75214	5.54076	28.9344	1.45338	3.13121	6.74600	.325733
3.08	9.4864	1.75499	5.54977	29.2181	1.45496	3.13461	6.75331	.324675
3.09	9.5481	1.75784	5.55878	29.5036	1.45653	3.13800	6.76061	.323625
3.10	9.6100	1.76068	5.56776	29.7910	1.45810	3.14138	6.76790	.322581
3.11	9.6721	1.76352	5.57674	30.0802	1.45967	3.14475	6.77517	.321543
3.12	9.7344	1.76635	5.58570	30.3713	1.46123	3.14812	6.78242	.320513
3.13	9.7969	1.76918	5.59464	30.6643	1.46279	3.15148	6.78966	.319489
3.14	9.8596	1.77200	5.60357	30.9591	1.46434	3.15483	6.79688	.318471
3.15	9.9225	1.77482	5.61249	31.2559	1.46590	3.15818	6.80409	.317460
3.16	9.9856	1.77764	5.62139	31.5545	1.46745	3.16152	6.81128	.316456
3.17	10.0489	1.78045	5.63028	31.8550	1.46899	3.16485	6.81846	.315457
3.18	10.1124	1.78326	5.63915	32.1574	1.47054	3.16817	6.82562	.314465
3.19	10.1761	1.78606	5.64801	32.4618	1.47208	3.17149	6.83277	.313480
3.20	10.2400	1.78885	5.65685	32.7680	1.47361	3.17480	6.83990	.312500
3.21	10.3041	1.79165	5.66569	33.0762	1.47515	3.17811	6.84702	.311526
3.22	10.3684	1.79444	5.67450	33.3862	1.47668	3.18140	6.85412	.310559
3.23	10.4329	1.79722	5.68331	33.6983	1.47820	3.18469	6.86121	.309598
3.24	10.4976	1.80000	5.69210	34.0122	1.47973	3.18798	6.86829	.308642
3.25	10.5625	1.80278	5.70088	34.3281	1.48125	3.19125	6.87534	.307692
3.26	10.6276	1.80555	5.70964	34.6460	1.48277	3.19452	6.88239	.306748
3.27	10.6929	1.80831	5.71839	34.9658	1.48428	3.19778	6.88942	.305810
3.28	10.7584	1.81108	5.72713	35.2876	1.48579	3.20104	6.89643	.304878
3.29	10.8241	1.81384	5.73585	35.6113	1.48730	3.20429	6.90344	.303951
3.30	10.8900	1.81659	5.74456	35.9370	1.48881	3.20753	6.91042	.303030
3.31	10.9561	1.81934	5.75326	36.2647	1.49031	3.21077	6.91740	.302115
3.32	11.0224	1.82209	5.76194	36.5944	1.49181	3.21400	6.92436	.301205
3.33	11.0889	1.82483	5.77062	36.9260	1.49330	3.21722	6.93130	.300300
3.34	11.1556	1.82757	5.77927	37.2597	1.49480	3.22044	6.93823	.299401
3.35	11.2225	1.83030	5.78792	37.5954	1.49629	3.22365	6.94515	.298507
3.36	11.2896	1.83303	5.79655	37.9331	1.49777	3.22686	6.95205	.297619
3.37	11.3569	1.83576	5.80517	38.2728	1.49926	3.23006	6.95894	.296736
3.38	11.4244	1.83848	5.81378	38.6145	1.50074	3.23325	6.96582	.295858
3.39	11.4921	1.84120	5.82237	38.9582	1.50222	3.23643	6.97268	.294985
3.40	11.5600	1.84391	5.83095	39.3040	1.50369	3.23961	6.97953	.294118
3.41	11.6281	1.84662	5.83952	39.6518	1.50517	3.24278	6.98637	.293255
3.42	11.6964	1.84932	5.84808	40.0017	1.50664	3.24595	6.99319	.292398
3.43	11.7649	1.85203	5.85662	40.3536	1.50810	3.24911	7.00000	.291545
3.44	11.8336	1.85472	5.86515	40.7076	1.50957	3.25227	7.00680	.290698
3.45	11.9025	1.85742	5.87367	41.0636	1.51103	3.25542	7.01358	.289855
3.46	11.9716	1.86011	5.88218	41.4217	1.51249	3.25856	7.02035	.289017
3.47	12.0409	1.86279	5.89067	41.7819	1.51394	3.26169	7.02711	.288184
3.48	12.1104	1.86548	5.89915	42.1442	1.51540	3.26482	7.03385	.287356
3.49	12.1801	1.86815	5.90762	42.5085	1.51685	3.26795	7.04058	.286533
3.50	12.2500	1.87083	5.91608	42.8750	1.51829	3.27107	7.04730	.285714
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10\,n}$	$\sqrt[3]{100\,n}$	$1/n$
3.50	12.2500	1.87083	5.91608	42.8750	1.51829	3.27107	7.04730	.285714
3.51	12.3201	1.87350	5.92453	43.2436	1.51974	3.27418	7.05400	.284900
3.52	12.3904	1.87617	5.93296	43.6142	1.52118	3.27729	7.06070	.284091
3.53	12.4609	1.87883	5.94138	43.9870	1.52262	3.28039	7.06738	.283286
3.54	12.5316	1.88149	5.94979	44.3619	1.52406	3.28348	7.07404	.282486
3.55	12.6025	1.88414	5.95819	44.7389	1.52549	3.28657	7.08070	.281690
3.56	12.6736	1.88680	5.96657	45.1180	1.52692	3.28965	7.08734	.280899
3.57	12.7449	1.88944	5.97495	45.4993	1.52835	3.29273	7.09397	.280112
3.58	12.8164	1.89209	5.98331	45.8827	1.52978	3.29580	7.10059	.279330
3.59	12.8881	1.89473	5.99166	46.2683	1.53120	3.29887	7.10719	.278552
3.60	12.9600	1.89737	6.00000	46.6560	1.53262	3.30193	7.11379	.277778
3.61	13.0321	1.90000	6.00833	47.0459	1.53404	3.30498	7.12037	.277008
3.62	13.1044	1.90263	6.01664	47.4379	1.53545	3.30803	7.12694	.276243
3.63	13.1769	1.90526	6.02495	47.8321	1.53686	3.31107	7.13349	.275482
3.64	13.2496	1.90788	6.03324	48.2285	1.53827	3.31411	7.14004	.274725
3.65	13.3225	1.91050	6.04152	48.6271	1.53968	3.31714	7.14657	.273973
3.66	13.3956	1.91311	6.04979	49.0279	1.54109	3.32017	7.15309	.273224
3.67	13.4689	1.91572	6.05805	49.4309	1.54249	3.32319	7.15960	.272480
3.68	13.5424	1.91833	6.06630	49.8360	1.54389	3.32621	7.16610	.271739
3.69	13.6161	1.92094	6.07454	50.2434	1.54529	3.32922	7.17258	.271003
3.70	13.6900	1.92354	6.08276	50.6530	1.54668	3.33222	7.17905	.270270
3.71	13.7641	1.92614	6.09098	51.0648	1.54807	3.33522	7.18552	.269542
3.72	13.8384	1.92873	6.09918	51.4788	1.54946	3.33822	7.19197	.268817
3.73	13.9129	1.93132	6.10737	51.8951	1.55085	3.34120	7.19840	.268097
3.74	13.9876	1.93391	6.11555	52.3136	1.55223	3.34419	7.20483	.267380
3.75	14.0625	1.93649	6.12372	52.7344	1.55362	3.34716	7.21125	.266667
3.76	14.1376	1.93907	6.13188	53.1574	1.55500	3.35014	7.21765	.265957
3.77	14.2129	1.94165	6.14003	53.5826	1.55637	3.35310	7.22405	.265252
3.78	14.2884	1.94422	6.14817	54.0102	1.55775	3.35607	7.23043	.264550
3.79	14.3641	1.94679	6.15630	54.4399	1.55912	3.35902	7.23680	.263852
3.80	14.4400	1.94936	6.16441	54.8720	1.56049	3.36198	7.24316	.263158
3.81	14.5161	1.95192	6.17252	55.3063	1.56186	3.36492	7.24950	.262467
3.82	14.5924	1.95448	6.18061	55.7430	1.56322	3.36786	7.25584	.261780
3.83	14.6689	1.95704	6.18870	56.1819	1.56459	3.37080	7.26217	.261097
3.84	14.7456	1.95959	6.19677	56.6231	1.56595	3.37373	7.26848	.260417
3.85	14.8225	1.96214	6.20484	57.0666	1.56731	3.37666	7.27479	.259740
3.86	14.8996	1.96469	6.21289	57.5125	1.56866	3.37958	7.28108	.259067
3.87	14.9769	1.96723	6.22093	57.9606	1.57001	3.38249	7.28736	.258398
3.88	15.0544	1.96977	6.22896	58.4111	1.57137	3.38540	7.29363	.257732
3.89	15.1321	1.97231	6.23699	58.8639	1.57271	3.38831	7.29989	.257069
3.90	15.2100	1.97484	6.24500	59.3190	1.57406	3.39121	7.30614	.256410
3.91	15.2881	1.97737	6.25300	59.7765	1.57541	3.39411	7.31238	.255754
3.92	15.3664	1.97990	6.26099	60.2363	1.57675	3.39700	7.31861	.255102
3.93	15.4449	1.98242	6.26897	60.6985	1.57809	3.39988	7.32483	.254453
3.94	15.5236	1.98494	6.27694	61.1630	1.57942	3.40277	7.33104	.253807
3.95	15.6025	1.98746	6.28490	61.6299	1.58076	3.40564	7.33723	.253165
3.96	15.6816	1.98997	6.29285	62.0991	1.58209	3.40851	7.34342	.252525
3.97	15.7609	1.99249	6.30079	62.5708	1.58342	3.41138	7.34960	.251889
3.98	15.8404	1.99499	6.30872	63.0448	1.58475	3.41424	7.35576	.251256
3.99	15.9201	1.99750	6.31664	63.5212	1.58608	3.41710	7.36192	.250627
4.00	16.0000	2.00000	6.32456	64.0000	1.58740	3.41995	7.36806	.250000
n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10\,n}$	$\sqrt[3]{100\,n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
4.00	16.0000	2.00000	6.32456	64.0000	1.58740	3.41995	7.36806	.250000
4.01	16.0801	2.00250	6.33246	64.4812	1.58872	3.42280	7.37420	.249377
4.02	16.1604	2.00499	6.34035	64.9648	1.59004	3.42564	7.38032	.248756
4.03	16.2409	2.00749	6.34823	65.4508	1.59136	3.42848	7.38644	.248139
4.04	16.3216	2.00998	6.35610	65.9393	1.59267	3.43131	7.39254	.247525
4.05	16.4025	2.01246	6.36396	66.4301	1.59399	3.43414	7.39864	.246914
4.06	16.4836	2.01494	6.37181	66.9234	1.59530	3.43697	7.40472	.246305
4.07	16.5649	2.01742	6.37966	67.4191	1.59661	3.43979	7.41080	.245700
4.08	16.6464	2.01990	6.38749	67.9173	1.59791	3.44260	7.41686	.245098
4.09	16.7281	2.02237	6.39531	68.4179	1.59922	3.44541	7.42291	.244499
4.10	16.8100	2.02485	6.40312	68.9210	1.60052	3.44822	7.42896	.243902
4.11	16.8921	2.02731	6.41093	69.4265	1.60182	3.45102	7.43499	.243309
4.12	16.9744	2.02978	6.41872	69.9345	1.60312	3.45382	7.44102	.242718
4.13	17.0569	2.03224	6.42651	70.4450	1.60441	3.45661	7.44703	.242131
4.14	17.1396	2.03470	6.43428	70.9579	1.60571	3.45939	7.45304	.241546
4.15	17.2225	2.03715	6.44205	71.4734	1.60700	3.46218	7.45904	.240964
4.16	17.3056	2.03961	6.44981	71.9913	1.60829	3.46496	7.46502	.240385
4.17	17.3889	2.04206	6.45755	72.5117	1.60958	3.46773	7.47100	.239808
4.18	17.4724	2.04450	6.46529	73.0346	1.61086	3.47050	7.47697	.239234
4.19	17.5561	2.04695	6.47302	73.5601	1.61215	3.47327	7.48292	.238663
4.20	17.6400	2.04939	6.48074	74.0880	1.61343	3.47603	7.48887	.238095
4.21	17.7241	2.05183	6.48845	74.6185	1.61471	3.47878	7.49481	.237530
4.22	17.8084	2.05426	6.49615	75.1514	1.61599	3.48154	7.50074	.236967
4.23	17.8929	2.05670	6.50384	75.6870	1.61726	3.48428	7.50666	.236407
4.24	17.9776	2.05913	6.51153	76.2250	1.61853	3.48703	7.51257	.235849
4.25	18.0625	2.06155	6.51920	76.7656	1.61981	3.48977	7.51847	.235294
4.26	18.1476	2.06398	6.52687	77.3088	1.62108	3.49250	7.52437	.234742
4.27	18.2329	2.06640	6.53452	77.8545	1.62234	3.49523	7.53025	.234192
4.28	18.3184	2.06882	6.54217	78.4028	1.62361	3.49796	7.53612	.233645
4.29	18.4041	2.07123	6.54981	78.9536	1.62487	3.50068	7.54199	.233100
4.30	18.4900	2.07364	6.55744	79.5070	1.62613	3.50340	7.54784	.232558
4.31	18.5761	2.07605	6.56506	80.0630	1.62739	3.50611	7.55369	.232019
4.32	18.6624	2.07846	6.57267	80.6216	1.62865	3.50882	7.55953	.231481
4.33	18.7489	2.08087	6.58027	81.1827	1.62991	3.51153	7.56535	.230947
4.34	18.8356	2.08327	6.58787	81.7465	1.63116	3.51423	7.57117	.230415
4.35	18.9225	2.08567	6.59545	82.3129	1.63241	3.51692	7.57698	.229885
4.36	19.0096	2.08806	6.60303	82.8819	1.63366	3.51962	7.58279	.229358
4.37	19.0969	2.09045	6.61060	83.4535	1.63491	3.52231	7.58858	.228833
4.38	19.1844	2.09284	6.61816	84.0277	1.63619	3.52499	7.59436	.228311
4.39	19.2721	2.09523	6.62571	84.6045	1.63740	3.52767	7.60014	.227790
4.40	19.3600	2.09762	6.63325	85.1840	1.63864	3.53035	7.60590	.227273
4.41	19.4481	2.10000	6.64078	85.7661	1.63988	3.53302	7.61166	.226757
4.42	19.5364	2.10238	6.64831	86.3509	1.64112	3.53569	7.61741	.226244
4.43	19.6249	2.10476	6.65582	86.9383	1.64236	3.53835	7.62315	.225734
4.44	19.7136	2.10713	6.66333	87.5284	1.64359	3.54101	7.62888	.225225
4.45	19.8025	2.10950	6.67083	88.1211	1.64483	3.54367	7.63461	.224719
4.46	19.8916	2.11187	6.67832	88.7165	1.64606	3.54632	7.64032	.224215
4.47	19.9809	2.11424	6.68581	89.3146	1.64729	3.54897	7.64603	.223714
4.48	20.0704	2.11660	6.69328	89.9154	1.64851	3.55162	7.65172	.223214
4.49	20.1601	2.11896	6.70075	90.5188	1.64974	3.55426	7.65741	.222717
4.50	20.2500	2.12132	6.70820	91.1250	1.65096	3.55689	7.66309	.222222
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
4.50	20.2500	2.12132	6.70820	91.1250	1.65096	3.55689	7.66309	.222222
4.51	20.3401	2.12368	6.71565	91.7339	1.65219	3.55953	7.66877	.221729
4.52	20.4304	2.12603	6.72309	92.3454	1.65341	3.56215	7.67443	.221239
4.53	20.5209	2.12838	6.73053	92.9597	1.65462	3.56478	7.68009	.220751
4.54	20.6116	2.13073	6.73795	93.5767	1.65584	3.56740	7.68573	.220264
4.55	20.7025	2.13307	6.74537	94.1964	1.65706	3.57002	7.69137	.219780
4.56	20.7936	2.13542	6.75278	94.8188	1.65827	3.57263	7.69700	.219298
4.57	20.8849	2.13776	6.76018	95.4440	1.65948	3.57524	7.70262	.218818
4.58	20.9764	2.14009	6.76757	96.0719	1.66069	3.57785	7.70824	.218341
4.59	21.0681	2.14243	6.77495	96.7026	1.66190	3.58045	7.71384	.217865
4.60	21.1600	2.14476	6.78233	97.3360	1.66310	3.58305	7.71944	.217391
4.61	21.2521	2.14709	6.78970	97.9722	1.66431	3.58564	7.72503	.216920
4.62	21.3444	2.14942	6.79706	98.6111	1.66551	3.58823	7.73061	.216450
4.63	21.4369	2.15174	6.80441	99.2528	1.66671	3.59082	7.73619	.215983
4.64	21.5296	2.15407	6.81175	99.8973	1.66791	3.59340	7.74175	.215517
4.65	21.6225	2.15639	6.81909	100.545	1.66911	3.59598	7.74731	.215054
4.66	21.7156	2.15870	6.82642	101.195	1.67030	3.59856	7.75286	.214592
4.67	21.8089	2.16102	6.83374	101.848	1.67150	3.60113	7.75840	.214133
4.68	21.9024	2.16333	6.84105	102.503	1.67269	3.60370	7.76394	.213675
4.69	21.9961	2.16564	6.84836	103.162	1.67388	3.60626	7.76946	.213220
4.70	22.0900	2.16795	6.85565	103.823	1.67507	3.60883	7.77498	.212766
4.71	22.1841	2.17025	6.86294	104.487	1.67626	3.61138	7.78049	.212314
4.72	22.2784	2.17256	6.87023	105.154	1.67744	3.61394	7.78599	.211864
4.73	22.3729	2.17486	6.87750	105.824	1.67863	3.61649	7.79149	.211416
4.74	22.4676	2.17715	6.88477	106.496	1.67981	3.61903	7.79697	.210970
4.75	22.5625	2.17945	6.89202	107.172	1.68099	3.62158	7.80245	.210526
4.76	22.6576	2.18174	6.89928	107.850	1.68217	3.62412	7.80793	.210084
4.77	22.7529	2.18403	6.90652	108.531	1.68334	3.62665	7.81339	.209644
4.78	22.8484	2.18632	6.91375	109.215	1.68452	3.62919	7.81885	.209205
4.79	22.9441	2.18861	6.92098	109.902	1.68569	3.63172	7.82429	.208768
4.80	23.0400	2.19089	6.92820	110.592	1.68687	3.63424	7.82974	.208333
4.81	23.1361	2.19317	6.93542	111.285	1.68804	3.63676	7.83517	.207900
4.82	23.2324	2.19545	6.94262	111.980	1.68920	3.63928	7.84059	.207469
4.83	23.3289	2.19773	6.94982	112.679	1.69037	3.64180	7.84601	.207039
4.84	23.4256	2.20000	6.95701	113.380	1.69154	3.64431	7.85142	.206612
4.85	23.5225	2.20227	6.96419	114.084	1.69270	3.64682	7.85683	.206186
4.86	23.6196	2.20454	6.97137	114.791	1.69386	3.64932	7.86222	.205761
4.87	23.7169	2.20681	6.97854	115.501	1.69503	3.65182	7.86761	.205339
4.88	23.8144	2.20907	6.98570	116.214	1.69619	3.65432	7.87299	.204918
4.89	23.9121	2.21133	6.99285	116.930	1.69734	3.65681	7.87837	.204499
4.90	24.0100	2.21359	7.00000	117.649	1.69850	3.65931	7.88374	.204082
4.91	24.1081	2.21585	7.00714	118.371	1.69965	3.66179	7.88909	.203666
4.92	24.2064	2.21811	7.01427	119.095	1.70081	3.66428	7.89445	.203252
4.93	24.3049	2.22036	7.02140	119.823	1.70196	3.66676	7.89979	.202840
4.94	24.4036	2.22261	7.02851	120.554	1.70311	3.66924	7.90513	.202429
4.95	24.5025	2.22486	7.03562	121.287	1.70426	3.67171	7.91046	.202020
4.96	24.6016	2.22711	7.04273	122.024	1.70540	3.67418	7.91578	.201613
4.97	24.7009	2.22935	7.04982	122.763	1.70655	3.67665	7.92110	.201207
4.98	24.8004	2.23159	7.05691	123.506	1.70769	3.67911	7.92641	.200803
4.99	24.9001	2.23383	7.06399	124.251	1.70884	3.68157	7.93171	.200401
5.00	25.0000	2.23607	7.07107	125.000	1.70998	3.68403	7.93701	.200000
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
5.00	25.0000	2.23607	7.07107	125.000	1.70998	3.68403	7.93701	.200000
5.01	25.1001	2.23830	7.07814	125.752	1.71112	3.68649	7.94229	.199601
5.02	25.2004	2.24054	7.08520	126.506	1.71225	3.68894	7.94757	.199203
5.03	25.3009	2.24277	7.09225	127.264	1.71339	3.69138	7.95285	.198807
5.04	25.4016	2.24499	7.09930	128.024	1.71452	3.69383	7.95811	.198413
5.05	25.5025	2.24722	7.10634	128.788	1.71566	3.69627	7.96337	.198020
5.06	25.6036	2.24944	7.11337	129.554	1.71679	3.69871	7.96863	.197628
5.07	25.7049	2.25167	7.12039	130.324	1.71792	3.70114	7.97387	.197239
5.08	25.8064	2.25389	7.12741	131.097	1.71905	3.70357	7.97911	.196850
5.09	25.9081	2.25610	7.13442	131.872	1.72017	3.70600	7.98434	.196464
5.10	26.0100	2.25832	7.14143	132.651	1.72130	3.70843	7.98957	.196078
5.11	26.1121	2.26053	7.14843	133.433	1.72242	3.71085	7.99479	.195695
5.12	26.2144	2.26274	7.15542	134.218	1.72355	3.71327	8.00000	.195312
5.13	26.3169	2.26495	7.16240	135.006	1.72467	3.71569	8.00520	.194932
5.14	26.4196	2.26716	7.16938	135.797	1.72579	3.71810	8.01040	.194553
5.15	26.5225	2.26936	7.17635	136.591	1.72691	3.72051	8.01559	.194175
5.16	26.6256	2.27156	7.18331	137.388	1.72802	3.72292	8.02078	.193798
5.17	26.7289	2.27376	7.19027	138.188	1.72914	3.72532	8.02596	.193424
5.18	26.8324	2.27596	7.19722	138.992	1.73025	3.72772	8.03113	.193050
5.19	26.9361	2.27816	7.20417	139.798	1.73137	3.73012	8.03629	.192678
5.20	27.0400	2.28035	7.21110	140.608	1.73248	3.73251	8.04145	.192308
5.21	27.1441	2.28254	7.21803	141.421	1.73359	3.73490	8.04660	.191939
5.22	27.2484	2.28473	7.22496	142.237	1.73470	3.73729	8.05175	.191571
5.23	27.3529	2.28692	7.23187	143.056	1.73580	3.73968	8.05689	.191205
5.24	27.4576	2.28910	7.23878	143.878	1.73691	3.74206	8.06202	.190840
5.25	27.5625	2.29129	7.24569	144.703	1.73801	3.74443	8.06714	.190476
5.26	27.6676	2.29347	7.25259	145.532	1.73912	3.74681	8.07226	.190114
5.27	27.7729	2.29565	7.25948	146.363	1.74022	3.74918	8.07737	.189753
5.28	27.8784	2.29783	7.26636	147.198	1.74132	3.75155	8.08248	.189394
5.29	27.9841	2.30000	7.27324	148.036	1.74242	3.75392	8.08758	.189036
5.30	28.0900	2.30217	7.28011	148.877	1.74351	3.75629	8.09267	.188679
5.31	28.1961	2.30434	7.28697	149.721	1.74461	3.75865	8.09776	.188324
5.32	28.3024	2.30651	7.29383	150.569	1.74570	3.76101	8.10284	.187970
5.33	28.4089	2.30868	7.30068	151.419	1.74680	3.76336	8.10791	.187617
5.34	28.5156	2.31084	7.30753	152.273	1.74789	3.76571	8.11298	.187266
5.35	28.6225	2.31301	7.31437	153.130	1.74898	3.76806	8.11804	.186916
5.36	28.7296	2.31517	7.32120	153.991	1.75007	3.77041	8.12310	.186567
5.37	28.8369	2.31733	7.32803	154.854	1.75116	3.77275	8.12814	.186220
5.38	28.9444	2.31948	7.33485	155.721	1.75224	3.77509	8.13319	.185874
5.39	29.0521	2.32164	7.34166	156.591	1.75333	3.77743	8.13822	.185529
5.40	29.1600	2.32379	7.34847	157.464	1.75441	3.77976	8.14325	.185185
5.41	29.2681	2.32594	7.35527	158.340	1.75549	3.78209	8.14828	.184843
5.42	29.3764	2.32809	7.36206	159.220	1.75657	3.78442	8.15329	.184502
5.43	29.4849	2.33024	7.36885	160.103	1.75765	3.78675	8.15831	.184162
5.44	29.5936	2.33238	7.37564	160.989	1.75873	3.78907	8.16331	.183824
5.45	29.7025	2.33452	7.38241	161.879	1.75981	3.79139	8.16831	.183486
5.46	29.8116	2.33666	7.38918	162.771	1.76088	3.79371	8.17330	.183150
5.47	29.9209	2.33880	7.39594	163.667	1.76196	3.79603	8.17829	.182815
5.48	30.0304	2.34094	7.40270	164.567	1.76303	3.79834	8.18327	.182482
5.49	30.1401	2.34307	7.40945	165.469	1.76410	3.80065	8.18824	.182149
5.50	30.2500	2.34521	7.41620	166.375	1.76517	3.80295	8.19321	.181818
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
5.50	30.2500	2.34521	7.41620	166.375	1.76517	3.80295	8.19321	.181818
5.51	30.3601	2.34734	7.42294	167.284	1.76624	3.80526	8.19818	.181488
5.52	30.4704	2.34947	7.42967	168.197	1.76731	3.80756	8.20313	.181159
5.53	30.5809	2.35160	7.43640	169.112	1.76838	3.80985	8.20808	.180832
5.54	30.6916	2.35372	7.44312	170.031	1.76944	3.81215	8.21303	.180505
5.55	30.8025	2.35584	7.44983	170.954	1.77051	3.81444	8.21797	.180180
5.56	30.9136	2.35797	7.45654	171.880	1.77157	3.81673	8.22290	.179856
5.57	31.0249	2.36008	7.46324	172.809	1.77263	3.81902	8.22783	.179533
5.58	31.1364	2.36220	7.46994	173.741	1.77369	3.82130	8.23275	.179211
5.59	31.2481	2.36432	7.47663	174.677	1.77475	3.82358	8.23766	.178891
5.60	31.3600	2.36643	7.48331	175.616	1.77581	3.82586	8.24257	.178571
5.61	31.4721	2.36854	7.48999	176.558	1.77686	3.82814	8.24747	.178253
5.62	31.5844	2.37065	7.49667	177.504	1.77792	3.83041	8.25237	.177936
5.63	31.6969	2.37276	7.50333	178.454	1.77897	3.83268	8.25726	.177620
5.64	31.8096	2.37487	7.50999	179.406	1.78003	3.83495	8.26215	.177305
5.65	31.9225	2.37697	7.51665	180.362	1.78108	3.83722	8.26703	.176991
5.66	32.0356	2.37908	7.52330	181.321	1.78213	3.83948	8.27190	.176678
5.67	32.1489	2.38118	7.52994	182.284	1.78318	3.84174	8.27677	.176367
5.68	32.2624	2.38328	7.53658	183.250	1.78422	3.84399	8.28164	.176056
5.69	32.3761	2.38537	7.54321	184.220	1.78527	3.84625	8.28649	.175747
5.70	32.4900	2.38747	7.54983	185.193	1.78632	3.84850	8.29134	.175439
5.71	32.6041	2.38956	7.55645	186.169	1.78736	3.85075	8.29619	.175131
5.72	32.7184	2.39165	7.56307	187.149	1.78840	3.85300	8.30103	.174825
5.73	32.8329	2.39374	7.56968	188.133	1.78944	3.85524	8.30587	.174520
5.74	32.9476	2.39583	7.57628	189.119	1.79048	3.85748	8.31069	.174216
5.75	33.0625	2.39792	7.58288	190.109	1.79152	3.85972	8.31552	.173913
5.76	33.1776	2.40000	7.58947	191.103	1.79256	3.86196	8.32034	.173611
5.77	33.2929	2.40208	7.59605	192.100	1.79360	3.86419	8.32515	.173310
5.78	33.4084	2.40416	7.60263	193.101	1.79463	3.86642	8.32995	.173010
5.79	33.5241	2.40624	7.60920	194.105	1.79567	3.86865	8.33476	.172712
5.80	33.6400	2.40832	7.61577	195.112	1.79670	3.87088	8.33955	.172414
5.81	33.7561	2.41039	7.62234	196.123	1.79773	3.87310	8.34434	.172117
5.82	33.8724	2.41247	7.62889	197.137	1.79876	3.87532	8.34913	.171821
5.83	33.9889	2.41454	7.63544	198.155	1.79979	3.87754	8.35390	.171527
5.84	34.1056	2.41661	7.64199	199.177	1.80082	3.87975	8.35868	.171233
5.85	34.2225	2.41868	7.64853	200.202	1.80185	3.88197	8.36345	.170940
5.86	34.3396	2.42074	7.65506	201.230	1.80288	3.88418	8.36821	.170649
5.87	34.4569	2.42281	7.66159	202.262	1.80390	3.88639	8.37297	.170358
5.88	34.5744	2.42487	7.66812	203.297	1.80492	3.88859	8.37772	.170068
5.89	34.6921	2.42693	7.67463	204.336	1.80595	3.89080	8.38247	.169779
5.90	34.8100	2.42899	7.68115	205.379	1.80697	3.89300	8.38721	.169492
5.91	34.9281	2.43105	7.68765	206.425	1.80799	3.89519	8.39194	.169205
5.92	35.0464	2.43311	7.69415	207.475	1.80901	3.89739	8.39667	.168919
5.93	35.1649	2.43516	7.70065	208.528	1.81003	3.89958	8.40140	.168634
5.94	35.2836	2.43721	7.70714	209.585	1.81104	3.90177	8.40612	.168350
5.95	35.4025	2.43926	7.71362	210.645	1.81206	3.90396	8.41083	.168067
5.96	35.5216	2.44131	7.72010	211.709	1.81307	3.90615	8.41554	.167785
5.97	35.6409	2.44336	7.72658	212.776	1.81409	3.90833	8.42025	.167504
5.98	35.7604	2.44540	7.73305	213.847	1.81510	3.91051	8.42494	.167224
5.99	35.8801	2.44745	7.73951	214.922	1.81611	3.91269	8.42964	.166945
6.00	36.0000	2.44949	7.74597	216.000	1.81712	3.91487	8.43433	.166667
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
6.00	36.0000	2.44949	7.74597	216.000	1.81712	3.91487	8.43433	.166667
6.01	36.1201	2.45153	7.75242	217.082	1.81813	3.91704	8.43901	.166389
6.02	36.2404	2.45357	7.75887	218.167	1.81914	3.91921	8.44369	.166113
6.03	36.3609	2.45561	7.76531	219.256	1.82014	3.92138	8.44836	.165837
6.04	36.4816	2.45764	7.77174	220.349	1.82115	3.92355	8.45303	.165563
6.05	36.6025	2.45967	7.77817	221.445	1.82215	3.92571	8.45769	.165289
6.06	36.7236	2.46171	7.78460	222.545	1.82316	3.92787	8.46235	.165017
6.07	36.8449	2.46374	7.79102	223.649	1.82416	3.93003	8.46700	.164745
6.08	36.9664	2.46577	7.79744	224.756	1.82516	3.93219	8.47165	.164474
6.09	37.0881	2.46779	7.80385	225.867	1.82616	3.93434	8.47629	.164204
6.10	37.2100	2.46982	7.81025	226.981	1.82716	3.93650	8.48093	.163934
6.11	37.3321	2.47184	7.81665	228.099	1.82816	3.93865	8.48556	.163666
6.12	37.4544	2.47386	7.82304	229.221	1.82915	3.94079	8.49018	.163399
6.13	37.5769	2.47588	7.82943	230.346	1.83015	3.94294	8.49481	.163132
6.14	37.6996	2.47790	7.83582	231.476	1.83115	3.94508	8.49942	.162866
6.15	37.8225	2.47992	7.84219	232.608	1.83214	3.94722	8.50403	.162602
6.16	37.9456	2.48193	7.84857	233.745	1.83313	3.94936	8.50864	.162338
6.17	38.0689	2.48395	7.85493	234.885	1.83412	3.95150	8.51324	.162075
6.18	38.1924	2.48596	7.86130	236.029	1.83511	3.95363	8.51784	.161812
6.19	38.3161	2.48797	7.86766	237.177	1.83610	3.95576	8.52243	.161551
6.20	38.4400	2.48998	7.87401	238.328	1.83709	3.95789	8.52702	.161290
6.21	38.5641	2.49199	7.88036	239.483	1.83808	3.96002	8.53160	.161031
6.22	38.6884	2.49399	7.88670	240.642	1.83906	3.96214	8.53618	.160772
6.23	38.8129	2.49600	7.89303	241.804	1.84005	3.96427	8.54075	.160514
6.24	38.9376	2.49800	7.89937	242.971	1.84103	3.96638	8.54532	.160256
6.25	39.0625	2.50000	7.90569	244.141	1.84202	3.96850	8.54988	.160000
6.26	39.1876	2.50200	7.91202	245.314	1.84300	3.97062	8.55444	.159744
6.27	39.3129	2.50400	7.91833	246.492	1.84398	3.97273	8.55899	.159490
6.28	39.4384	2.50599	7.92465	247.673	1.84496	3.97484	8.56354	.159236
6.29	39.5641	2.50799	7.93095	248.858	1.84594	3.97695	8.56808	.158983
6.30	39.6900	2.50998	7.93725	250.047	1.84691	3.97906	8.57262	.158730
6.31	39.8161	2.51197	7.94355	251.240	1.84789	3.98116	8.57715	.158479
6.32	39.9424	2.51396	7.94984	252.436	1.84887	3.98326	8.58168	.158228
6.33	40.0689	2.51595	7.95613	253.636	1.84984	3.98536	8.58620	.157978
6.34	40.1956	2.51794	7.96241	254.840	1.85082	3.98746	8.59072	.157729
6.35	40.3225	2.51992	7.96869	256.048	1.85179	3.98956	8.59524	.157480
6.36	40.4496	2.52190	7.97496	257.259	1.85276	3.99165	8.59975	.157233
6.37	40.5769	2.52389	7.98123	258.475	1.85373	3.99374	8.60425	.156986
6.38	40.7044	2.52587	7.98749	259.694	1.85470	3.99583	8.60875	.156740
6.39	40.8321	2.52784	7.99375	260.917	1.85567	3.99792	8.61325	.156495
6.40	40.9600	2.52982	8.00000	262.144	1.85664	4.00000	8.61774	.156250
6.41	41.0881	2.53180	8.00625	263.375	1.85760	4.00208	8.62222	.156006
6.42	41.2164	2.53377	8.01249	264.609	1.85857	4.00416	8.62671	.155763
6.43	41.3449	2.53574	8.01873	265.848	1.85953	4.00624	8.63118	.155521
6.44	41.4736	2.53772	8.02496	267.090	1.86050	4.00832	8.63566	.155280
6.45	41.6025	2.53969	8.03119	268.336	1.86146	4.01039	8.64012	.155039
6.46	41.7316	2.54165	8.03741	269.586	1.86242	4.01246	8.64459	.154799
6.47	41.8609	2.54362	8.04363	270.840	1.86338	4.01453	8.64904	.154560
6.48	41.9904	2.54558	8.04984	272.098	1.86434	4.01660	8.65350	.154321
6.49	42.1201	2.54755	8.05605	273.359	1.86530	4.01866	8.65795	.154083
6.50	42.2500	2.54951	8.06226	274.625	1.86626	4.02073	8.66239	.153846
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10\,n}$	$\sqrt[3]{100\,n}$	$1/n$
6.50	42.2500	2.54951	8.06226	274.625	1.86626	4.02073	8.66239	.153846
6.51	42.3801	2.55147	8.06846	275.894	1.86721	4.02279	8.66683	.153610
6.52	42.5104	2.55343	8.07465	277.168	1.86817	4.02485	8.67127	.153374
6.53	42.6409	2.55539	8.08084	278.445	1.86912	4.02690	8.67570	.153139
6.54	42.7716	2.55734	8.08703	279.726	1.87008	4.02896	8.68012	.152905
6.55	42.9025	2.55930	8.09321	281.011	1.87103	4.03101	8.68455	.152672
6.56	43.0336	2.56125	8.09938	282.300	1.87198	4.03306	8.68896	.152439
6.57	43.1649	2.56320	8.10555	283.593	1.87293	4.03511	8.69338	.152207
6.58	43.2964	2.56515	8.11172	284.890	1.87388	4.03715	8.69778	.151976
6.59	43.4281	2.56710	8.11788	286.191	1.87483	4.03920	8.70219	.151745
6.60	43.5600	2.56905	8.12404	287.496	1.87578	4.04124	8.70659	.151515
6.61	43.6921	2.57099	8.13019	288.805	1.87672	4.04328	8.71098	.151286
6.62	43.8244	2.57294	8.13634	290.118	1.87767	4.04532	8.71537	.151057
6.63	43.9569	2.57488	8.14248	291.434	1.87862	4.04735	8.71976	.150830
6.64	44.0896	2.57682	8.14862	292.755	1.87956	4.04939	8.72414	.150602
6.65	44.2225	2.57876	8.15475	294.080	1.88050	4.05142	8.72852	.150376
6.66	44.3556	2.58070	8.16088	295.408	1.88144	4.05345	8.73289	.150150
6.67	44.4889	2.58263	8.16701	296.741	1.88239	4.05548	8.73726	.149925
6.68	44.6224	2.58457	8.17313	298.078	1.88333	4.05750	8.74162	.149701
6.69	44.7561	2.58650	8.17924	299.418	1.88427	4.05953	8.74598	.149477
6.70	44.8900	2.58844	8.18535	300.763	1.88520	4.06155	8.75034	.149254
6.71	45.0241	2.59037	8.19146	302.112	1.88614	4.06357	8.75469	.149031
6.72	45.1584	2.59230	8.19756	303.464	1.88708	4.06559	8.75904	.148810
6.73	45.2929	2.59422	8.20366	304.821	1.88801	4.06760	8.76338	.148588
6.74	45.4276	2.59615	8.20975	306.182	1.88895	4.06961	8.76772	.148368
6.75	45.5625	2.59808	8.21584	307.547	1.88988	4.07163	8.77205	.148148
6.76	45.6976	2.60000	8.22192	308.916	1.89081	4.07364	8.77638	.147929
6.77	45.8329	2.60192	8.22800	310.289	1.89175	4.07564	8.78071	.147710
6.78	45.9684	2.60384	8.23408	311.666	1.89268	4.07765	8.78503	.147493
6.79	46.1041	2.60576	8.24015	313.047	1.89361	4.07965	8.78935	.147275
6.80	46.2400	2.60768	8.24621	314.432	1.89454	4.08166	8.79366	.147059
6.81	46.3761	2.60960	8.25227	315.821	1.89546	4.08365	8.79797	.146843
6.82	46.5124	2.61151	8.25833	317.215	1.89639	4.08565	8.80227	.146628
6.83	46.6489	2.61343	8.26438	318.612	1.89732	4.08765	8.80657	.146413
6.84	46.7856	2.61534	8.27043	320.014	1.89824	4.08964	8.81087	.146199
6.85	46.9225	2.61725	8.27647	321.419	1.89917	4.09163	8.81516	.145985
6.86	47.0596	2.61916	8.28251	322.829	1.90009	4.09362	8.81945	.145773
6.87	47.1969	2.62107	8.28855	324.243	1.90102	4.09561	8.82373	.145560
6.88	47.3344	2.62298	8.29458	325.661	1.90194	4.09760	8.82801	.145349
6.89	47.4721	2.62488	8.30060	327.083	1.90286	4.09958	8.83228	.145138
6.90	47.6100	2.62679	8.30662	328.509	1.90378	4.10157	8.83656	.144928
6.91	47.7481	2.62869	8.31264	329.939	1.90470	4.10355	8.84082	.144718
6.92	47.8864	2.63059	8.31865	331.374	1.90562	4.10552	8.84509	.144509
6.93	48.0249	2.63249	8.32466	332.813	1.90653	4.10750	8.84934	.144300
6.94	48.1636	2.63439	8.33067	334.255	1.90745	4.10948	8.85360	.144092
6.95	48.3025	2.63629	8.33667	335.702	1.90837	4.11145	8.85785	.143885
6.96	48.4416	2.63818	8.34266	337.154	1.90928	4.11342	8.86210	.143678
6.97	48.5809	2.64008	8.34865	338.609	1.91019	4.11539	8.86634	.143472
6.98	48.7204	2.64197	8.35464	340.068	1.91111	4.11736	8.87058	.143266
6.99	48.8601	2.64386	8.36062	341.532	1.91202	4.11932	8.87481	.143062
7.00	49.0000	2.64575	8.36660	343.000	1.91293	4.12129	8.87904	.142857
n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10\,n}$	$\sqrt[3]{100\,n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10\,n}$	$\sqrt[3]{100\,n}$	$1/n$
7.00	49.0000	2.64575	8.36660	343.000	1.91293	4.12129	8.87904	.142857
7.01	49.1401	2.64764	8.37257	344.472	1.91384	4.12325	8.88327	.142653
7.02	49.2804	2.64953	8.37854	345.948	1.91475	4.12521	8.88749	.142450
7.03	49.4209	2.65141	8.38451	347.429	1.91566	4.12716	8.89171	.142248
7.04	49.5616	2.65330	8.39047	348.914	1.91657	4.12912	8.89592	.142045
7.05	49.7025	2.65518	8.39643	350.403	1.91747	4.13107	8.90013	.141844
7.06	49.8436	2.65707	8.40238	351.896	1.91838	4.13303	8.90434	.141643
7.07	49.9849	2.65895	8.40833	353.393	1.91929	4.13498	8.90854	.141443
7.08	50.1264	2.66083	8.41427	354.895	1.92019	4.13693	8.91274	.141243
7.09	50.2681	2.66271	8.42021	356.401	1.92109	4.13887	8.91693	.141044
7.10	50.4100	2.66458	8.42615	357.911	1.92200	4.14082	8.92112	.140845
7.11	50.5521	2.66646	8.43208	359.425	1.92290	4.14276	8.92531	.140647
7.12	50.6944	2.66833	8.43801	360.944	1.92380	4.14470	8.92949	.140449
7.13	50.8369	2.67021	8.44393	362.467	1.92470	4.14664	8.93367	.140252
7.14	50.9796	2.67208	8.44985	363.994	1.92560	4.14858	8.93784	.140056
7.15	51.1225	2.67395	8.45577	365.526	1.92650	4.15052	8.94201	.139860
7.16	51.2656	2.67582	8.46168	367.062	1.92740	4.15245	8.94618	.139665
7.17	51.4089	2.67769	8.46759	368.602	1.92829	4.15438	8.95034	.139470
7.18	51.5524	2.67955	8.47349	370.146	1.92919	4.15631	8.95450	.139276
7.19	51.6961	2.68142	8.47939	371.695	1.93008	4.15824	8.95866	.139082
7.20	51.8400	2.68328	8.48528	373.248	1.93098	4.16017	8.96281	.138889
7.21	51.9841	2.68514	8.49117	374.805	1.93187	4.16209	8.96696	.138696
7.22	52.1284	2.68701	8.49706	376.367	1.93277	4.16402	8.97110	.138504
7.23	52.2729	2.68887	8.50294	377.933	1.93366	4.16594	8.97524	.138313
7.24	52.4176	2.69072	8.50882	379.503	1.93455	4.16786	8.97938	.138122
7.25	52.5625	2.69258	8.51469	381.078	1.93544	4.16978	8.98351	.137931
7.26	52.7076	2.69444	8.52056	382.657	1.93633	4.17169	8.98764	.137741
7.27	52.8529	2.69629	8.52643	384.241	1.93722	4.17361	8.99176	.137552
7.28	52.9984	2.69815	8.53229	385.828	1.93810	4.17552	8.99588	.137363
7.29	53.1441	2.70000	8.53815	387.420	1.93899	4.17743	9.00000	.137174
7.30	53.2900	2.70185	8.54400	389.017	1.93988	4.17934	9.00411	.136986
7.31	53.4361	2.70370	8.54985	390.618	1.94076	4.18125	9.00822	.136799
7.32	53.5824	2.70555	8.55570	392.223	1.94165	4.18315	9.01233	.136612
7.33	53.7289	2.70740	8.56154	393.833	1.94253	4.18506	9.01643	.136426
7.34	53.8756	2.70924	8.56738	395.447	1.94341	4.18696	9.02053	.136240
7.35	54.0225	2.71109	8.57321	397.065	1.94430	4.18886	9.02462	.136054
7.36	54.1696	2.71293	8.57904	398.688	1.94518	4.19076	9.02871	.135870
7.37	54.3169	2.71477	8.58487	400.316	1.94606	4.19266	9.03280	.135685
7.38	54.4644	2.71662	8.59069	401.947	1.94694	4.19455	9.03689	.135501
7.39	54.6121	2.71846	8.59651	403.583	1.94782	4.19644	9.04097	.135318
7.40	54.7600	2.72029	8.60233	405.224	1.94870	4.19834	9.04504	.135135
7.41	54.9081	2.72213	8.60814	406.869	1.94957	4.20023	9.04911	.134953
7.42	55.0564	2.72397	8.61394	408.518	1.95045	4.20212	9.05318	.134771
7.43	55.2049	2.72580	8.61974	410.172	1.95132	4.20400	9.05725	.134590
7.44	55.3536	2.72764	8.62554	411.831	1.95220	4.20589	9.06131	.134409
7.45	55.5025	2.72947	8.63134	413.494	1.95307	4.20777	9.06537	.134228
7.46	55.6516	2.73130	8.63713	415.161	1.95395	4.20965	9.06942	.134048
7.47	55.8009	2.73313	8.64292	416.833	1.95482	4.21153	9.07347	.133869
7.48	55.9504	2.73496	8.64870	418.509	1.95569	4.21341	9.07752	.133690
7.49	56.1001	2.73679	8.65448	420.190	1.95656	4.21529	9.08156	.133511
7.50	56.2500	2.73861	8.66025	421.875	1.95743	4.21716	9.08560	.133333
n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10\,n}$	$\sqrt[3]{100\,n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
7.50	56.2500	2.73861	8.66025	421.875	1.95743	4.21716	9.08560	.133333
7.51	56.4001	2.74044	8.66603	423.565	1.95830	4.21904	9.08964	.133156
7.52	56.5504	2.74226	8.67179	425.259	1.95917	4.22091	9.09367	.132979
7.53	56.7009	2.74408	8.67756	426.958	1.96004	4.22278	9.09770	.132802
7.54	56.8516	2.74591	8.68332	428.661	1.96091	4.22465	9.10173	.132626
7.55	57.0025	2.74773	8.68907	430.369	1.96177	4.22651	9.10575	.132450
7.56	57.1536	2.74955	8.69483	432.081	1.96264	4.22838	9.10977	.132275
7.57	57.3049	2.75136	8.70057	433.798	1.96350	4.23024	9.11378	.132100
7.58	57.4564	2.75318	8.70632	435.520	1.96437	4.23210	9.11779	.131926
7.59	57.6081	2.75500	8.71206	437.245	1.96523	4.23396	9.12180	.131752
7.60	57.7600	2.75681	8.71780	438.976	1.96610	4.23582	9.12581	.131579
7.61	57.9121	2.75862	8.72353	440.711	1.96696	4.23768	9.12981	.131406
7.62	58.0644	2.76043	8.72926	442.451	1.96782	4.23954	9.13380	.131234
7.63	58.2169	2.76225	8.73499	444.195	1.96868	4.24139	9.13780	.131062
7.64	58.3696	2.76405	8.74071	445.944	1.96954	4.24324	9.14179	.130890
7.65	58.5225	2.76586	8.74643	447.697	1.97040	4.24509	9.14577	.130719
7.66	58.6756	2.76767	8.75214	449.455	1.97126	4.24694	9.14976	.130548
7.67	58.8289	2.76948	8.75785	451.218	1.97211	4.24879	9.15374	.130378
7.68	58.9824	2.77128	8.76356	452.985	1.97297	4.25063	9.15771	.130208
7.69	59.1361	2.77308	8.76926	454.757	1.97383	4.25248	9.16169	.130039
7.70	59.2900	2.77489	8.77496	456.533	1.97468	4.25432	9.16566	.129870
7.71	59.4441	2.77669	8.78066	458.314	1.97554	4.25616	9.16962	.129702
7.72	59.5984	2.77849	8.78635	460.100	1.97639	4.25800	9.17359	.129534
7.73	59.7529	2.78029	8.79204	461.890	1.97724	4.25984	9.17754	.129366
7.74	59.9076	2.78209	8.79773	463.685	1.97809	4.26167	9.18150	.129199
7.75	60.0625	2.78388	8.80341	465.484	1.97895	4.26351	9.18545	.129032
7.76	60.2176	2.78568	8.80909	467.289	1.97980	4.26534	9.18940	.128866
7.77	60.3729	2.78747	8.81476	469.097	1.98065	4.26717	9.19335	.128700
7.78	60.5284	2.78927	8.82043	470.911	1.98150	4.26900	9.19729	.128535
7.79	60.6841	2.79106	8.82610	472.729	1.98234	4.27083	9.20123	.128370
7.80	60.8400	2.79285	8.83176	474.552	1.98319	4.27266	9.20516	.128205
7.81	60.9961	2.79464	8.83742	476.380	1.98404	4.27448	9.20910	.128041
7.82	61.1524	2.79643	8.84308	478.212	1.98489	4.27631	9.21302	.127877
7.83	61.3089	2.79821	8.84873	480.049	1.98573	4.27813	9.21695	.127714
7.84	61.4656	2.80000	8.85438	481.890	1.98658	4.27995	9.22087	.127551
7.85	61.6225	2.80179	8.86002	483.737	1.98742	4.28177	9.22479	.127389
7.86	61.7796	2.80357	8.86566	485.588	1.98826	4.28359	9.22871	.127226
7.87	61.9369	2.80535	8.87130	487.443	1.98911	4.28540	9.23262	.127065
7.88	62.0944	2.80713	8.87694	489.304	1.98995	4.28722	9.23653	.126904
7.89	62.2521	2.80891	8.88257	491.169	1.99079	4.28903	9.24043	.126743
7.90	62.4100	2.81069	8.88819	493.039	1.99163	4.29084	9.24434	.126582
7.91	62.5681	2.81247	8.89382	494.914	1.99247	4.29265	9.24823	.126422
7.92	62.7264	2.81425	8.89944	496.793	1.99331	4.29446	9.25213	.126263
7.93	62.8849	2.81603	8.90505	498.677	1.99415	4.29627	9.25602	.126103
7.94	63.0436	2.81780	8.91067	500.566	1.99499	4.29807	9.25991	.125945
7.95	63.2025	2.81957	8.91628	502.460	1.99582	4.29987	9.26380	.125786
7.96	63.3616	2.82135	8.92188	504.358	1.99666	4.30168	9.26768	.125628
7.97	63.5209	2.82312	8.92749	506.262	1.99750	4.30348	9.27156	.125471
7.98	63.6804	2.82489	8.93308	508.170	1.99833	4.30528	9.27544	.125313
7.99	63.8401	2.82666	8.93868	510.082	1.99917	4.30707	9.27931	.125156
8.00	64.0000	2.82843	8.94427	512.000	2.00000	4.30887	9.28318	.125000
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

n	n^2	$\sqrt{n}$		n^3	$\sqrt[3]{n}$			$1/n$
8.00	64.0000	2.82843	8.94427	512.000	2.00000	4.30887	9.28318	.125000
8.01	64.1601	2.83019	8.94986	513.922	2.00083	4.31066	9.28704	.124844
8.02	64.3204	2.83196	8.95545	515.850	2.00167	4.31246	9.29091	.124688
8.03	64.4809	2.83373	8.96103	517.782	2.00250	4.31425	9.29477	.124533
8.04	64.6416	2.83549	8.96660	519.718	2.00333	4.31604	9.29862	.124378
8.05	64.8025	2.83725	8.97218	521.660	2.00416	4.31783	9.30248	.124224
8.06	64.9636	2.83901	8.97775	523.607	2.00499	4.31961	9.30633	.124069
8.07	65.1249	2.84077	8.98332	525.558	2.00582	4.32140	9.31018	.123916
8.08	65.2864	2.84253	8.98888	527.514	2.00664	4.32318	9.31402	.123762
8.09	65.4481	2.84429	8.99444	529.475	2.00747	4.32497	9.31786	.123609
8.10	65.6100	2.84605	9.00000	531.441	2.00830	4.32675	9.32170	.123457
8.11	65.7721	2.84781	9.00555	533.412	2.00912	4.32853	9.32553	.123305
8.12	65.9344	2.84956	9.01110	535.387	2.00995	4.33031	9.32936	.123153
8.13	66.0969	2.85132	9.01665	537.368	2.01078	4.33208	9.33319	.123001
8.14	66.2596	2.85307	9.02219	539.353	2.01160	4.33386	9.33702	.122850
8.15	66.4225	2.85482	9.02774	541.343	2.01242	4.33563	9.34084	.122699
8.16	66.5856	2.85657	9.03327	543.338	2.01325	4.33741	9.34466	.122549
8.17	66.7489	2.85832	9.03881	545.339	2.01407	4.33918	9.34847	.122399
8.18	66.9124	2.86007	9.04434	547.343	2.01489	4.34095	9.35229	.122249
8.19	67.0761	2.86182	9.04986	549.353	2.01571	4.34271	9.35610	.122100
8.20	67.2400	2.86356	9.05539	551.368	2.01653	4.34448	9.35990	.121951
8.21	67.4041	2.86531	9.06091	553.388	2.01735	4.34625	9.36370	.121803
8.22	67.5684	2.86705	9.06642	555.412	2.01817	4.34801	9.36751	.121655
8.23	67.7329	2.86880	9.07193	557.442	2.01899	4.34977	9.37130	.121507
8.24	67.8976	2.87054	9.07744	559.476	2.01980	4.35153	9.37510	.121359
8.25	68.0625	2.87228	9.08295	561.516	2.02062	4.35329	9.37889	.121212
8.26	68.2276	2.87402	9.08845	563.560	2.02144	4.35505	9.38268	.121065
8.27	68.3929	2.87576	9.09395	565.609	2.02225	4.35681	9.38646	.120919
	68.5584	2.87750	9.09945	567.664	2.02307	4.35856	9.39024	.120773
	68.7241	2.87924	9.10494	569.723	2.02388	4.36032	9.39402	.120627
	68.8900	2.88097	9.11043	571.787	2.02469	4.36207	9.39780	.120482
	69.0561	2.88271	9.11592	573.856	2.02551	4.36382	9.40157	.120337
	69.2224	2.88444	9.12140	575.930	2.02632	4.36557	9.40534	.120192
8.33	69.3889	2.88617	9.12688	578.010	2.02713	4.36732	9.40911	.120048
8.34	69.5556	2.88791	9.13236	580.094	2.02794	4.36907	9.41287	.119904
8.35	69.7225	2.88964	9.13783	582.183	2.02875	4.37081	9.41663	.119760
8.36	69.8896	2.89137	9.14330	584.277	2.02956	4.37256	9.42039	.119617
8.37	70.0569	2.89310	9.14877	586.376	2.03037	4.37430	9.42414	.119474
8.38	70.2244	2.89482	9.15423	588.480	2.03118	4.37604	9.42789	.119332
8.39	70.3921	2.89655	9.15969	590.590	2.03199	4.37778	9.43164	.119190
8.40	70.5600	2.89828	9.16515	592.704	2.03279	4.37952	9.43539	.119048
8.41	70.7281	2.90000	9.17061	594.823	2.03360	4.38126	9.43913	.118906
8.42	70.8964	2.90172	9.17606	596.948	2.03440	4.38299	9.44287	.118765
8.43	71.0649	2.90345	9.18150	599.077	2.03521	4.38473	9.44661	.118624
8.44	71.2336	2.90517	9.18695	601.212	2.03601	4.38646	9.45034	.118483
8.45	71.4025	2.90689	9.19239	603.351	2.03682	4.38819	9.45407	.118343
8.46	71.5716	2.90861	9.19783	605.496	2.03762	4.38992	9.45780	.118203
8.47	71.7409	2.91033	9.20326	607.645	2.03842	4.39165	9.46152	.118064
8.48	71.9104	2.91204	9.20869	609.800	2.03923	4.39338	9.46525	.117925
8.49	72.0801	2.91376	9.21412	611.960	2.04003	4.39510	9.46897	.117786
8.50	72.2500	2.91548	9.21954	614.125	2.04083	4.39683	9.47268	.117647
n	n^2	$\sqrt{n}$			$\sqrt[3]{n}$	$\sqrt[3]{10\,n}$		$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
8.50	72.2500	2.91548	9.21954	614.125	2.04083	4.39683	9.47268	.117647
8.51	72.4201	2.91719	9.22497	616.295	2.04163	4.39855	9.47640	.117509
8.52	72.5904	2.91890	9.23038	618.470	2.04243	4.40028	9.48011	.117371
8.53	72.7609	2.92062	9.23580	620.650	2.04323	4.40200	9.48381	.117233
8.54	72.9316	2.92233	9.24121	622.836	2.04402	4.40372	9.48752	.117096
8.55	73.1025	2.92404	9.24662	625.026	2.04482	4.40543	9.49122	.116959
8.56	73.2736	2.92575	9.25203	627.222	2.04562	4.40715	9.49492	.116822
8.57	73.4449	2.92746	9.25743	629.423	2.04641	4.40887	9.49861	.116686
8.58	73.6164	2.92916	9.26283	631.629	2.04721	4.41058	9.50231	.116550
8.59	73.7881	2.93087	9.26823	633.840	2.04801	4.41229	9.50600	.116414
8.60	73.9600	2.93258	9.27362	636.056	2.04880	4.41400	9.50969	.116279
8.61	74.1321	2.93428	9.27901	638.277	2.04959	4.41571	9.51337	.116144
8.62	74.3044	2.93598	9.28440	640.504	2.05039	4.41742	9.51705	.116009
8.63	74.4769	2.93769	9.28978	642.736	2.05118	4.41913	9.52073	.115875
8.64	74.6496	2.93939	9.29516	644.973	2.05197	4.42084	9.52441	.115741
8.65	74.8225	2.94109	9.30054	647.215	2.05276	4.42254	9.52808	.115607
8.66	74.9956	2.94279	9.30591	649.462	2.05355	4.42425	9.53175	.115473
8.67	75.1689	2.94449	9.31128	651.714	2.05434	4.42595	9.53542	.115340
8.68	75.3424	2.94618	9.31665	653.972	2.05513	4.42765	9.53908	.115207
8.69	75.5161	2.94788	9.32202	656.235	2.05592	4.42935	9.54274	.115075
8.70	75.6900	2.94958	9.32738	658.503	2.05671	4.43105	9.54640	.114943
8.71	75.8641	2.95127	9.33274	660.776	2.05750	4.43274	9.55006	.114811
8.72	76.0384	2.95296	9.33809	663.055	2.05828	4.43444	9.55371	.114679
8.73	76.2129	2.95466	9.34345	665.339	2.05907	4.43613	9.55736	.114548
8.74	76.3876	2.95635	9.34880	667.628	2.05986	4.43783	9.56101	.114416
8.75	76.5625	2.95804	9.35414	669.922	2.06064	4.43952	9.56466	.114286
8.76	76.7376	2.95973	9.35949	672.221	2.06143	4.44121	9.56830	.114155
8.77	76.9129	2.96142	9.36483	674.526	2.06221	4.44290	9.57194	.114025
8.78	77.0884	2.96311	9.37017	676.836	2.06299	4.44459	9.57557	.113895
8.79	77.2641	2.96479	9.37550	679.151	2.06378	4.44627	9.57921	.113766
8.80	77.4400	2.96648	9.38083	681.472	2.06456	4.44796	9.58284	.113636
8.81	77.6161	2.96816	9.38616	683.798	2.06534	4.44964	9.58647	.113507
8.82	77.7924	2.96985	9.39149	686.129	2.06612	4.45133	9.59009	.113379
8.83	77.9689	2.97153	9.39681	688.465	2.06690	4.45301	9.59372	.113250
8.84	78.1456	2.97321	9.40213	690.807	2.06768	4.45469	9.59734	.113122
8.85	78.3225	2.97489	9.40744	693.154	2.06846	4.45637	9.60095	.112994
8.86	78.4996	2.97658	9.41276	695.506	2.06924	4.45805	9.60457	.112867
8.87	78.6769	2.97825	9.41807	697.864	2.07002	4.45972	9.60818	.112740
8.88	78.8544	2.97993	9.42338	700.227	2.07080	4.46140	9.61179	.112613
8.89	79.0321	2.98161	9.42868	702.595	2.07157	4.46307	9.61540	.112486
8.90	79.2100	2.98329	9.43398	704.969	2.07235	4.46475	9.61900	.112360
8.91	79.3881	2.98496	9.43928	707.348	2.07313	4.46642	9.62260	.112233
8.92	79.5664	2.98664	9.44458	709.732	2.07390	4.46809	9.62620	.112108
8.93	79.7449	2.98831	9.44987	712.122	2.07468	4.46976	9.62980	.111982
8.94	79.9236	2.98998	9.45516	714.517	2.07545	4.47142	9.63339	.111857
8.95	80.1025	2.99166	9.46044	716.917	2.07622	4.47309	9.63698	.111732
8.96	80.2816	2.99333	9.46573	719.323	2.07700	4.47476	9.64057	.111607
8.97	80.4609	2.99500	9.47101	721.734	2.07777	4.47642	9.64415	.111483
8.98	80.6404	2.99666	9.47629	724.151	2.07854	4.47808	9.64774	.111359
8.99	80.8201	2.99833	9.48156	726.573	2.07931	4.47974	9.65132	.111235
9.00	81.0000	3.00000	9.48683	729.000	2.08008	4.48140	9.65489	.111111
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$			$1/n$
9.00	81.0000	3.00000	9.48683	729.000	2.08008	4.48140	9.65489	.111111
9.01	81.1801	3.00167	9.49210	731.433	2.08085	4.48306	9.65847	.110988
9.02	81.3604	3.00333	9.49737	733.871	2.08162	4.48472	9.66204	.110865
9.03	81.5409	3.00500	9.50263	736.314	2.08239	4.48638	9.66561	.110742
9.04	81.7216	3.00666	9.50789	738.763	2.08316	4.48803	9.66918	.110619
9.05	81.9025	3.00832	9.51315	741.218	2.08393	4.48969	9.67274	.110497
9.06	82.0836	3.00998	9.51840	743.677	2.08470	4.49134	9.67630	.110375
9.07	82.2649	3.01164	9.52365	746.143	2.08546	4.49299	9.67986	.110254
9.08	82.4464	3.01330	9.52890	748.613	2.08623	4.49464	9.68342	.110132
9.09	82.6281	3.01496	9.53415	751.089	2.08699	4.49629	9.68697	.110011
9.10	82.8100	3.01662	9.53939	753.571	2.08776	4.49794	9.69052	.109890
9.11	82.9921	3.01828	9.54463	756.058	2.08852	4.49959	9.69407	.109769
9.12	83.1744	3.01993	9.54987	758.551	2.08929	4.50123	9.69762	.109649
9.13	83.3569	3.02159	9.55510	761.048	2.09005	4.50288	9.70116	.109529
9.14	83.5396	3.02324	9.56033	763.552	2.09081	4.50452	9.70470	.109409
9.15	83.7225	3.02490	9.56556	766.061	2.09158	4.50616	9.70824	.109290
9.16	83.9056	3.02655	9.57079	768.575	2.09234	4.50781	9.71177	.109170
9.17	84.0889	3.02820	9.57601	771.095	2.09310	4.50945	9.71531	.109051
9.18	84.2724	3.02985	9.58123	773.621	2.09386	4.51108	9.71884	.108932
9.19	84.4561	3.03150	9.58645	776.152	2.09462	4.51272	9.72236	.108814
9.20	84.6400	3.03315	9.59166	778.688	2.09538	4.51436	9.72589	.108696
9.21	84.8241	3.03480	9.59687	781.230	2.09614	4.51599	9.72941	.108578
9.22	85.0084	3.03645	9.60208	783.777	2.09690	4.51763	9.73293	.108460
9.23	85.1929	3.03809	9.60729	786.330	2.09765	4.51926	9.73645	.108342
9.24	85.3776	3.03974	9.61249	788.889	2.09841	4.52089	9.73996	.108225
9.25	85.5625	3.04138	9.61769	791.453	2.09917	4.52252	9.74348	.108108
9.26	85.7476	3.04302	9.62289	794.023	2.09992	4.52415	9.74699	.107991
9.27	85.9329	3.04467	9.62808	796.598	2.10068	4.52578	9.75049	.107875
9.28	86.1184	3.04631	9.63328	799.179	2.10144	4.52740	9.75400	.107759
9.29	86.3041	3.04795	9.63846	801.765	2.10219	4.52903	9.75750	.107643
9.30	86.4900	3.04959	9.64365	804.357	2.10294	4.53065	9.76100	.107527
9.31	86.6761	3.05123	9.64883	806.954	2.10370	4.53228	9.76450	.107411
9.32	86.8624	3.05287	9.65401	809.558	2.10445	4.53390	9.76799	.107296
9.33	87.0489	3.05450	9.65919	812.166	2.10520	4.53552	9.77148	.107181
9.34	87.2356	3.05614	9.66437	814.781	2.10595	4.53714	9.77497	.107066
9.35	87.4225	3.05778	9.66954	817.400	2.10671	4.53876	9.77846	.106952
9.36	87.6096	3.05941	9.67471	820.026	2.10746	4.54038	9.78195	.106838
9.37	87.7969	3.06105	9.67988	822.657	2.10821	4.54199	9.78543	.106724
9.38	87.9844	3.06268	9.68504	825.294	2.10896	4.54361	9.78891	.106610
9.39	88.1721	3.06431	9.69020	827.936	2.10971	4.54522	9.79239	.106496
9.40	88.3600	3.06594	9.69536	830.584	2.11045	4.54684	9.79586	.106383
9.41	88.5481	3.06757	9.70052	833.238	2.11120	4.54845	9.79933	.106270
9.42	88.7364	3.06920	9.70567	835.897	2.11195	4.55006	9.80280	.106157
9.43	88.9249	3.07083	9.71082	838.562	2.11270	4.55167	9.80627	.106045
9.44	89.1136	3.07246	9.71597	841.232	2.11344	4.55328	9.80974	.105932
9.45	89.3025	3.07409	9.72111	843.909	2.11419	4.55488	9.81320	.105820
9.46	89.4916	3.07571	9.72625	846.591	2.11494	4.55649	9.81666	.105708
9.47	89.6809	3.07734	9.73139	849.278	2.11568	4.55809	9.82012	.105597
9.48	89.8704	3.07896	9.73653	851.971	2.11642	4.55970	9.82357	.105485
9.49	90.0601	3.08058	9.74166	854.670	2.11717	4.56130	9.82703	.105374
9.50	90.2500	3.08221	9.74679	857.375	2.11791	4.56290	9.83048	.105263
n	n^2	$\sqrt{n}$		n^3	$\sqrt[3]{n}$			$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10\,n}$	$\sqrt[3]{100\,n}$	$1/n$
9.50	90.2500	3.08221	9.74679	857.375	2.11791	4.56290	9.83048	.105263
9.51	90.4401	3.08383	9.75192	860.085	2.11865	4.56450	9.83392	.105152
9.52	90.6304	3.08545	9.75705	862.801	2.11940	4.56610	9.83737	.105042
9.53	90.8209	3.08707	9.76217	865.523	2.12014	4.56770	9.84081	.104932
9.54	91.0116	3.08869	9.76729	868.251	2.12088	4.56930	9.84425	.104822
9.55	91.2025	3.09031	9.77241	870.984	2.12162	4.57089	9.84769	.104712
9.56	91.3936	3.09192	9.77753	873.723	2.12236	4.57249	9.85113	.104603
9.57	91.5849	3.09354	9.78264	876.467	2.12310	4.57408	9.85456	.104493
9.58	91.7764	3.09516	9.78775	879.218	2.12384	4.57567	9.85799	.104384
9.59	91.9681	3.09677	9.79285	881.974	2.12458	4.57727	9.86142	.104275
9.60	92.1600	3.09839	9.79796	884.736	2.12532	4.57886	9.86485	.104167
9.61	92.3521	3.10000	9.80306	887.504	2.12605	4.58045	9.86827	.104058
9.62	92.5444	3.10161	9.80816	890.277	2.12679	4.58204	9.87169	.103950
9.63	92.7369	3.10322	9.81326	893.056	2.12753	4.58362	9.87511	.103842
9.64	92.9296	3.10483	9.81835	895.841	2.12826	4.58521	9.87853	.103734
9.65	93.1225	3.10644	9.82344	898.632	2.12900	4.58679	9.88195	.103627
9.66	93.3156	3.10805	9.82853	901.429	2.12974	4.58838	9.88536	.103520
9.67	93.5089	3.10966	9.83362	904.231	2.13047	4.58996	9.88877	.103413
9.68	93.7024	3.11127	9.83870	907.039	2.13120	4.59154	9.89217	.103306
9.69	93.8961	3.11288	9.84378	909.853	2.13194	4.59312	9.89558	.103199
9.70	94.0900	3.11448	9.84886	912.673	2.13267	4.59470	9.89898	.103093
9.71	94.2841	3.11609	9.85393	915.499	2.13340	4.59628	9.90238	.102987
9.72	94.4784	3.11769	9.85901	918.330	2.13414	4.59786	9.90578	.102881
9.73	94.6729	3.11929	9.86408	921.167	2.13487	4.59943	9.90918	.102775
9.74	94.8676	3.12090	9.86914	924.010	2.13560	4.60101	9.91257	.102669
9.75	95.0625	3.12250	9.87421	926.859	2.13633	4.60258	9.91596	.102564
9.76	95.2576	3.12410	9.87927	929.714	2.13706	4.60416	9.91935	.102459
9.77	95.4529	3.12570	9.88433	932.575	2.13779	4.60573	9.92274	.102354
9.78	95.6484	3.12730	9.88939	935.441	2.13852	4.60730	9.92612	.102249
9.79	95.8441	3.12890	9.89444	938.314	2.13925	4.60887	9.92950	.102145
9.80	96.0400	3.13050	9.89949	941.192	2.13997	4.61044	9.93288	.102041
9.81	96.2361	3.13209	9.90454	944.076	2.14070	4.61200	9.93626	.101937
9.82	96.4324	3.13369	9.90959	946.966	2.14143	4.61357	9.93964	.101833
9.83	96.6289	3.13528	9.91464	949.862	2.14216	4.61514	9.94301	.101729
9.84	96.8256	3.13688	9.91968	952.764	2.14288	4.61670	9.94638	.101626
9.85	97.0225	3.13847	9.92472	955.672	2.14361	4.61826	9.94975	.101523
9.86	97.2196	3.14006	9.92975	958.585	2.14433	4.61983	9.95311	.101420
9.87	97.4169	3.14166	9.93479	961.505	2.14506	4.62139	9.95648	.101317
9.88	97.6144	3.14325	9.93982	964.430	2.14578	4.62295	9.95984	.101215
9.89	97.8121	3.14484	9.94485	967.362	2.14651	4.62451	9.96320	.101112
9.90	98.0100	3.14643	9.94987	970.299	2.14723	4.62607	9.96655	.101010
9.91	98.2081	3.14802	9.95490	973.242	2.14795	4.62762	9.96991	.100908
9.92	98.4064	3.14960	9.95992	976.191	2.14867	4.62918	9.97326	.100806
9.93	98.6049	3.15119	9.96494	979.147	2.14940	4.63073	9.97661	.100705
9.94	98.8036	3.15278	9.96995	982.108	2.15012	4.63229	9.97996	.100604
9.95	99.0025	3.15436	9.97497	985.075	2.15084	4.63384	9.98331	.100503
9.96	99.2016	3.15595	9.97998	988.048	2.15156	4.63539	9.98665	.100402
9.97	99.4009	3.15753	9.98499	991.027	2.15228	4.63694	9.98999	.100301
9.98	99.6004	3.15911	9.98999	994.012	2.15300	4.63849	9.99333	.100200
9.99	99.8001	3.16070	9.99500	997.003	2.15372	4.64004	9.99667	.100100
10.00	100.000	3.16228	10.0000	1000.00	2.15443	4.64159	10.0000	.100000
n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10\,n}$	$\sqrt[3]{100\,n}$	$1/n$

			2							
0.0		5.395	6.088	6.493	6.781	7.004	7.187	7.341	7.474	7.592
	7.697	7.793	7.880	7.960	8.034	8.103	8.167	8.228	8.285	8.339
	8.391	8.439	8.486	8.530	8.573	8.614	8.653	8.691	8.727	8.762
	8.796	8.829	8.861	8.891	8.921	8.950	8.978	9.006	9.032	9.058
	9.084	9.108	9.132	9.156	9.179	9.201	9.223	9.245	9.266	9.287
	9.307	9.327	9.346	9.365	9.384	9.402	9.420	9.438	9.455	9.472
	9.489	9.506	9.522	9.538	9.554	9.569	9.584	9.600	9.614	9.629
		9.658	9.671	9.685	9.699	9.712	9.726	9.739	9.752	9.764
	9.777	9.789	9.802	9.814	9.826	9.837	9.849	9.861	9.872	9.883
	9.895	9.906	9.917	9.927	9.938	9.949	9.959	9.970	9.980	9.990
		0995	1980	2956	3922	4879	5827	6766	7696	8618
	9531	*0436	*1333	*2222	*3103	*3976	*4842	*5700	*6551	*7395
	0.1 8232	9062	9885	*0701	*1511	*2314	*3111	*3902	*4686	*5464
	0.2 6236	7003	7763	8518	9267	*0010	*0748	*1481	*2208	*2930
	0.3 3647	4359	5066	5767	6464	7156	7844	8526	9204	9878
	0.4 0547	1211	1871	2527	3178	3825	4469	5108	5742	6373
	7000	7623	8243	8858	9470	*0078	*0682	*1282	*1879	*2473
	0.5 3063	3649	4232	4812	5389	5962	6531	7098	7661	8222
	8779	9333	9884	*0432	*0977	*1519	*2058	*2594	*3127	*3658
	0.6 4185	4710	5233	5752	6269	6783	7294	7803	8310	8813
2.0	9315	9813	*0310	*0804				*2755	*3237	*3716
2.1	0.7 4194	4669	5142	5612	6081	6547	7011	7473	7932	8390
2.2	8846	9299	9751	*0200	*0648	*1093	*1536	*1978	*2418	*2855
2.3	0.8 3291	3725	4157	4587	5015	5442	5866	6289	6710	7129
2.4	7547	7963	8377	8789	9200	9609	*0016	*0422	*0826	*1228
2.5	0.9 1629	2028	2426	2822	3216	3609	4001	4391	4779	5166
2.6	5551	5935	6317	6698	7078	7456	7833	8208	8582	8954
2.7	9325	9695	*0063	*0430	*0796	*1160	*1523	*1885	*2245	*2604
2.8	1.0 2962	3318	3674	4028	4380	4732	5082	5431	5779	6126
2.9	6471	6815	7158	7500	7841	8181	8519	8856	9192	9527
3.0	9861	*0194	*0526	*0856	*1186	*1514	*1841	*2168	*2493	*2817
3.1	1.1 3140	3462	3783	4103	4422	4740	5057	5373	5688	6002
3.2	6315	6627	6938	7248	7557	7865	8173	8479	8784	9089
3.3	9392	9695	9996	*0297	*0597	*0896	*1194	*1491	*1788	*2083
3.4	1.2 2378	2671	2964	3256	3547	3837	4127	4415	4703	4990
3.5	5276	5562	5846	6130	6413	6695	6976	7257	7536	7815
3.6	8093	8371	8647	8923	9198	9473	9746	*0019	*0291	*0563
3.7	1.3 0833	1103	1372	1641	1909	2176	2442	2708	2972	3237
3.8	3500	3763	4025	4286	4547	4807	5067	5325	5584	5841
3.9	6098	6354	6609	6864	7118	7372	7624	7877	8128	8379
4.0	8629	8879	9128	9377	9624	9872	*0118	*0364	*0610	*0854
4.1	1.4 1099	1342	1585	1828	2070	2311	2552	2792	3031	3270
4.2	3508	3746	3984	4220	4456	4692	4927	5161	5395	5629
4.3	5862	6094	6326	6557	6787	7018	7247	7476	7705	7933
4.4	8160	8387	8614	8840	9065	9290	9515	9739	9962	*0185
4.5	1.5 0408	0630	0851	1072	1293	1513	1732	1951	2170	2388
4.6	2606	2823	3039	3256	3471	3687	3902	4116	4330	4543
4.7	4756	4969	5181	5393	5604	5814	6025	6235	6444	6653
4.8	6862	7070	7277	7485	7691	7898	8104	8309	8515	8719
4.9	8924	9127	9331	9534	9737	9939	*0141	*0342	*0543	*0744
5.0	1.6 0944	1144	1343	1542	1741	1939	2137	2334	2531	2728
	0			3					8	9

N	0	1	2	3	4	5	6	7	8	9
5.0	1.6 0944	1144	1343	1542	1741	1939	2137	2334	2531	2728
5.1	2924	3120	3315	3511	3705	3900	4094	4287	4481	4673
5.2	4866	5058	5250	5441	5632	5823	6013	6203	6393	6582
5.3	6771	6959	7147	7335	7523	7710	7896	8083	8269	8455
5.4	8640	8825	9010	9194	9378	9562	9745	9928	*0111	*0293
5.5	1.7 0475	0656	0838	1019	1199	1380	1560	1740	1919	2098
5.6	2277	2455	2633	2811	2988	3166	3342	3519	3695	3871
5.7	4047	4222	4397	4572	4746	4920	5094	5267	5440	5613
5.8	5786	5958	6130	6302	6473	6644	6815	6985	7156	7326
5.9	7495	7665	7834	8002	8171	8339	8507	8675	8842	9009
6.0	9176	9342	9509	9675	9840	*0006	*0171	*0336	*0500	*0665
6.1	1.8 0829	0993	1156	1319	1482	1645	1808	1970	2132	2294
6.2	2455	2616	2777	2938	3098	3258	3418	3578	3737	3896
6.3	4055	4214	4372	4530	4688	4845	5003	5160	5317	5473
6.4	5630	5786	5942	6097	6253	6408	6563	6718	6872	7026
6.5	7180	7334	7487	7641	7794	7947	8099	8251	8403	8555
6.6	8707	8858	9010	9160	9311	9462	9612	9762	9912	*0061
6.7	1.9 0211	0360	0509	0658	0806	0954	1102	1250	1398	1545
6.8	1692	1839	1986	2132	2279	2425	2571	2716	2862	3007
6.9	3152	3297	3442	3586	3730	3874	4018	4162	4305	4448
7.0	4591	4734	4876	5019	5161	5303	5445	5586	5727	5869
7.1	6009	6150	6291	6431	6571	6711	6851	6991	7130	7269
7.2	7408	7547	7685	7824	7962	8100	8238	8376	8513	8650
7.3	8787	8924	9061	9198	9334	9470	9606	9742	9877	*0013
7.4	2.0 0148	0283	0418	0553	0687	0821	0956	1089	1223	1357
7.5	1490	1624	1757	1890	2022	2155	2287	2419	2551	2683
7.6	2815	2946	3078	3209	3340	3471	3601	3732	3862	3992
7.7	4122	4252	4381	4511	4640	4769	4898	5027	5156	5284
7.8	5412	5540	5668	5796	5924	6051	6179	6306	6433	6560
7.9	6686	6813	6939	7065	7191	7317	7443	7568	7694	7819
8.0	7944	8069	8194	8318	8443	8567	8691	8815	8939	9063
8.1	9186	9310	9433	9556	9679	9802	9924	*0047	*0169	*0291
8.2	2.1 0413	0535	0657	0779	0900	1021	1142	1263	1384	1505
8.3	1626	1746	1866	1986	2106	2226	2346	2465	2585	2704
8.4	2823	2942	3061	3180	3298	3417	3535	3653	3771	3889
8.5	4007	4124	4242	4359	4476	4593	4710	4827	4943	5060
8.6	5176	5292	5409	5524	5640	5756	5871	5987	6102	6217
8.7	6332	6447	6562	6677	6791	6905	7020	7134	7248	7361
8.8	7475	7589	7702	7816	7929	8042	8155	8267	8380	8493
8.9	8605	8717	8830	8942	9054	9165	9277	9389	9500	9611
9.0	9722	9834	9944	*0055	*0166	*0276	*0387	*0497	*0607	*0717
9.1	2.2 0827	0937	1047	1157	1266	1375	1485	1594	1703	1812
9.2	1920	2029	2138	2246	2354	2462	2570	2678	2786	2894
9.3	3001	3109	3216	3324	3431	3538	3645	3751	3858	3965
9.4	4071	4177	4284	4390	4496	4601	4707	4813	4918	5024
9.5	5129	5234	5339	5444	5549	5654	5759	5863	5968	6072
9.6	6176	6280	6384	6488	6592	6696	6799	6903	7006	7109
9.7	7213	7316	7419	7521	7624	7727	7829	7932	8034	8136
9.8	8238	8340	8442	8544	8646	8747	8849	8950	9051	9152
9.9	9253	9354	9455	9556	9657	9757	9858	9958	*0058	*0158
10.0	2.3 0259	0358	0458	0558	0658	0757	0857	0956	1055	1154
N	0	1	2	3	4	5	6	7	8	9

10	2.30259	**25**	3.21888	**40**	3.68888	**55**	4.00733	**70**	4.24850	**85**	4.44265
11	2.39790	26	3.25810	41	3.71357	56	4.02535	71	4.26268	86	4.45435
12	2.48491	27	3.29584	42	3.73767	57	4.04305	72	4.27667	87	4.46591
13	2.56495	28	3.33220	43	3.76120	58	4.06044	73	4.29046	88	4.47734
14	2.63906	29	3.36730	44	3.78419	59	4.07754	74	4.30407	89	4.48864
15	2.70805	**30**	3.40120	**45**	3.80666	**60**	4.09434	**75**	4.31749	**90**	4.49981
16	2.77259	31	3.43399	46	3.82864	61	4.11087	76	4.33073	91	4.51086
17	2.83321	32	3.46574	47	3.85015	62	4.12713	77	4.34381	92	4.52179
18	2.89037	33	3.49651	48	3.87120	63	4.14313	78	4.35671	93	4.53260
19	2.94444	34	3.52636	49	3.89182	64	4.15888	79	4.36945	94	4.54329
20	2.99573	**35**	3.55535	**50**	3.91202	**65**	4.17439	**80**	4.38203	**95**	4.55388
21	3.04452	36	3.58352	51	3.93183	66	4.18965	81	4.39445	96	4.56435
22	3.09104	37	3.61092	52	3.95124	67	4.20469	82	4.40672	97	4.57471
23	3.13549	38	3.63759	53	3.97029	68	4.21951	83	4.41884	98	4.58497
24	3.17805	39	3.66356	54	3.98898	69	4.23411	84	4.43082	99	4.59512

Napierian or Natural Logarithms — 100 to 409

N	0	1	2	3	4	5	6	7	8	9
10	4.6 0517	1512	2497	3473	4439	5396	6344	7283	8213	9135
11	4.7 0048	0953	1850	2739	3620	4493	5359	6217	7068	7912
12	8749	9579	*0402	*1218	*2028	*2831	*3628	*4419	*5203	*5981
13	4.8 6753	7520	8280	9035	9784	*0527	*1265	*1998	*2725	*3447
14	4.9 4164	4876	5583	6284	6981	7673	8361	9043	9721	*0395
15	5.0 1064	1728	2388	3044	3695	4343	4986	5625	6260	6890
16	7517	8140	8760	9375	9987	*0595	*1199	*1799	*2396	*2990
17	5.1 3580	4166	4749	5329	5906	6479	7048	7615	8178	8739
18	9296	9850	*0401	*0949	*1494	*2036	*2575	*3111	*3644	*4175
19	5.2 4702	5227	5750	6269	6786	7300	7811	8320	8827	9330
20	9832	*0330	*0827	*1321	*1812	*2301	*2788	*3272	*3754	*4233
21	5.3 4711	5186	5659	6129	6598	7064	7528	7990	8450	8907
22	9363	9816	*0268	*0717	*1165	*1610	*2053	*2495	*2935	*3372
23	5.4 3808	4242	4674	5104	5532	5959	6383	6806	7227	7646
24	8064	8480	8894	9306	9717	*0126	*0533	*0939	*1343	*1745
25	5.5 2146	2545	2943	3339	3733	4126	4518	4908	5296	5683
26	6068	6452	6834	7215	7595	7973	8350	8725	9099	9471
27	9842	*0212	*0580	*0947	*1313	*1677	*2040	*2402	*2762	*3121
28	5.6 3479	3835	4191	4545	4897	5249	5599	5948	6296	6643
29	6988	7332	7675	8017	8358	8698	9036	9373	9709	*0044
30	5.7 0378	0711	1043	1373	1703	2031	2359	2685	3010	3334
31	3657	3979	4300	4620	4939	5257	5574	5890	6205	6519
32	6832	7144	7455	7765	8074	8383	8690	8996	9301	9606
33	9909	*0212	*0513	*0814	*1114	*1413	*1711	*2008	*2305	*2600
34	5.8 2895	3188	3481	3773	4064	4354	4644	4932	5220	5507
35	5793	6079	6363	6647	6930	7212	7493	7774	8053	8332
36	8610	8888	9164	9440	9715	9990	*0263	*0536	*0808	*1080
37	5.9 1350	1620	1889	2158	2426	2693	2959	3225	3489	3754
38	4017	4280	4542	4803	5064	5324	5584	5842	6101	6358
39	6615	6871	7126	7381	7635	7889	8141	8394	8645	8896
40	9146	9396	9645	9894	*0141	*0389	*0635	*0881	*1127	*1372
N	0	1	2	3	4	5	6	7	8	9

Above 409, use the formula $\log_e 10n = \log_e n + \log_e 10 = \log_e n + 2.30258509$,
or the formula $\log_e n = \log_e 10 \cdot \log_{10} n = 2.30258509 \log_{10} n$.

N	$N \cdot M$	N	$N \cdot M$
0	0.00000 000	**50**	21.71472 410
1	0.43429 448	51	22.14901 858
2	0.86858 896	52	22.58331 306
3	1.30288 345	53	23.01760 754
4	1.73717 793	54	23.45190 202
5	2.17147 241	55	23.88619 650
6	2.60576 689	56	24.32049 099
7	3.04006 137	57	24.75478 547
8	3.47435 586	58	25.18907 995
9	3.90865 034	59	25.62337 443
10	4.34294 482	**60**	26.05766 891
11	4.77723 930	61	26.49196 340
12	5.21153 378	62	26.92625 788
13	5.64582 826	63	27.36055 236
14	6.08012 275	64	27.79484 684
15	6.51441 723	65	28.22914 132
16	6.94871 171	66	28.66343 581
17	7.38300 619	67	29.09773 029
18	7.81730 067	68	29.53202 477
19	8.25159 516	69	29.96631 925
20	8.68588 964	**70**	30.40061 373
21	9.12018 412	71	30.83490 822
22	9.55447 860	72	31.26920 270
23	9.98877 308	73	31.70349 718
24	10.42306 757	74	32.13779 166
25	10.85736 205	75	32.57208 614
26	11.29165 653	76	33.00638 062
27	11.72595 101	77	33.44067 511
28	12.16024 549	78	33.87496 959
29	12.59453 998	79	34.30926 407
30	13.02883 446	**80**	34.74355 855
31	13.46312 894	81	35.17785 303
32	13.89742 342	82	35.61214 752
33	14.33171 790	83	36.04644 200
34	14.76601 238	84	36.48073 648
35	15.20030 687	85	36.91503 096
36	15.63460 135	86	37.34932 544
37	16.06889 583	87	37.78361 993
38	16.50319 031	88	38.21791 441
39	16.93748 479	89	38.65220 889
40	17.37177 928	**90**	39.08650 337
41	17.80607 376	91	39.52079 785
42	18.24036 824	92	39.95509 234
43	18.67466 272	93	40.38938 682
44	19.10895 720	94	40.82368 130
45	19.54325 169	95	41.25797 578
46	19.97754 617	96	41.69227 026
47	20.41184 065	97	42.12656 474
48	20.84613 513	98	42.56085 923
49	21.28042 961	99	42.99515 371
50	21.71472 410	**100**	43.42944 819

N	$N \div M$	N	$N \div M$
0	0.00000 000	**50**	115.12925 465
1	2.30258 509	51	117.43183 974
2	4.60517 019	52	119.73442 484
3	6.90775 528	53	122.03700 993
4	9.21034 037	54	124.33959 502
5	11.51292 546	55	126.64218 011
6	13.81551 056	56	128.94476 521
7	16.11809 565	57	131.24735 030
8	18.42068 074	58	133.54993 539
9	20.72326 584	59	135.85252 049
10	23.02585 093	**60**	138.15510 558
11	25.32843 602	61	140.45769 067
12	27.63102 112	62	142.76027 577
13	29.93360 621	63	145.06286 086
14	32.23619 130	64	147.36544 595
15	34.53877 639	65	149.66803 104
16	36.84136 149	66	151.97061 614
17	39.14394 658	67	154.27320 123
18	41.44653 167	68	156.57578 632
19	43.74911 677	69	158.87837 142
20	46.05170 186	**70**	161.18095 651
21	48.35428 695	71	163.48354 160
22	50.65687 205	72	165.78612 670
23	52.95945 714	73	168.08871 179
24	55.26204 223	74	170.39129 688
25	57.56462 732	75	172.69388 197
26	59.86721 242	76	174.99646 707
27	62.16979 751	77	177.29905 216
28	64.47238 260	78	179.60163 725
29	66.77496 770	79	181.90422 235
30	69.07755 279	**80**	184.20680 744
31	71.38013 788	81	186.50939 253
32	73.68272 298	82	188.81197 763
33	75.98530 807	83	191.11456 272
34	78.28789 316	84	193.41714 781
35	80.59047 825	85	195.71973 290
36	82.89306 335	86	198.02231 800
37	85.19564 844	87	200.32490 309
38	87.49823 353	88	202.62748 818
39	89.80081 863	89	204.93007 328
40	92.10340 372	**90**	207.23265 837
41	94.40598 881	91	209.53524 346
42	96.70857 391	92	211.83782 856
43	99.01115 900	93	214.14041 365
44	101.31374 409	94	216.44299 874
45	103.61632 918	95	218.74558 383
46	105.91891 428	96	221.04816 893
47	108.22149 937	97	223.35075 402
48	110.52408 446	98	225.65333 911
49	112.82666 956	99	227.95592 421
50	115.12925 465	**100**	230.25850 930

$M = \log_{10} e = .43429\ 44819\ 03251\ 82765$

$\log_{10} n = \log_e n \cdot \log_{10} e = M \log_e n.$

$\log_{10} e^x = x \cdot \log_{10} e = x \cdot M.$

$1/M = \log_e 10 = 2.30258\ 50929\ 94045\ 68402$

$\log_e n = \log_{10} n \cdot \log_e 10 = (1/M) \log_{10} n.$

$\log_e (10^n \cdot x) = \log_e x + n(1/M).$

x	e^x Value	e^x Log_{10}	e^{-x} Value	Sinh x Value	Sinh x Log_{10}	Cosh x Value	Cosh x Log_{10}	Tanh x Value
0.00	1.0000	.00000	1.0000	0.0000	$-\infty$	1.0000	.00000	.00000
0.01	1.0101	.00434	.99005	0.0100	.00001	1.0001	.00002	.01000
0.02	1.0202	.00869	.98020	0.0200	.30106	1.0002	.00009	.02000
0.03	1.0305	.01303	.97045	0.0300	.47719	1.0005	.00020	.02999
0.04	1.0408	.01737	.96079	0.0400	.60218	1.0008	.00035	.03998
0.05	1.0513	.02171	.95123	0.0500	.69915	1.0013	.00054	.04996
0.06	1.0618	.02606	.94176	0.0600	.77841	1.0018	.00078	.05993
0.07	1.0725	.03040	.93239	0.0701	.84545	1.0025	.00106	.06989
0.08	1.0833	.03474	.92312	0.0801	.90355	1.0032	.00139	.07983
0.09	1.0942	.03909	.91393	0.0901	.95483	1.0041	.00176	.08976
0.10	1.1052	.04343	.90484	0.1002	.00072	1.0050	.00217	.09967
0.11	1.1163	.04777	.89583	0.1102	.04227	1.0061	.00262	.10956
0.12	1.1275	.05212	.88692	0.1203	.08022	1.0072	.00312	.11943
0.13	1.1388	.05646	.87810	0.1304	.11517	1.0085	.00366	.12927
0.14	1.1503	.06080	.86936	0.1405	.14755	1.0098	.00424	.13909
0.15	1.1618	.06514	.86071	0.1506	.17772	1.0113	.00487	.14889
0.16	1.1735	.06949	.85214	0.1607	.20597	1.0128	.00554	.15865
0.17	1.1853	.07383	.84366	0.1708	.23254	1.0145	.00625	.16838
0.18	1.1972	.07817	.83527	0.1810	.25762	1.0162	.00700	.17808
0.19	1.2092	.08252	.82696	0.1911	.28136	1.0181	.00779	.18775
0.20	1.2214	.08686	.81873	0.2013	.30392	1.0201	.00863	.19738
0.21	1.2337	.09120	.81058	0.2115	.32541	1.0221	.00951	.20697
0.22	1.2461	.09554	.80252	0.2218	.34592	1.0243	.01043	.21652
0.23	1.2586	.09989	.79453	0.2320	.36555	1.0266	.01139	.22603
0.24	1.2712	.10423	.78663	0.2423	.38437	1.0289	.01239	.23550
0.25	1.2840	.10857	.77880	0.2526	.40245	1.0314	.01343	.24492
0.26	1.2969	.11292	.77105	0.2629	.41986	1.0340	.01452	.25430
0.27	1.3100	.11726	.76338	0.2733	.43663	1.0367	.01564	.26362
0.28	1.3231	.12160	.75578	0.2837	.45282	1.0395	.01681	.27291
0.29	1.3364	.12595	.74826	0.2941	.46847	1.0423	.01801	.28213
0.30	1.3499	.13029	.74082	0.3045	.48362	1.0453	.01926	.29131
0.31	1.3634	.13463	.73345	0.3150	.49830	1.0484	.02054	.30044
0.32	1.3771	.13897	.72615	0.3255	.51254	1.0516	.02187	.30951
0.33	1.3910	.14332	.71892	0.3360	.52637	1.0549	.02323	.31852
0.34	1.4049	.14766	.71177	0.3466	.53981	1.0584	.02463	.32748
0.35	1.4191	.15200	.70469	0.3572	.55290	1.0619	.02607	.33638
0.36	1.4333	.15635	.69768	0.3678	.56564	1.0655	.02755	.34521
0.37	1.4477	.16069	.69073	0.3785	.57807	1.0692	.02907	.35399
0.38	1.4623	.16503	.68386	0.3892	.59019	1.0731	.03063	.36271
0.39	1.4770	.16937	.67706	0.4000	.60202	1.0770	.03222	.37136
0.40	1.4918	.17372	.67032	0.4108	.61358	1.0811	.03385	.37995
0.41	1.5068	.17806	.66365	0.4216	.62488	1.0852	.03552	.38847
0.42	1.5220	.18240	.65705	0.4325	.63594	1.0895	.03723	.39693
0.43	1.5373	.18675	.65051	0.4434	.64677	1.0939	.03897	.40532
0.44	1.5527	.19109	.64404	0.4543	.65738	1.0984	.04075	.41364
0.45	1.5683	.19543	.63763	0.4653	.66777	1.1030	.04256	.42190
0.46	1.5841	.19978	.63128	0.4764	.67797	1.1077	.04441	.43008
0.47	1.6000	.20412	.62500	0.4875	.68797	1.1125	.04630	.43820
0.48	1.6161	.20846	.61878	0.4986	.69779	1.1174	.04822	.44624
0.49	1.6323	.21280	.61263	0.5098	.70744	1.1225	.05018	.45422
0.50	1.6487	.21715	.60653	0.5211	.71692	1.1276	.05217	.46212

x	e^x Value	e^x Log_{10}	e^{-x} Value	Sinh x Value	Sinh x Log_{10}	Cosh x Value	Cosh x Log_{10}	Tanh x Value
0.50	1.6487	.21715	.60653	0.5211	.71692	1.1276	.05217	.46212
0.51	1.6653	.22149	.60050	0.5324	.72624	1.1329	.05419	.46995
0.52	1.6820	.22583	.59452	0.5438	.73540	1.1383	.05625	.47770
0.53	1.6989	.23018	.58860	0.5552	.74442	1.1438	.05834	.48538
0.54	1.7160	.23452	.58275	0.5666	.75330	1.1494	.06046	.49299
0.55	1.7333	.23886	.57695	0.5782	.76204	1.1551	.06262	.50052
0.56	1.7507	.24320	.57121	0.5897	.77065	1.1609	.06481	.50798
0.57	1.7683	.24755	.56553	0.6014	.77914	1.1669	.06703	.51536
0.58	1.7860	.25189	.55990	0.6131	.78751	1.1730	.06929	.52267
0.59	1.8040	.25623	.55433	0.6248	.79576	1.1792	.07157	.52990
0.60	1.8221	.26058	.54881	0.6367	.80390	1.1855	.07389	.53705
0.61	1.8404	.26492	.54335	0.6485	.81194	1.1919	.07624	.54413
0.62	1.8589	.26926	.53794	0.6605	.81987	1.1984	.07861	.55113
0.63	1.8776	.27361	.53259	0.6725	.82770	1.2051	.08102	.55805
0.64	1.8965	.27795	.52729	0.6846	.83543	1.2119	.08346	.56490
0.65	1.9155	.28229	.52205	0.6967	.84308	1.2188	.08593	.57167
0.66	1.9348	.28663	.51685	0.7090	.85063	1.2258	.08843	.57836
0.67	1.9542	.29098	.51171	0.7213	.85809	1.2330	.09095	.58498
0.68	1.9739	.29532	.50662	0.7336	.86548	1.2402	.09351	.59152
0.69	1.9937	.29966	.50158	0.7461	.87278	1.2476	.09609	.59798
0.70	2.0138	.30401	.49659	0.7586	.88000	1.2552	.09870	.60437
0.71	2.0340	.30835	.49164	0.7712	.88715	1.2628	.10134	.61068
0.72	2.0544	.31269	.48675	0.7838	.89423	1.2706	.10401	.61691
0.73	2.0751	.31703	.48191	0.7966	.90123	1.2785	.10670	.62307
0.74	2.0959	.32138	.47711	0.8094	.90817	1.2865	.10942	.62915
0.75	2.1170	.32572	.47237	0.8223	.91504	1.2947	.11216	.63515
0.76	2.1383	.33006	.46767	0.8353	.92185	1.3030	.11493	.64108
0.77	2.1598	.33441	.46301	0.8484	.92859	1.3114	.11773	.64693
0.78	2.1815	.33875	.45841	0.8615	.93527	1.3199	.12055	.65271
0.79	2.2034	.34309	.45384	0.8748	.94190	1.3286	.12340	.65841
0.80	2.2255	.34744	.44933	0.8881	.94846	1.3374	.12627	.66404
0.81	2.2479	.35178	.44486	0.9015	.95498	1.3464	.12917	.66959
0.82	2.2705	.35612	.44043	0.9150	.96144	1.3555	.13209	.67507
0.83	2.2933	.36046	.43605	0.9286	.96784	1.3647	.13503	.68048
0.84	2.3164	.36481	.43171	0.9423	.97420	1.3740	.13800	.68581
0.85	2.3396	.36915	.42741	0.9561	.98051	1.3835	.14099	.69107
0.86	2.3632	.37349	.42316	0.9700	.98677	1.3932	.14400	.69626
0.87	2.3869	.37784	.41895	0.9840	.99299	1.4029	.14704	.70137
0.88	2.4109	.38218	.41478	0.9981	.99916	1.4128	.15009	.70642
0.89	2.4351	.38652	.41066	1.0122	.00528	1.4229	.15317	.71139
0.90	2.4596	.39087	.40657	1.0265	.01137	1.4331	.15627	.71630
0.91	2.4843	.39521	.40252	1.0409	.01741	1.4434	.15939	.72113
0.92	2.5093	.39955	.39852	1.0554	.02341	1.4539	.16254	.72590
0.93	2.5345	.40389	.39455	1.0700	.02937	1.4645	.16570	.73059
0.94	2.5600	.40824	.39063	1.0847	.03530	1.4753	.16888	.73522
0.95	2.5857	.41258	.38674	1.0995	.04119	1.4862	.17208	.73978
0.96	2.6117	.41692	.38289	1.1144	.04704	1.4973	.17531	.74428
0.97	2.6379	.42127	.37908	1.1294	.05286	1.5085	.17855	.74870
0.98	2.6645	.42561	.37531	1.1446	.05864	1.5199	.18181	.75307
0.99	2.6912	.42995	.37158	1.1598	.06439	1.5314	.18509	.75736
1.00	2.7183	.43429	.36788	1.1752	.07011	1.5431	.18839	.76159

x	e^x Value	e^x $\log_{10}$	e^{-x} Value	Sinh x Value	Sinh x $\log_{10}$	Cosh x Value	Cosh x $\log_{10}$	Tanh x Value
1.00	2.7183	.43429	.36788	1.1752	.07011	1.5431	.18839	.76159
1.01	2.7456	.43864	.36422	1.1907	.07580	1.5549	.19171	.76576
1.02	2.7732	.44298	.36060	1.2063	.08146	1.5669	.19504	.76987
1.03	2.8011	.44732	.35701	1.2220	.08708	1.5790	.19839	.77391
1.04	2.8292	.45167	.35345	1.2379	.09268	1.5913	.20176	.77789
1.05	2.8577	.45601	.34994	1.2539	.09825	1.6038	.20515	.78181
1.06	2.8864	.46035	.34646	1.2700	.10379	1.6164	.20855	.78566
1.07	2.9154	.46470	.34301	1.2862	.10930	1.6292	.21197	.78946
1.08	2.9447	.46904	.33960	1.3025	.11479	1.6421	.21541	.79320
1.09	2.9743	.47338	.33622	1.3190	.12025	1.6552	.21886	.79688
1.10	3.0042	.47772	.33287	1.3356	.12569	1.6685	.22233	.80050
1.11	3.0344	.48207	.32956	1.3524	.13111	1.6820	.22582	.80406
1.12	3.0649	.48641	.32628	1.3693	.13649	1.6956	.22931	.80757
1.13	3.0957	.49075	.32303	1.3863	.14186	1.7093	.23283	.81102
1.14	3.1268	.49510	.31982	1.4035	.14720	1.7233	.23636	.81441
1.15	3.1582	.49944	.31664	1.4208	.15253	1.7374	.23990	.81775
1.16	3.1899	.50378	.31349	1.4382	.15783	1.7517	.24346	.82104
1.17	3.2220	.50812	.31037	1.4558	.16311	1.7662	.24703	.82427
1.18	3.2544	.51247	.30728	1.4735	.16836	1.7808	.25062	.82745
1.19	3.2871	.51681	.30422	1.4914	.17360	1.7957	.25422	.83058
1.20	3.3201	.52115	.30119	1.5095	.17882	1.8107	.25784	.83365
1.21	3.3535	.52550	.29820	1.5276	.18402	1.8258	.26146	.83668
1.22	3.3872	.52984	.29523	1.5460	.18920	1.8412	.26510	.83965
1.23	3.4212	.53418	.29229	1.5645	.19437	1.8568	.26876	.84258
1.24	3.4556	.53853	.28938	1.5831	.19951	1.8725	.27242	.84546
1.25	3.4903	.54287	.28650	1.6019	.20464	1.8884	.27610	.84828
1.26	3.5254	.54721	.28365	1.6209	.20975	1.9045	.27979	.85106
1.27	3.5609	.55155	.28083	1.6400	.21485	1.9208	.28349	.85380
1.28	3.5966	.55590	.27804	1.6593	.21993	1.9373	.28721	.85648
1.29	3.6328	.56024	.27527	1.6788	.22499	1.9540	.29093	.85913
1.30	3.6693	.56458	.27253	1.6984	.23004	1.9709	.29467	.86172
1.31	3.7062	.56893	.26982	1.7182	.23507	1.9880	.29842	.86428
1.32	3.7434	.57327	.26714	1.7381	.24009	2.0053	.30217	.86678
1.33	3.7810	.57761	.26448	1.7583	.24509	2.0228	.30594	.86925
1.34	3.8190	.58195	.26185	1.7786	.25008	2.0404	.30972	.87167
1.35	3.8574	.58630	.25924	1.7991	.25505	2.0583	.31352	.87405
1.36	3.8962	.59064	.25666	1.8198	.26002	2.0764	.31732	.87639
1.37	3.9354	.59498	.25411	1.8406	.26496	2.0947	.32113	.87869
1.38	3.9749	.59933	.25158	1.8617	.26990	2.1132	.32495	.88095
1.39	4.0149	.60367	.24908	1.8829	.27482	2.1320	.32878	.88317
1.40	4.0552	.60801	.24660	1.9043	.27974	2.1509	.33262	.88535
1.41	4.0960	.61236	.24414	1.9259	.28464	2.1700	.33647	.88749
1.42	4.1371	.61670	.24171	1.9477	.28952	2.1894	.34033	.88960
1.43	4.1787	.62104	.23931	1.9697	.29440	2.2090	.34420	.89167
1.44	4.2207	.62538	.23693	1.9919	.29926	2.2288	.34807	.89370
1.45	4.2631	.62973	.23457	2.0143	.30412	2.2488	.35196	.89569
1.46	4.3060	.63407	.23224	2.0369	.30896	2.2691	.35585	.89765
1.47	4.3492	.63841	.22993	2.0597	.31379	2.2896	.35976	.89958
1.48	4.3929	.64276	.22764	2.0827	.31862	2.3103	.36367	.90147
1.49	4.4371	.64710	.22537	2.1059	.32343	2.3312	.36759	.90332
1.50	4.4817	.65144	.22313	2.1293	.32823	2.3524	.37151	.90515

x	e^x Value	e^x Log10	e^{-x} Value	Sinh x Value	Sinh x Log10	Cosh x Value	Cosh x Log10	Tanh x Value
1.50	4.4817	.65144	.22313	2.1293	.32823	2.3524	.37151	.90515
1.51	4.5267	.65578	.22091	2.1529	.33303	2.3738	.37545	.90694
1.52	4.5722	.66013	.21871	2.1768	.33781	2.3955	.37939	.90870
1.53	4.6182	.66447	.21654	2.2008	.34258	2.4174	.38334	.91042
1.54	4.6646	.66881	.21438	2.2251	.34735	2.4395	.38730	.91212
1.55	4.7115	.67316	.21225	2.2496	.35211	2.4619	.39126	.91379
1.56	4.7588	.67750	.21014	2.2743	.35686	2.4845	.39524	.91542
1.57	4.8066	.68184	.20805	2.2993	.36160	2.5073	.39921	.91703
1.58	4.8550	.68619	.20598	2.3245	.36633	2.5305	.40320	.91860
1.59	4.9037	.69053	.20393	2.3499	.37105	2.5538	.40719	.92015
1.60	4.9530	.69487	.20190	2.3756	.37577	2.5775	.41119	.92167
1.61	5.0028	.69921	.19989	2.4015	.38048	2.6013	.41520	.92316
1.62	5.0531	.70356	.19790	2.4276	.38518	2.6255	.41921	.92462
1.63	5.1039	.70790	.19593	2.4540	.38987	2.6499	.42323	.92606
1.64	5.1552	.71224	.19398	2.4806	.39456	2.6746	.42725	.92747
1.65	5.2070	.71659	.19205	2.5075	.39923	2.6995	.43129	.92886
1.66	5.2593	.72093	.19014	2.5346	.40391	2.7247	.43532	.93022
1.67	5.3122	.72527	.18825	2.5620	.40857	2.7502	.43937	.93155
1.68	5.3656	.72961	.18637	2.5896	.41323	2.7760	.44341	.93286
1.69	5.4195	.73396	.18452	2.6175	.41788	2.8020	.44747	.93415
1.70	5.4739	.73830	.18268	2.6456	.42253	2.8283	.45153	.93541
1.71	5.5290	.74264	.18087	2.6740	.42717	2.8549	.45559	.93665
1.72	5.5845	.74699	.17907	2.7027	.43180	2.8818	.45966	.93786
1.73	5.6407	.75133	.17728	2.7317	.43643	2.9090	.46374	.93906
1.74	5.6973	.75567	.17552	2.7609	.44105	2.9364	.46782	.94023
1.75	5.7546	.76002	.17377	2.7904	.44567	2.9642	.47191	.94138
1.76	5.8124	.76436	.17204	2.8202	.45028	2.9922	.47600	.94250
1.77	5.8709	.76870	.17033	2.8503	.45488	3.0206	.48009	.94361
1.78	5.9299	.77304	.16864	2.8806	.45948	3.0492	.48419	.94470
1.79	5.9895	.77739	.16696	2.9112	.46408	3.0782	.48830	.94576
1.80	6.0496	.78173	.16530	2.9422	.46867	3.1075	.49241	.94681
1.81	6.1104	.78607	.16365	2.9734	.47325	3.1371	.49652	.94783
1.82	6.1719	.79042	.16203	3.0049	.47783	3.1669	.50064	.94884
1.83	6.2339	.79476	.16041	3.0367	.48241	3.1972	.50476	.94983
1.84	6.2965	.79910	.15882	3.0689	.48698	3.2277	.50889	.95080
1.85	6.3598	.80344	.15724	3.1013	.49154	3.2585	.51302	.95175
1.86	6.4237	.80779	.15567	3.1340	.49610	3.2897	.51716	.95268
1.87	6.4883	.81213	.15412	3.1671	.50066	3.3212	.52130	.95359
1.88	6.5535	.81647	.15259	3.2005	.50521	3.3530	.52544	.95449
1.89	6.6194	.82082	.15107	3.2341	.50976	3.3852	.52959	.95537
1.90	6.6859	.82516	.14957	3.2682	.51430	3.4177	.53374	.95624
1.91	6.7531	.82950	.14808	3.3025	.51884	3.4506	.53789	.95709
1.92	6.8210	.83385	.14661	3.3372	.52338	3.4838	.54205	.95792
1.93	6.8895	.83819	.14515	3.3722	.52791	3.5173	.54621	.95873
1.94	6.9588	.84253	.14370	3.4075	.53244	3.5512	.55038	.95953
1.95	7.0287	.84687	.14227	3.4432	.53696	3.5855	.55455	.96032
1.96	7.0993	.85122	.14086	3.4792	.54148	3.6201	.55872	.96109
1.97	7.1707	.85556	.13946	3.5156	.54600	3.6551	.56290	.96185
1.98	7.2427	.85990	.13807	3.5523	.55051	3.6904	.56707	.96259
1.99	7.3155	.86425	.13670	3.5894	.55502	3.7261	.57126	.96331
2.00	7.3891	.86859	.13534	3.6269	.55953	3.7622	.57544	.96403

x	e^x Value	e^x Log10	e^{-x} Value	Sinh x Value	Sinh x Log10	Cosh x Value	Cosh x Log10	Tanh x Value
2.00	7.3891	.86859	.13534	3.6269	.55953	3.7622	.57544	.96403
2.01	7.4633	.87293	.13399	3.6647	.56403	3.7987	.57963	.96473
2.02	7.5383	.87727	.13266	3.7028	.56853	3.8355	.58382	.96541
2.03	7.6141	.88162	.13134	3.7414	.57303	3.8727	.58802	.96609
2.04	7.6906	.88596	.13003	3.7803	.57753	3.9103	.59221	.96675
2.05	7.7679	.89030	.12873	3.8196	.58202	3.9483	.59641	.96740
2.06	7.8460	.89465	.12745	3.8593	.58650	3.9867	.60061	.96803
2.07	7.9248	.89899	.12619	3.8993	.59099	4.0255	.60482	.96865
2.08	8.0045	.90333	.12493	3.9398	.59547	4.0647	.60903	.96926
2.09	8.0849	.90768	.12369	3.9806	.59995	4.1043	.61324	.96986
2.10	8.1662	.91202	.12246	4.0219	.60443	4.1443	.61745	.97045
2.11	8.2482	.91636	.12124	4.0635	.60890	4.1847	.62167	.97103
2.12	8.3311	.92070	.12003	4.1056	.61337	4.2256	.62589	.97159
2.13	8.4149	.92505	.11884	4.1480	.61784	4.2669	.63011	.97215
2.14	8.4994	.92939	.11765	4.1909	.62231	4.3085	.63433	.97269
2.15	8.5849	.93373	.11648	4.2342	.62677	4.3507	.63856	.97323
2.16	8.6711	.93808	.11533	4.2779	.63123	4.3932	.64278	.97375
2.17	8.7583	.94242	.11418	4.3221	.63569	4.4362	.64701	.97426
2.18	8.8463	.94676	.11304	4.3666	.64015	4.4797	.65125	.97477
2.19	8.9352	.95110	.11192	4.4116	.64460	4.5236	.65548	.97526
2.20	9.0250	.95545	.11080	4.4571	.64905	4.5679	.65972	.97574
2.21	9.1157	.95979	.10970	4.5030	.65350	4.6127	.66396	.97622
2.22	9.2073	.96413	.10861	4.5494	.65795	4.6580	.66820	.97668
2.23	9.2999	.96848	.10753	4.5962	.66240	4.7037	.67244	.97714
2.24	9.3933	.97282	.10646	4.6434	.66684	4.7499	.67668	.97759
2.25	9.4877	.97716	.10540	4.6912	.67128	4.7966	.68093	.97803
2.26	9.5831	.98151	.10435	4.7394	.67572	4.8437	.68518	.97846
2.27	9.6794	.98585	.10331	4.7880	.68016	4.8914	.68943	.97888
2.28	9.7767	.99019	.10228	4.8372	.68459	4.9395	.69368	.97929
2.29	9.8749	.99453	.10127	4.8868	.68903	4.9881	.69794	.97970
2.30	9.9742	.99888	.10026	4.9370	.69346	5.0372	.70219	.98010
2.31	10.074	.00322	.09926	4.9876	.69789	5.0868	.70645	.98049
2.32	10.176	.00756	.09827	5.0387	.70232	5.1370	.71071	.98087
2.33	10.278	.01191	.09730	5.0903	.70675	5.1876	.71497	.98124
2.34	10.381	.01625	.09633	5.1425	.71117	5.2388	.71923	.98161
2.35	10.486	.02059	.09537	5.1951	.71559	5.2905	.72349	.98197
2.36	10.591	.02493	.09442	5.2483	.72002	5.3427	.72776	.98233
2.37	10.697	.02928	.09348	5.3020	.72444	5.3954	.73203	.98267
2.38	10.805	.03362	.09255	5.3562	.72885	5.4487	.73630	.98301
2.39	10.913	.03796	.09163	5.4109	.73327	5.5026	.74056	.98335
2.40	11.023	.04231	.09072	5.4662	.73769	5.5569	.74484	.98367
2.41	11.134	.04665	.08982	5.5221	.74210	5.6119	.74911	.98400
2.42	11.246	.05099	.08892	5.5785	.74652	5.6674	.75338	.98431
2.43	11.359	.05534	.08804	5.6354	.75093	5.7235	.75766	.98462
2.44	11.473	.05968	.08716	5.6929	.75534	5.7801	.76194	.98492
2.45	11.588	.06402	.08629	5.7510	.75975	5.8373	.76621	.98522
2.46	11.705	.06836	.08543	5.8097	.76415	5.8951	.77049	.98551
2.47	11.822	.07271	.08458	5.8689	.76856	5.9535	.77477	.98579
2.48	11.941	.07705	.08374	5.9288	.77296	6.0125	.77906	.98607
2.49	12.061	.08139	.08291	5.9892	.77737	6.0721	.78334	.98635
2.50	12.182	.08574	.08208	6.0502	.78177	6.1323	.78762	.98661

x	e^x Value	e^x Log_{10}	e^{-x} Value	Sinh x Value	Sinh x Log_{10}	Cosh x Value	Cosh x Log_{10}	Tanh x Value
2.50	12.182	.08574	.08208	6.0502	.78177	6.1323	.78762	.98661
2.51	12.305	.09008	.08127	6.1118	.78617	6.1931	.79191	.98688
2.52	12.429	.09442	.08046	6.1741	.79057	6.2545	.79619	.98714
2.53	12.554	.09877	.07966	6.2369	.79497	6.3166	.80048	.98739
2.54	12.680	.10311	.07887	6.3004	.79937	6.3793	.80477	.98764
2.55	12.807	.10745	.07808	6.3645	.80377	6.4426	.80906	.98788
2.56	12.936	.11179	.07730	6.4293	.80816	6.5066	.81335	.98812
2.57	13.066	.11614	.07654	6.4946	.81256	6.5712	.81764	.98835
2.58	13.197	.12048	.07577	6.5607	.81695	6.6365	.82194	.98858
2.59	13.330	.12482	.07502	6.6274	.82134	6.7024	.82623	.98881
2.60	13.464	.12917	.07427	6.6947	.82573	6.7690	.83052	.98903
2.61	13.599	.13351	.07353	6.7628	.83012	6.8363	.83482	.98924
2.62	13.736	.13785	.07280	6.8315	.83451	6.9043	.83912	.98946
2.63	13.874	.14219	.07208	6.9008	.83890	6.9729	.84341	.98966
2.64	14.013	.14654	.07136	6.9709	.84329	7.0423	.84771	.98987
2.65	14.154	.15088	.07065	7.0417	.84768	7.1123	.85201	.99007
2.66	14.296	.15522	.06995	7.1132	.85206	7.1831	.85631	.99026
2.67	14.440	.15957	.06925	7.1854	.85645	7.2546	.86061	.99045
2.68	14.585	.16391	.06856	7.2583	.86083	7.3268	.86492	.99064
2.69	14.732	.16825	.06788	7.3319	.86522	7.3998	.86922	.99083
2.70	14.880	.17260	.06721	7.4063	.86960	7.4735	.87352	.99101
2.71	15.029	.17694	.06654	7.4814	.87398	7.5479	.87783	.99118
2.72	15.180	.18128	.06587	7.5572	.87836	7.6231	.88213	.99136
2.73	15.333	.18562	.06522	7.6338	.88274	7.6991	.88644	.99153
2.74	15.487	.18997	.06457	7.7112	.88712	7.7758	.89074	.99170
2.75	15.643	.19431	.06393	7.7894	.89150	7.8533	.89505	.99186
2.76	15.800	.19865	.06329	7.8683	.89588	7.9316	.89936	.99202
2.77	15.959	.20300	.06266	7.9480	.90026	8.0106	.90367	.99218
2.78	16.119	.20734	.06204	8.0285	.90463	8.0905	.90798	.99233
2.79	16.281	.21168	.06142	8.1098	.90901	8.1712	.91229	.99248
2.80	16.445	.21602	.06081	8.1919	.91339	8.2527	.91660	.99263
2.81	16.610	.22037	.06020	8.2749	.91776	8.3351	.92091	.99278
2.82	16.777	.22471	.05961	8.3586	.92213	8.4182	.92522	.99292
2.83	16.945	.22905	.05901	8.4432	.92651	8.5022	.92953	.99306
2.84	17.116	.23340	.05843	8.5287	.93088	8.5871	.93385	.99320
2.85	17.288	.23774	.05784	8.6150	.93525	8.6728	.93816	.99333
2.86	17.462	.24208	.05727	8.7021	.93963	8.7594	.94247	.99346
2.87	17.637	.24643	.05670	8.7902	.94400	8.8469	.94679	.99359
2.88	17.814	.25077	.05613	8.8791	.94837	8.9352	.95110	.99372
2.89	17.993	.25511	.05558	8.9689	.95274	9.0244	.95542	.99384
2.90	18.174	.25945	.05502	9.0596	.95711	9.1146	.95974	.99396
2.91	18.357	.26380	.05448	9.1512	.96148	9.2056	.96405	.99408
2.92	18.541	.26814	.05393	9.2437	.96584	9.2976	.96837	.99420
2.93	18.728	.27248	.05340	9.3371	.97021	9.3905	.97269	.99431
2.94	18.916	.27683	.05287	9.4315	.97458	9.4844	.97701	.99443
2.95	19.106	.28117	.05234	9.5268	.97895	9.5791	.98133	.99454
2.96	19.298	.28551	.05182	9.6231	.98331	9.6749	.98565	.99464
2.97	19.492	.28985	.05130	9.7203	.98768	9.7716	.98997	.99475
2.98	19.688	.29420	.05079	9.8185	.99205	9.8693	.99429	.99485
2.99	19.886	.29854	.05029	9.9177	.99641	9.9680	.99861	.99496
3.00	20.086	.30288	.04979	10.018	.00078	10.068	.00293	.99505

x	e^x Value	e^x Log$_{10}$	e^{-x} Value	Sinh x Value	Sinh x Log$_{10}$	Cosh x Value	Cosh x Log$_{10}$	Tanh x Value
3.00	20.086	.30288	.04979	10.018	.00078	10.068	.00293	.99505
3.05	21.115	.32460	.04736	10.534	.02259	10.581	.02454	.99552
3.10	22.198	.34631	.04505	11.076	.04440	11.122	.04616	.99595
3.15	23.336	.36803	.04285	11.647	.06620	11.689	.06779	.99633
3.20	24.533	.38974	.04076	12.246	.08799	12.287	.08943	.99668
3.25	25.790	.41146	.03877	12.876	.10977	12.915	.11108	.99700
3.30	27.113	.43317	.03688	13.538	.13155	13.575	.13273	.99728
3.35	28.503	.45489	.03508	14.234	.15332	14.269	.15439	.99754
3.40	29.964	.47660	.03337	14.965	.17509	14.999	.17605	.99777
3.45	31.500	.49832	.03175	15.734	.19685	15.766	.19772	.99799
3.50	33.115	.52003	.03020	16.543	.21860	16.573	.21940	.99818
3.55	34.813	.54175	.02872	17.392	.24036	17.421	.24107	.99835
3.60	36.598	.56346	.02732	18.285	.26211	18.313	.26275	.99851
3.65	38.475	.58517	.02599	19.224	.28385	19.250	.28444	.99865
3.70	40.447	.60689	.02472	20.211	.30559	20.236	.30612	.99878
3.75	42.521	.62860	.02352	21.249	.32733	21.272	.32781	.99889
3.80	44.701	.65032	.02237	22.339	.34907	22.362	.34951	.99900
3.85	46.993	.67203	.02128	23.486	.37081	23.507	.37120	.99909
3.90	49.402	.69375	.02024	24.691	.39254	24.711	.39290	.99918
3.95	51.935	.71546	.01925	25.958	.41427	25.977	.41459	.99926
4.00	54.598	.73718	.01832	27.290	.43600	27.308	.43629	.99933
4.10	60.340	.78061	.01657	30.162	.47946	30.178	.47970	.99945
4.20	66.686	.82404	.01500	33.336	.52291	33.351	.52310	.99955
4.30	73.700	.86747	.01357	36.843	.56636	36.857	.56652	.99963
4.40	81.451	.91090	.01228	40.719	.60980	40.732	.60993	.99970
4.50	90.017	.95433	.01111	45.003	.65324	45.014	.65335	.99975
4.60	99.484	.99775	.01005	49.737	.69668	49.747	.69677	.99980
4.70	109.95	.04118	.00910	54.969	.74012	54.978	.74019	.99983
4.80	121.51	.08461	.00823	60.751	.78355	60.759	.78361	.99986
4.90	134.29	.12804	.00745	67.141	.82699	67.149	.82704	.99989
5.00	148.41	.17147	.00674	74.203	.87042	74.210	.87046	.99991
5.10	164.02	.21490	.00610	82.008	.91386	82.014	.91389	.99993
5.20	181.27	.25833	.00552	90.633	.95729	90.639	.95731	.99994
5.30	200.34	.30176	.00499	100.17	.00072	100.17	.00074	.99995
5.40	221.41	.34519	.00452	110.70	.04415	110.71	.04417	.99996
5.50	244.69	.38862	.00409	122.34	.08758	122.35	.08760	.99997
5.60	270.43	.43205	.00370	135.21	.13101	135.22	.13103	.99997
5.70	298.87	.47548	.00335	149.43	.17444	149.44	.17445	.99998
5.80	330.30	.51891	.00303	165.15	.21787	165.15	.21788	.99998
5.90	365.04	.56234	.00274	182.52	.26130	182.52	.26131	.99998
6.00	403.43	.60577	.00248	201.71	.30473	201.72	.30474	.99999
6.25	518.01	.71434	.00193	259.01	.41331	259.01	.41331	.99999
6.50	665.14	.82291	.00150	332.57	.52188	332.57	.52189	1.0000
6.75	854.06	.93149	.00117	427.03	.63046	427.03	.63046	1.0000
7.00	1096.6	.04006	.00091	548.32	.73903	548.32	.73903	1.0000
7.50	1808.0	.25721	.00055	904.02	.95618	904.02	.95618	1.0000
8.00	2981.0	.47436	.00034	1490.5	.17333	1490.5	.17333	1.0000
8.50	4914.8	.69150	.00020	2457.4	.39047	2457.4	.39047	1.0000
9.00	8103.1	.90865	.00012	4051.5	.60762	4051.5	.60762	1.0000
9.50	13360.	.12580	.00007	6679.9	.82477	6679.9	.82477	1.0000
10.00	22026.	.34294	.00005	11013.	.04191	11013.	.04191	1.0000

[Characteristics of Logarithms omitted—determine by rule from the value]

°	0′ Value	0′ Log₁₀	10′ Value	10′ Log₁₀	20′ Value	20′ Log₁₀	30′ Value	30′ Log₁₀	40′ Value	40′ Log₁₀	50′ Value	50′ Log₁₀
0	.0000		.0000	4.3254	.0000	4.9275	.0000	5.2796	.0000	5.5295	.0001	5.7223
1	.0001	5.8817	.0001	6.0156	.0001	6.1315	.0002	.2338	.0002	.3254	.0003	.4081
2	.0003	.4837	.0004	.5532	.0004	.6176	.0005	.6775	.0005	.7336	.0006	.7862
3	.0007	.8358	.0008	.8828	.0008	.9273	.0009	.9697	.0010	.0101	.0011	.0487
4	.0012	.0856	.0013	.1211	.0014	.1551	.0015	.1879	.0017	.2195	.0018	.2499
5	.0019	.2793	.0020	.3078	.0022	.3354	.0023	.3621	.0024	.3880	.0026	.4132
6	.0027	.4376	.0029	.4614	.0031	.4845	.0032	.5071	.0034	.5290	.0036	.5504
7	.0037	.5713	.0039	.5918	.0041	.6117	.0043	.6312	.0045	.6503	.0047	.6689
8	.0049	.6872	.0051	.7051	.0053	.7226	.0055	.7397	.0057	.7566	.0059	.7731
9	.0062	.7893	.0064	.8052	.0066	.8208	.0069	.8361	.0071	.8512	.0073	.8660
10	.0076	.8806	.0079	.8949	.0081	.9090	.0084	.9229	.0086	.9365	.0089	.9499
11	.0092	.9631	.0095	.9762	.0097	.9890	.0100	.0016	.0103	.0141	.0106	.0264
12	.0109	.0385	.0112	.0504	.0115	.0622	.0119	.0738	.0122	.0853	.0125	.0966
13	.0128	.1077	.0131	.1187	.0135	.1296	.0138	.1404	.0142	.1510	.0145	.1614
14	.0149	.1718	.0152	.1820	.0156	.1921	.0159	.2021	.0163	.2120	.0167	.2218
15	.0170	.2314	.0174	.2409	.0178	.2504	.0182	.2597	.0186	.2689	.0190	.2781
16	.0194	.2871	.0198	.2961	.0202	.3049	.0206	.3137	.0210	.3223	.0214	.3309
17	.0218	.3394	.0223	.3478	.0227	.3561	.0231	.3644	.0236	.3726	.0240	.3806
18	.0245	.3887	.0249	.3966	.0254	.4045	.0258	.4123	.0263	.4200	.0268	.4276
19	.0272	.4352	.0277	.4427	.0282	.4502	.0287	.4576	.0292	.4649	.0297	.4721
20	.0302	.4793	.0307	.4865	.0312	.4936	.0317	.5006	.0322	.5075	.0327	.5144
21	.0332	.5213	.0337	.5281	.0343	.5348	.0348	.5415	.0353	.5481	.0359	.5547
22	.0364	.5612	.0370	.5677	.0375	.5741	.0381	.5805	.0386	.5868	.0392	.5931
23	.0397	.5993	.0403	.6055	.0409	.6116	.0415	.6177	.0421	.6238	.0426	.6298
24	.0432	.6357	.0438	.6417	.0444	.6476	.0450	.6534	.0456	.6592	.0462	.6650
25	.0468	.6707	.0475	.6764	.0481	.6820	.0487	.6876	.0493	.6932	.0500	.6987
26	.0506	.7042	.0512	.7096	.0519	.7151	.0525	.7204	.0532	.7258	.0538	.7311
27	.0545	.7364	.0552	.7416	.0558	.7468	.0565	.7520	.0572	.7572	.0578	.7623
28	.0585	.7673	.0592	.7724	.0599	.7774	.0606	.7824	.0613	.7874	.0620	.7923
29	.0627	.7972	.0634	.8020	.0641	.8069	.0648	.8117	.0655	.8165	.0663	.8213
30	.0670	.8260	.0677	.8307	.0684	.8354	.0692	.8400	.0699	.8446	.0707	.8492
31	.0714	.8538	.0722	.8583	.0729	.8629	.0737	.8673	.0744	.8718	.0752	.8763
32	.0760	.8807	.0767	.8851	.0775	.8894	.0783	.8938	.0791	.8981	.0799	.9024
33	.0807	.9067	.0815	.9109	.0823	.9152	.0831	.9194	.0839	.9236	.0847	.9277
34	.0855	.9319	.0863	.9360	.0871	.9401	.0879	.9442	.0888	.9482	.0896	.9523
35	.0904	.9563	.0913	.9603	.0921	.9643	.0929	.9682	.0938	.9722	.0946	.9761
36	.0955	.9800	.0963	.9838	.0972	.9877	.0981	.9915	.0989	.9954	.0998	.9992
37	.1007	.0030	.1016	.0067	.1024	.0105	.1033	.0142	.1042	.0179	.1051	.0216
38	.1060	.0253	.1069	.0289	.1078	.0326	.1087	.0362	.1096	.0398	.1105	.0434
39	.1114	.0470	.1123	.0505	.1133	.0541	.1142	.0576	.1151	.0611	.1160	.0646
40	.1170	.0681	.1179	.0716	.1189	.0750	.1198	.0784	.1207	.0817	.1217	.0853
41	.1226	.0887	.1236	.0920	.1246	.0954	.1255	.0987	.1265	.1021	.1275	.1054
42	.1284	.1087	.1294	.1119	.1304	.1152	.1314	.1185	.1323	.1217	.1333	.1249
43	.1343	.1282	.1353	.1314	.1363	.1345	.1373	.1377	.1383	.1409	.1393	.1440
44	.1403	.1472	.1413	.1503	.1424	.1534	.1434	.1565	.1444	.1596	.1454	.1626
45	.1464	.1657	.1475	.1687	.1485	.1718	.1495	.1748	.1506	.1778	.1516	.1808
46	.1527	.1838	.1538	.1867	.1548	.1897	.1558	.1926	.1569	.1956	.1579	.1985
47	.1590	.2014	.1600	.2043	.1611	.2072	.1622	.2101	.1633	.2129	.1644	.2158
48	.1654	.2186	.1665	.2215	.1676	.2243	.1687	.2271	.1698	.2299	.1709	.2327
49	.1720	.2355	.1731	.2382	.1742	.2410	.1753	.2437	.1764	.2465	.1775	.2492
50	.1786	.2519	.1797	.2546	.1808	.2573	.1820	.2600	.1831	.2627	.1842	.2653
51	.1853	.2680	.1865	.2706	.1876	.2732	.1887	.2759	.1899	.2785	.1910	.2811
52	.1922	.2837	.1933	.2863	.1945	.2888	.1956	.2914	.1968	.2940	.1979	.2965
53	.1991	.2991	.2003	.3016	.2014	.3041	.2026	.3066	.2038	.3091	.2049	.3116
54	.2061	.3141	.2073	.3166	.2085	.3190	.2096	.3215	.2108	.3239	.2120	.3264
55	.2132	.3288	.2144	.3312	.2156	.3336	.2168	.3361	.2180	.3384	.2192	.3408
56	.2204	.3432	.2216	.3456	.2228	.3480	.2240	.3503	.2252	.3527	.2265	.3550
57	.2277	.3573	.2289	.3596	.2301	.3620	.2314	.3643	.2326	.3666	.2338	.3689
58	.2350	.3711	.2363	.3734	.2375	.3757	.2388	.3779	.2400	.3802	.2412	.3824
59	.2425	.3847	.2437	.3869	.2450	.3891	.2462	.3913	.2475	.3935	.2487	.3957

[Characteristics of Logarithms omitted—determine by rule from the value]

°	0′ Value	0′ Log10	10′ Value	10′ Log10	20′ Value	20′ Log10	30′ Value	30′ Log10	40′ Value	40′ Log10	50′ Value	50′ Log10
60	.2500	.3979	.2513	.4001	.2525	.4023	.2538	.4045	.2551	.4066	.2563	.4088
61	.2576	.4109	.2589	.4131	.2601	.4152	.2614	.4173	.2627	.4195	.2640	.4216
62	.2653	.4237	.2665	.4258	.2678	.4279	.2691	.4300	.2704	.4320	.2717	.4341
63	.2730	.4362	.2743	.4382	.2756	.4403	.2769	.4423	.2782	.4444	.2795	.4464
64	.2808	.4484	.2821	.4504	.2834	.4524	.2847	.4545	.2861	.4565	.2874	.4584
65	.2887	.4604	.2900	.4624	.2913	.4644	.2927	.4664	.2940	.4683	.2953	.4703
66	.2966	.4722	.2980	.4742	.2993	.4761	.3006	.4780	.3020	.4799	.3033	.4819
67	.3046	.4838	.3060	.4857	.3073	.4876	.3087	.4895	.3100	.4914	.3113	.4932
68	.3127	.4951	.3140	.4970	.3154	.4989	.3167	.5007	.3181	.5026	.3195	.5044
69	.3208	.5063	.3222	.5081	.3235	.5099	.3249	.5117	.3263	.5136	.3276	.5154
70	.3290	.5172	.3304	.5190	.3317	.5208	.3331	.5226	.3345	.5244	.3358	.5261
71	.3372	.5279	.3386	.5297	.3400	.5314	.3413	.5332	.3427	.5349	.3441	.5367
72	.3455	.5384	.3469	.5402	.3483	.5419	.3496	.5436	.3510	.5454	.3524	.5471
73	.3538	.5488	.3552	.5505	.3566	.5522	.3580	.5539	.3594	.5556	.3608	.5572
74	.3622	.5589	.3636	.5606	.3650	.5623	.3664	.5639	.3678	.5656	.3692	.5672
75	.3706	.5689	.3720	.5705	.3734	.5722	.3748	.5738	.3762	.5754	.3776	.5771
76	.3790	.5787	.3805	.5803	.3819	.5819	.3833	.5835	.3847	.5851	.3861	.5867
77	.3875	.5883	.3889	.5899	.3904	.5915	.3918	.5930	.3932	.5946	.3946	.5962
78	.3960	.5977	.3975	.5993	.3989	.6009	.4003	.6024	.4017	.6039	.4032	.6055
79	.4046	.6070	.4060	.6085	.4075	.6101	.4089	.6116	.4103	.6131	.4117	.6146
80	.4132	.6161	.4146	.6176	.4160	.6191	.4175	.6206	.4189	.6221	.4203	.6236
81	.4218	.6251	.4232	.6266	.4247	.6280	.4261	.6295	.4275	.6310	.4290	.6324
82	.4304	.6339	.4319	.6353	.4333	.6368	.4347	.6382	.4362	.6397	.4376	.6411
83	.4391	.6425	.4405	.6440	.4420	.6454	.4434	.6468	.4448	.6482	.4463	.6496
84	.4477	.6510	.4492	.6524	.4506	.6538	.4521	.6552	.4535	.6566	.4550	.6580
85	.4564	.6594	.4579	.6607	.4593	.6621	.4608	.6635	.4622	.6649	.4637	.6662
86	.4651	.6676	.4666	.6689	.4680	.6703	.4695	.6716	.4709	.6730	.4724	.6743
87	.4738	.6756	.4753	.6770	.4767	.6783	.4782	.6796	.4796	.6809	.4811	.6822
88	.4826	.6835	.4840	.6848	.4855	.6862	.4869	.6875	.4884	.6887	.4898	.6900
89	.4913	.6913	.4937	.6926	.4942	.6939	.4956	.6952	.4971	.6964	.4985	.6977
90	.5000	.6990	.5015	.7002	.5029	.7015	.5044	.7027	.5058	.7040	.5073	.7052
91	.5087	.7065	.5102	.7077	.5116	.7090	.5131	.7102	.5145	.7114	.5160	.7126
92	.5174	.7139	.5189	.7151	.5204	.7163	.5218	.7175	.5233	.7187	.5247	.7199
93	.5262	.7211	.5276	.7223	.5291	.7235	.5305	.7247	.5320	.7259	.5334	.7271
94	.5349	.7283	.5363	.7294	.5378	.7306	.5392	.7318	.5407	.7329	.5421	.7341
95	.5436	.7353	.5450	.7364	.5465	.7376	.5479	.7387	.5494	.7399	.5508	.7410
96	.5523	.7421	.5537	.7433	.5552	.7444	.5566	.7455	.5580	.7467	.5595	.7478
97	.5609	.7489	.5624	.7500	.5638	.7511	.5653	.7523	.5667	.7534	.5682	.7545
98	.5696	.7556	.5710	.7567	.5725	.7577	.5739	.7588	.5753	.7599	.5768	.7610
99	.5782	.7621	.5797	.7632	.5811	.7642	.5825	.7653	.5840	.7664	.5854	.7674
100	.5868	.7685	.5883	.7696	.5897	.7706	.5911	.7717	.5925	.7727	.5940	.7738
101	.5954	.7748	.5968	.7759	.5983	.7769	.5997	.7779	.6011	.7790	.6025	.7800
102	.6040	.7810	.6054	.7820	.6068	.7830	.6082	.7841	.6096	.7851	.6111	.7861
103	.6125	.7871	.6139	.7881	.6153	.7891	.6167	.7901	.6181	.7911	.6195	.7921
104	.6210	.7931	.6224	.7940	.6238	.7950	.6252	.7960	.6266	.7970	.6280	.7980
105	.6294	.7989	.6308	.7999	.6322	.8009	.6336	.8018	.6350	.8028	.6364	.8037
106	.6378	.8047	.6392	.8056	.6406	.8066	.6420	.8075	.6434	.8085	.6448	.8094
107	.6462	.8104	.6476	.8113	.6490	.8122	.6504	.8131	.6517	.8141	.6531	.8150
108	.6545	.8159	.6559	.8168	.6573	.8177	.6587	.8187	.6600	.8196	.6614	.8205
109	.6628	.8214	.6642	.8223	.6655	.8232	.6669	.8241	.6683	.8250	.6696	.8258
110	.6710	.8267	.6724	.8276	.6737	.8285	.6751	.8294	.6765	.8302	.6778	.8311
111	.6792	.8320	.6805	.8329	.6819	.8337	.6833	.8346	.6846	.8354	.6860	.8363
112	.6873	.8371	.6887	.8380	.6900	.8388	.6913	.8397	.6927	.8405	.6940	.8414
113	.6954	.8422	.6967	.8430	.6980	.8439	.6994	.8447	.7007	.8455	.7020	.8464
114	.7034	.8472	.7047	.8480	.7060	.8488	.7073	.8496	.7087	.8504	.7100	.8513
115	.7113	.8521	.7126	.8529	.7139	.8537	.7153	.8545	.7166	.8553	.7179	.8561
116	.7192	.8568	.7205	.8576	.7218	.8584	.7231	.8592	.7244	.8600	.7257	.8608
117	.7270	.8615	.7283	.8623	.7296	.8631	.7309	.8638	.7322	.8646	.7335	.8654
118	.7347	.8661	.7360	.8669	.7373	.8676	.7386	.8684	.7399	.8691	.7411	.8699
119	.7424	.8706	.7437	.8714	.7449	.8721	.7462	.8729	.7475	.8736	.7487	.8743

[Characteristics of Logarithms omitted—determine by rule from the value]

	0′		10′		20′		30′		40′		50′	
	Value	Log_{10}	Value	Log_{10}	Value	Log_{10}	Value	Log_{10}	Value	Log_{10}	Value	Log_{10}
120	.7500	.8751	.7513	.8758	.7525	.8765	.7538	.8772	.7550	.8780	.7563	.8787
121	.7575	.8794	.7588	.8801	.7600	.8808	.7612	.8815	.7625	.8822	.7637	.8829
122	.7650	.8836	.7662	.8843	.7674	.8850	.7686	.8857	.7699	.8864	.7711	.8871
123	.7723	.8878	.7735	.8885	.7748	.8892	.7760	.8898	.7772	.8905	.7784	.8912
124	.7796	.8919	.7808	.8925	.7820	.8932	.7832	.8939	.7844	.8945	.7856	.8952
125	.7868	.8959	.7880	.8965	.7892	.8972	.7904	.8978	.7915	.8985	.7927	.8991
126	.7939	.8998	.7951	.9004	.7962	.9010	.7974	.9017	.7986	.9023	.7997	.9030
127	.8009	.9036	.8021	.9042	.8032		.8044	.9055	.8055	.9061	.8067	.9067
128	.8078	.9073	.8090	.9079	.8101	.9085	.8113	.9092	.8124	.9098	.8135	.9104
129	.8147	.9110	.8158	.9116	.8169	.9122	.8180	.9128	.8192	.9134	.8203	.9140
130	.8214	.9146	.8225	.9151	.8236	.9157	.8247	.9163	.8258	.9169	.8269	.9175
131	.8280	.9180	.8291	.9186	.8302	.9192	.8313	.9198	.8324	.9203	.8335	.9209
132	.8346	.9215	.8356	.9220	.8367	.9226	.8378	.9231	.8389	.9237	.8399	.9242
133	.8410	.9248	.8421	.9253	.8431	.9259	.8442	.9264	.8452	.9270	.8463	.9275
134	.8473	.9281	.8484		.8494	.9291	.8501	.9297	.8515	.9302	.8525	.9307
135	.8536	.9312	.8546	.9318	.8556	.9323	.8566	.9328	.8576	.9333	.8587	.9338
136	.8597	.9343	.8607	.9348	.8617	.9353	.8627	.9359	.8637	.9364	.8647	.9369
137	.8657	.9374	.8667	.9379	.8677	.9383	.8686	.9388		.9393	.8706	.9398
138	.8716	.9403	.8725	.9408	.8735	.9413	.8745	.9417	.8754	.9422	.8764	.9427
139	.8774	.9432	.8783	.9436	.8793	.9441	.8802	.9446	.8811	.9450	.8821	.9455
140	.8830	.9460	.8840	.9464	.8849	.9469	.8858	.9473	.8867	.9478	.8877	.9482
141		.9487	.8895	.9491	.8904	.9496	.8913	.9500	.8922	.9505	.8931	.9509
142	.8940	.9513	.8949	.9518	.8958	.9522	.8967	.9526	.8976	.9531	.8984	.9535
143	.8993	.9539	.9002	.9543	.9011	.9548	.9019	.9552	.9028	.9556	.9037	.9560
144	.9045	.9564	.9054	.9568	.9062	.9572	.9071	.9576	.9079	.9580	.9087	.9584
145	.9096	.9588	.9104	.9592	.9112	.9596	.9121	.9600	.9129	.9604	.9137	.9608
146	.9145	.9612	.9153	.9616	.9161	.9620	.9169	.9623	.9177	.9627	.9185	.9631
147	.9193	.9635	.9201	.9638	.9209	.9642	.9217	.9646	.9225	.9650	.9233	.9653
148	.9240	.9657	.9248	.9660	.9256	.9664	.9263	.9668	.9271	.9671	.9278	.9675
149	.9286	.9678	.9293	.9682	.9301	.9685	.9308	.9689	.9316	.9692	.9323	.9695
150	.9330	.9699	.9337	.9702	.9345	.9706	.9352	.9709	.9359	.9712	.9366	.9716
151	.9373	.9719	.9380	.9722	.9387	.9725	.9394	.9729	.9401	.9732	.9408	.9735
152	.9415	.9738	.9422	.9741	.9428	.9744	.9435	.9747	.9442	.9751	.9448	.9754
153	.9455	.9757	.9462	.9760	.9468	.9763	.9475	.9766	.9481	.9769	.9488	.9772
154	.9494	.9774	.9500	.9777	.9507	.9780	.9513	.9783	.9519	.9786	.9525	.9789
155	.9532	.9792	.9538	.9794	.9544	.9797	.9550	.9800	.9556	.9803	.9562	.9805
156	.9568	.9808	.9574	.9811	.9579	.9813	.9585	.9816	.9591	.9819	.9597	.9821
157	.9603	.9824	.9608	.9826	.9614	.9829	.9619	.9831	.9625	.9834	.9630	.9836
158	.9636	.9839	.9641	.9841	.9647	.9844	.9652	.9846	.9657	.9849	.9663	.9851
159	.9668	.9853	.9673	.9856	.9678	.9858	.9683	.9860		.9863	.9693	.9865
160	.9698	.9867	.9703	.9869	.9708	.9871	.9713	.9874	.9718	.9876	.9723	.9878
161	.9728	.9880	.9732		.9737	.9884	.9742	.9886	.9746		.9751	.9890
162	.9755	.9892	.9760		.9764		.9769	.9898	.9773	.9900	.9777	.99[illegible]
163	.9782	.9904	.9786	.9906	.9790	.9908	.9794	.9910	.9798	.9911	.9802	
164	.9806	.9915	.9810	.9917	.9814	.9919	.9818	.9920	.9822	.9922	.9826	
165	.9830	.9925	.9833	.9927	.9837	.9929	.9841	.9930	.9844	.9932	.9848	
166	.9851	.9935	.9855	.9937		.9938	.9862	.9940	.9865	.9941	.9869	
167	.9872	.9944	.9875	.9945	.9878	.9947	.9881	.9948	.9885	.9950	.9888	
168	.9891	.9952		.9954	.9897	.9955	.9900	.9956	.9903	.9957	.9905	
169	.9908	.9960	.9911	.9961	.9914	.9962	.9916	.9963	.9919	.9965	.9921	
170	.9924	.9967	.9927	.9968	.9929	.9969	.9931	.9970	.9934	.9971	.9936	
171	.9938	.9973	.9941	.9974	.9943	.9975	.9945	.9976	.9947	.9977	.9949	
172	.9951	.9979	.9953	.9980	.9955	.9981	.9957	.9981	.9959	.9982	.9961	.998
173	.9963	.9984	.9964	.9984		.9985	.9968	.9986	.9969	.9987	.9971	
174	.9973	.9988	.9974		.9976	.9989	.9977	.9990	.9978	.9991	.9980	.9991
175	.9981	.9992	.9982	.9992	.9983	.9993	.9985	.9993	.9986	.9994	.9987	.9994
176	.9988	.9995	.9989		.9990		.9991	.9996	.9992	.9996	.9992	.9997
177	.9993	.9997	.9994	.9997	.9995		.9995	.9998	.9996	.9998		.9998
178	.9997	.9999	.9997	.9999	.9998	.9999	.9998	.9999	.9999	.9999		.9999
179	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.0000	1.0000	.0000

Table XI — Factor Table — Logarithms of Primes

[If N is prime, its logarithm is given. If N is not prime, its factors are given.]

N	*1*	*3*	*7*	*9*
10	0043213738	0128372247	0293837777	0374264979
11	$3\cdot 37$	0530784435	$3^2\cdot 13$	$7\cdot 17$
12	11^2	$3\cdot 41$	1038037210	$3\cdot 43$
13	1172712957	$7\cdot 19$	1367205672	1430148003
14	$3\cdot 47$	$11\cdot 13$	$3\cdot 7^2$	1731862684
15	1789769473	$3^2\cdot 17$	1958996524	$3\cdot 53$
16	$7\cdot 23$	2121876044	2227164711	13^2
17	$3^2\cdot 19$	2380461031	$3\cdot 59$	2528530310
18	2576785749	$3\cdot 61$	$11\cdot 17$	$3^3\cdot 7$
19	2810333672	2855573090	2944662262	2988530764
20	$3\cdot 67$	$7\cdot 29$	$3^2\cdot 23$	$11\cdot 19$
21	3242824553	$3\cdot 71$	$7\cdot 31$	$3\cdot 73$
22	$13\cdot 17$	3483048630	3560258572	3598354823
23	$3\cdot 7\cdot 11$	3673559210	$3\cdot 79$	3783979009
24	3820170426	3^5	$13\cdot 19$	$3\cdot 83$
25	3996737215	$11\cdot 23$	4099331233	$7\cdot 37$
26	$3^2\cdot 29$	4199557485	$3\cdot 89$	4297522800
27	4329692909	$3\cdot 7\cdot 13$	4424797691	$3^2\cdot 31$
28	4487063199	4517864355	$7\cdot 41$	17^2
29	$3\cdot 97$	4668676204	$3^3\cdot 11$	$13\cdot 23$
30	$7\cdot 43$	$3\cdot 101$	4871383755	$3\cdot 103$
31	4927603890	4955443375	5010592622	$11\cdot 29$
32	$3\cdot 107$	$17\cdot 19$	$3\cdot 109$	$7\cdot 47$
33	5198279938	$3^2\cdot 37$	5276299009	$3\cdot 113$
34	$11\cdot 31$	7^3	5403294748	5428254270
35	$3^3\cdot 13$	5477747054	$3\cdot 7\cdot 17$	5550944486
36	19^2	$3\cdot 11^2$	5646660643	$3^2\cdot 41$
37	$7\cdot 53$	5717088318	$13\cdot 29$	5786392100
38	$3\cdot 127$	5831987740	$3^2\cdot 43$	5809496013
39	$17\cdot 23$	$3\cdot 131$	5987905068	$3\cdot 7\cdot 19$
40	6031443726	$13\cdot 31$	$11\cdot 37$	6117233080
41	$3\cdot 137$	$7\cdot 59$	$3\cdot 139$	6222140230
42	6242820958	$3^2\cdot 47$	$7\cdot 61$	$3\cdot 11\cdot 13$
43	6344772702	6364878964	$19\cdot 23$	6424645202
44	$3^2\cdot 7^2$	6464037262	$3\cdot 149$	6522463410
45	$11\cdot 41$	$3\cdot 151$	6599162001	$3^3\cdot 17$
46	6637009254	6655809910	6693168806	$7\cdot 67$
47	$3\cdot 157$	$11\cdot 43$	$3^2\cdot 53$	6803355134
48	$13\cdot 37$	$3\cdot 7\cdot 23$	6875289612	$3\cdot 163$
49	6910814921	$17\cdot 29$	$7\cdot 71$	6981005456
50	$3\cdot 167$	7015679851	$3\cdot 13^2$	7067177823
51	$7\cdot 73$	$3^3\cdot 19$	$11\cdot 47$	$3\cdot 173$
52	7168377233	7185016889	$17\cdot 31$	23^2
53	$3^2\cdot 59$	$13\cdot 41$	$3\cdot 179$	$7^2\cdot 11$
54	7331972651	$3\cdot 181$	7379873263	$3^2\cdot 61$
55	$19\cdot 29$	$7\cdot 79$	7458551952	$13\cdot 43$
56	$3\cdot 11\cdot 17$	7505083949	$3^4\cdot 7$	7551122664
57	7566361082	$3\cdot 191$	7611758132	$3\cdot 193$
58	$7\cdot 83$	$11\cdot 53$	7686381012	$19\cdot 31$
59	$3\cdot 197$	7730546934	$3\cdot 199$	7774268224
60	7788744720	$3^2\cdot 67$	7831886911	$3\cdot 7\cdot 29$
61	$13\cdot 47$	7874604745	7902851640	7916906490
62	$3^3\cdot 23$	$7\cdot 89$	$3\cdot 11\cdot 19$	$17\cdot 37$
63	8000293592	$3\cdot 211$	$7^2\cdot 13$	$3^2\cdot 71$
64	8068580295	8082109729	8109042807	$11\cdot 59$
65	$3\cdot 7\cdot 31$	8149131813	$3^2\cdot 73$	8188854146
66	8202014595	$3\cdot 13\cdot 17$	$23\cdot 29$	$3\cdot 223$
67	$11\cdot 61$	8280150642	8305886687	$7\cdot 97$
68	$3\cdot 227$	8344207037	$3\cdot 229$	$13\cdot 53$
69	8394780474	$3^2\cdot 7\cdot 11$	$17\cdot 41$	$3\cdot 233$

N	*Log N*
2	301029995664
3	477121254720
5	698970004336
7	845098040014
11	041392685158
13	113943352307
17	230448921378
19	278753600953
23	361727836018
29	462397997899
31	491361693834
37	568201724067
41	612783856720
43	633468455580
47	672097857936
53	724275869601
59	770852011642
61	785329835011
67	826074802701
71	851258348719
73	863322860120
79	897627091290
83	919078092376
89	949390006645
97	986771734266
1301	1142772966
1303	1149444157
1307	1162755876
1319	1202447955
1321	1209028176
1327	1228709229
1361	1338581252
1367	1357685146
1373	1376705372
1381	1401936786
1399	1458177145
1409	1489109931
1423	1532049001
1427	1544239731
1429	1550322288
1433	1562461904
1439	1580607939
1447	1604685311
1451	1616674124
1453	1622656143
1459	1640552919
1471	1676126727
1481	1705550585
1483	1711411510
1487	1723109685
1489	1728946978
1493	1740598077
1499	1758016328
1511	1792644643
1523	1826999033
1531	1849751907
1543	1883659261
1549	1900514178
1553	1911714557
1559	1928461152

[If *N* is a prime, its logarithm is given. If *N* is not a prime, its factors are given.]

N	*1*	*3*	*7*	*9*
70	8457180180	19·37	7·101	8506462352
71	3^2·79	23·31	3·239	8567288904
72	7·103	3·241	8615344109	3^6
73	17·43	8651039746	11·67	8686444384
74	3·13·19	8709888138	3^2·83	7·107
75	8756399370	3·251	8790958795	3·11·23
76	8813846568	7·109	13·59	8859263398
77	3·257	8881794939	3·7·37	19·41
78	11·71	3^3·29	8959747324	3·263
79	7·113	13·61	9014583214	17·47
80	3^2·89	11·73	3·269	9079485216
81	9090208542	3·271	19·43	3^2·7·13
82	9143431571	9153998352	9175055096	9185545306
83	3·277	7^2·17	3^3·31	9237619608
84	29^2	3·281	7·11^2	3·283
85	23·37	9309490312	9329808219	9339931638
86	3·7·41	9360107957	3·17^2	11·79
87	13·67	3^2·97	9429995934	3·293
88	9449759084	9459607036	9479236198	7·127
89	3^4·11	19·47	3·13·23	29·31
90	17·53	3·7·43	9576072871	3^2·101
91	9595183770	11·83	7·131	9633155114
92	3·307	13·71	3^2·103	9680157140
93	7^2·19	3·311	9717395909	3·313
94	9735896234	23·41	9763499790	13·73
95	3·317	9790929006	3·11·29	7·137
96	31^2	3^2·107	9854264741	3·17·19
97	9872192299	7·139	9898945637	11·89
98	3^2·109	9925535178	3·7·47	23·43
99	9960736545	3·331	9986951583	3^3·37
100	7·11·13	17·59	19·53	0038911662
101	3·337	0056094454	3^2·113	0081741840
102	0090257421	3·11·31	13·79	3·7^3
103	0132586653	0141003215	17·61	0166155476
104	3·347	7·149	3·349	0207754882
105	0216027160	3^4·13	7·151	3·353
106	0257153839	0265332645	11·97	0289777052
107	3^2·7·17	29·37	3·359	13·83
108	23·47	3·19^2	0362295441	3^2·11^2
109	0378247506	0386201619	0402066276	7·157
110	3·367	0425755124	3^3·41	0449315461
111	11·101	3·7·53	0480531731	3·373
112	19·59	0503797563	7^2·23	0526939419
113	3·13·29	11·103	3·379	17·67
114	7·163	3^2·127	31·37	3·383
115	0610753236	0618293073	13·89	19·61
116	3^3·43	0655797147	3·389	7·167
117	0685568951	3·17·23	11·107	3^2·131
118	0722498976	7·13^2	0744507190	29·41
119	3·397	0766404437	3^2·7·19	11·109
120	0795430074	3·401	17·71	3·13·31
121	7·173	0838608009	0852905782	23·53
122	3·11·37	0874264570	3·409	0895518829
123	0902580529	3^2·137	0923696996	3·7·59
124	17·73	11·113	29·43	0965624384
125	3^2·139	7·179	3·419	1000257301
126	13·97	3·421	7·181	3^3·47
127	31·41	19·67	1061908973	1068705445
128	3·7·61	1082266564	3^2·11·13	1102529174
129	1109262423	3·431	1129399761	3·433

N	*Log N*
1567	1950689965
1571	1961761850
1579	1983821300
1583	1994809149
1597	2033049161
1601	2043913319
1607	2060158768
1609	2065560441
1613	2076343674
1619	2092468488
1621	2097830148
1627	2113875529
1637	2140486794
1657	2193225084
1663	2208922492
1667	2219355998
1669	2224563367
1693	2286569581
1697	2296818423
1699	2301933789
1709	2327420627
1721	2357808703
1723	2362852774
1733	2387985627
1741	2407987711
1747	2422929050
1753	2437819161
1759	2452658395
1777	2496874278
1783	2511513432
1787	2521245525
1789	2526103406
1801	2555137128
1811	2579184503
1823	2607866687
1831	2626883443
1847	2664668954
1861	2697463731
1867	2711443179
1871	2720737875
1873	2725377774
1877	2734642726
1879	2739267801
1889	2762319579
1901	2789821169
1907	2803506930
1913	2817149700
1931	2857822738
1933	2862318540
1949	2898118391
1951	2902572694
1973	2961270853
1979	2964457942
1987	2981978671
1993	2995072987
1997	3003780649
1999	3008127941
2003	3016809493
2011	3034120706
2017	3047058982

AMOUNT OF ONE DOLLAR PRINCIPAL AT COMPOUND INTEREST AFTER n YEARS

	2 %	2½ %	3 %	3½ %	4 %		5 %	6 %
	1.020	1.025	1.030	1.035	1.040	1.045	1.050	1.060
	1.0404	1.050	1.060	1.071	1.081	1.092	1.102	1.123
	1.0612	1.076	1.092	1.108	1.124	1.141	1.157	1.191
	1.082	1.103	1.125	1.147	1.169	1.192	1.215	1.262
	1.104	1.131	1.159	1.187	1.216	1.246	1.276	1.338
	1.126	1.159	1.194	1.229	1.265	1.302	1.340	1.418
	1.148	1.1887	1.229	1.272	1.315	1.360	1.4071	1.503
	1.171	1.2184	1.266	1.316	1.368	1.422	1.477	1.593
	1.195	1.2489	1.3048	1.362	1.423	1.486	1.551	1.689
	1.219	1.2801	1.343	1.410	1.480	1.553	1.628	1.790
1	1.2434	1.3121	1.3842	1.460	1.539	1.622	1.710	1.898
12	1.2682	1.3449	1.425	1.511	1.601	1.695	1.7959	2.0122
13	1.2936	1.3785	1.4685	1.564	1.6651	1.7722	1.8856	2.132
1	1.319	1.4130	1.5126	1.6187	1.7317	1.8519	1.9799	2.2609
1	1.3459	1.4483	1.5580	1.6753	1.8009	1.9353	2.0789	2.3966
16	1.3728	1.4845	1.6047	1.7340	1.873	2.0224	2.1829	2.5404
17	1.400	1.5216	1.6528	1.7947	1.9479	2.1134	2.2920	2.6928
18	1.4282	1.559	1.7024	1.8575	2.0258	2.2085	2.4066	2.8543
19	1.4568	1.5987	1.7535	1.922	2.1068	2.3079	2.5270	3.0256
20	1.4859	.6386	1.8061	1.9898	2.1911	2.4117	2.6533	3.2071
21	1.51	1.6796	1.8603	2.0594	2.2788	2.5202	2.7860	3.3996
2	1.5460	1.7216	1.9161	2.1315	2.3699	2.6337	2.9253	3.603
23	1.5769	1.7646	1.9736	2.2061	2.4647	2.7522	3.0715	3.8197
24	1.6084	1.8087	2.0328	2.2833	2.5633	2.8760	3.2251	4.0489
25	1.6406	1.8539	2.0938	2.3632	2.6658	3.0054	3.3864	4.2919
26	1.6734	1.9003	2.1566	2.4460	2.7725	3.1407	3.5557	4.5494
27	1.7069	1.9478	2.2213	2.5316	2.8834	3.2820	3.7335	4.8223
28	1.7410	1.9965	2.2879	2.6202	2.9987	3.4297	3.9201	5.1117
29	1.7758	2.0464	2.3566	2.7119	3.1187	3.5840	4.1161	5.4184
30	1.8114	2.0976	2.4273	2.8068	.2434	.7453	4.3219	5.7435
31	.8476	2.1500	2.5001	.9050	.3731	.9139	4.5380	6.0881
32	.8845	2.2038	2.5751	.0067	.5081	.0900	4.7649	6.4534
33	.9222	2.2589	2.6523	.1119	.6484	.2740	5.0032	6.8406
34	.9607	2.3153	2.7319	.2209	.7943	.4664	5.2533	7.2510
35	.9999	.3732	2.8139	.3336	.9461	.6673	5.5160	7.6861
36	.0399	2.4325	.8983	.4503	4.1039	.8774	5.7918	8.1473
37	.0807	.4933	.9852	.5710	.2681	.0969	6.0814	8.6361
38	.1223	.5557	.0748	.6960	.4388	.3262	6.3855	9.1543
39	.1647	.6196	.1670	.8254	.6164	.5659	6.7048	9.7035
40	.2080	.6851	.2620	.9593	.8010	.8164	7.0400	0.2857
41	.2522	.7522	.3599	.0978	.9931	.0781	7.3920	0.9029
42	.2972	.8210	.4607	.2413	.1928	.3516	7.7616	1.5570
43	.3432	.8915	.5645	.3897	.4005	.6374	8.1497	2.2505
44	.3901	.9638	.6715	.5433	.6165	.9361	8.5572	2.9855
45	.4379	.0379	.7816	.7024	.8412	.2482	8.9850	3.7646
46	.4866	.1139	.8950	.8669	.0748	.5744	9.4343	4.5905
47	.5363	.1917	.0119	.0373	.3178	.9153	9.9060	5.4659
48	.5871	.2715	1323	.2136	.5705	.2715	0.4013	6.3939
49	.6388	.3533	.2562	3961	.8333	.6437	0.9213	7.3775
50	.6916	.4371	.3839	.5849	.1067	.0326	1.4674	8.4202

 Table XII b — Compound Discount: $1/(1+r)^n$

PRESENT VALUE OF ONE DOLLAR DUE AT THE END OF n YEARS

n	2 %	2½ %	3 %	3½ %	4 %	4½ %	5 %	6 %	7 %
1	.98039	.97561	.97087	[illegible]	[illegible]	.95694	.95238	.94340	.93458
2	.96117	.95181	.94260	.93351	.92456	.91573	.90703	.89000	.87344
	.94232	.92860	.91514	.90194	.88900	.87630	.86384	.83962	.81630
	.92385	.90595	.88849	.87144	.85480	.83856	.82270	.79209	.76290
	.90573	.88385	.86261	.84197	.82193	.80245	.78353	.74726	.71299
	.88797	.86230	.83748	.81350	.79031	.76790	.74622	.70496	.66634
	.87056	.84127	.81309	.78599	.75992	.73483	.71068	.66506	.62275
	.85349	.82075	.78941	.75941	.73069	.70319	.67684	.62741	.58201
	.83676	.80073	.76642	.73373	.70259	.67290	.64461	.59190	.54393
10	.82035	.78120	.74409	.70892	.67556	.64393	.61391	.55839	.50835
	.80426	.76214	.72242	.68495	.64958	.61620	.58468	.52679	.47509
	.78849	.74356	.70138	.66178	.62460	.58966	.55684	.49697	.44401
	.77303	.72542	.68095	.63940	.60057	.56427	.53032	.46884	.41496
	.75788	.70773	.66112	.61778	.57748	.53997	.50507	.44230	.38782
	.74301	.69047	.64186	.59689	.55526	.51672	.48102	.41727	.36245
	.72845	.67362	.62317	.57671	.53391	.49447	.45811	.39365	.33873
	.71416	.65720	.60502	.55720	.51337	.47318	.43630	.37136	.31657
	.70016	.64117	.58739	.53836	.49363	.45280	.41552	.35034	.29586
	.68643	.62553	.57029	.52016	.47464	.43330	.39573	.33051	27651
20	.67297	.61027	.55368	.50257	.45639	.41464	.37689	.31180	.25842
21	.65978	.59539	.53755	.48557	[illegible]	[illegible]	[illegible]	.29416	.24151
22	.64684	.58086	.52189	.46915	.42196	.37970	.34185	.27751	.22571
	.63416	.56670	.50669	.45329	.40573	.36335	.32557	.26180	.21095
	.62172	.55288	.49193	.43796	.39012	.34770	.31007	.24698	.19715
	.60953	.53939	.47761	.42315	.37512	.33273	.29530	.23300	.18425
	.59758	.52623	.46369	.40884		.31840	.28124	.21981	.17220
	.58586	.51340	.45019	.39501	.34682	.30469	.26785	.20737	.16093
	.57437	.50088	.43708	.38165	.33348	.29157	.25509	.19563	.15040
	.56311		.42435	.36875	.32065	.27902	.24295	.18456	.14056
30	.55207	.47674	.41199	.35628	.30832	.26700	.23138	.17411	.13137
	.54125	.46511	.39999	.34423	.29646	.25550	.22036	.16425	.12277
	.53063	.45377	.38834	.33259	.28506	.24450	.20987	.15496	.11474
	.52023	.44270	.37703	.32134	.27409	.23397	.19987	.14619	.10723
	.51003	.43191	.36604	.31048	.26355	.22390	.19035	.13791	.10022
	.50003	.42137	.35538	.29998	.25342	.21425	.18129	.13011	.09366
	.49022	.41109	.34503	.28983	.24367	.20503	.17266	.12274	.08754
	.48061	.40107	.33498	.28003	.23430	.19620	.16444	.11580	.08181
	.47119	.39128	.32523	.27056	.22529	.18775	.15661	.10924	.07646
	.46195	.38174	.31575	.26141	.21662	.17967	.14915	.10306	.07146
40	.45289	.37243	.30656	.25257	.20829	.17193	.14205	.09722	.06678
	.44401	.36335	.29763	.24403	.20028	.16453	.13528	.09172	.06241
	.43530	.35448	.28896	.23578	.19257	.15744	.12884	.08653	.05833
	.42677	.34584	.28054	.22781	.18517	.15066	.12270	.08163	.05451
	.41840	.33740	.27237	.22010	.17805	.14417	.11686	.07701	.05095
	.41020	.32917	.26444	.21266	.17120	.13796	.11130	.07265	.04761
	.40215	.32115	.25674	.20547	.16461	.13202	.10600	.06854	.04450
	.39427	.31331	.24926	.19852	.15828	.12634	.10095	.06466	.04159
	.38654	.30567	.24200	.19181	.15219	.12090	.09614	.06100	.03887
	.37896	.29822	.23495	.18532	.14634	.11569	.09156	.05755	.03632
50	.37153	29094	.22811	.17905	.14071	.11071	.08720	.05429	.03395

Table XII c — Amount of an Annuity

AMOUNT OF AN ANNUITY OF ONE DOLLAR PER YEAR AFTER n YEARS

	2 %	2½ %	3 %	3½ %	4 %	4½ %	5 %	6 %	7 %
	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	2.0200	2.0250	2.0300	2.0350	2.0400	2.0450	2.0500	2.0600	2.0700
	3.0604	3.0756	3.0909	3.1062	3.1216	3.1370	3.1525	3.1836	3.2149
	4.1216	4.1525	4.1836	4.2149	4.2465	4.2782	4.3101	4.3746	4.4399
	5.2040	5.2563	5.3091	5.3625	5.4163	5.4707	5.5256	5.6371	5.7507
	6.3081	6.3877	6.4684	6.5502	6.6330	6.7169	6.8019	6.9753	7.1533
	7.4343	7.5474	7.6625	7.7794	7.8983	8.0192	8.1420	8.3938	8.6540
	8.5830	8.7361	8.8923	9.0517	9.2142	9.3800	9.5491	9.8975	10.2598
	9.7546	9.9545	10.1591	10.3685	10.5828	10.8021	11.0266	11.4913	11.9780
10	10.9497	11.2034	11.4639	11.7314	12.0061	12.2882	12.5779	13.1808	13.8164
11	12.1687	12.4835	12.8078	13.1420	13.4864	13.8412	14.2068	14.9716	15.7836
	13.4121	13.7956	14.1920	14.6020	15.0258	15.4640	15.9171	16.8699	17.8885
	14.6803	15.1404	15.6178	16.1130	16.6268	17.1599	17.7130	18.8821	20.1406
	15.9739	16.5190	17.0863	17.6770	18.2919	18.9321	19.5986	21.0151	22.5505
	17.2934	17.9319	18.5989	19.2957	20.0236	20.7841	21.5786	23.2760	25.1290
	18.6393	19.3802	20.1569	20.9710	21.8245	22.7193	23.6575	25.6725	27.8881
	20.0121	20.8647	21.7616	22.7050	23.6975	24.7417	25.8404	28.2129	30.8402
	21.4123	22.3863	23.4144	24.4997	25.6454	26.8551	28.1324	30.9057	33.9990
	22.8406	23.9460	25.1169	26.3572	27.6712	29.0636	30.5390	33.7600	37.3790
20	24.2974	25.5447	26.8704	28.2797	29.7781	31.3714	33.0660	36.7856	40.9955
21	25.7833	27.1833	28.676	30.2695	31.9692	33.7831	35.7193	39.9927	44.8652
	27.2990	28.8629	30.5368	32.3289	34.2480	36.3034	38.5052	43.3923	49.0057
	28.8450	30.5844	32.4529	34.4604	36.6179	38.9370	41.4305	46.9958	53.4361
	30.4219	32.3490	34.4265	36.6665	39.0826	41.6892	44.5020	50.8156	58.1767
	32.0303	34.1578	36.4593	38.9499	41.6459	44.5652	47.7271	54.8645	63.2490
	33.6709	36.0117	38.5530	41.3131	44.3117	47.5706	51.1135	59.1564	68.6765
	35.3443	37.9120	40.7096	43.7591	47.0842	50.7113	54.6691	63.7058	74.4838
	37.0512	39.8598	42.9309	46.2906	49.9676	53.9933	58.4026	68.5281	80.6977
29	38.792	41.8563	45.2189	48.9108	52.9663	57.4230	62.3227	73.6398	87.3465
30	40.5681	43.902	47.5754	51.6227	56.0849	61.0071	66.4388	79.0582	94.4608
	42.3794	46.0003	50.002	54.429	59.3283	64.7524	70.7608	84.8017	102.0730
	44.2270	48.1503	52.5028	57.3345	62.7015	68.666	75.2988	90.8898	110.2182
	46.1116	50.3540	55.0778	60.341	66.2095	72.7562	80.0638	97.3432	118.9334
34	48.0338	52.6129	57.7302	63.4532	69.8579	77.0303	85.0670	104.1838	128.2588
35	49.9945	54.9282	60.4621	66.6740	73.6522	81.4966	90.3203	111.4348	138.2369
	51.9944	57.3014	63.2759	70.0076	77.5983	86.1640	95.836	119.1209	148.9135
	54.0343	59.7339	66.1742	73.4579	81.7022	91.0413	101.6281	127.2681	160.3374
38	56.1149	62.2273	69.1594	77.0289	85.9703	96.1382	107.709	135.904	172.5610
39	58.237	64.7830	72.234	80.7249	90.4091	101.4644	114.0950	145.0585	185.6403
40	60.4020	67.4026	75.401	84.5503	95.0255	107.0303	120.7998	154.7620	199.6351
41	62.6100	70.0876	78.6633	88.5095	99.8265	112.846	127.8398	165.0477	214.6096
42	64.862	72.8398	82.0232	92.6074	104.8196	118.9248	135.2318	175.9505	230.6322
43	67.1595	75.6608	85.4839	96.8486	110.0124	125.2764	142.993	187.5076	247.7765
44	69.5027	78.5523	89.0484	101.2383	115.4129	131.9138	151.1430	199.7580	266.1209
	71.8927	81.5161	92.7199	105.781	121.0294	138.8500	159.7002	212.743	285.7493
46	74.3306	84.5540	96.5015	110.4840	126.8706	146.0982	168.685	226.5081	306.751
47	76.8172	87.6679	100.3965	115.3510	132.9454	153.6726	178.1194	241.0986	329.2244
	79.3535	90.8596	104.4084	120.3883	139.263	161.5879	188.0254	256.5645	353.2701
	81.9406	94.1311	108.5406	125.6018	145.8337	169.8594	198.426	272.9584	378.999
	84.5794	97.4843	112.796	130.9979	152.6671	178.5030	209.3480	290.3359	406.528

PRESENT VALUE OF ONE DOLLAR PER YEAR FOR n YEARS

n	2 %	$2\frac{1}{2}$ %	3 %	$3\frac{1}{2}$ %	4 %	$4\frac{1}{2}$ %	5 %	6 %	7%
1	.9804	.9756	.9709	.9662	.9615	.9569	.9524	.9434	.9346
2	1.9416	1.9274	1.9135	1.8997	1.8861	1.8727	1.8594	1.8334	1.8080
3	2.8839	2.8560	2.8286	2.8016	2.7751	2.7490	2.7232	2.6730	2.6243
4	3.8077	3.7620	3.7171	3.6731	3.6299	3.5875	3.5460	3.4651	3.3872
5	4.7135	4.6458	4.5797	4.5151	4.4518	4.3900	4.3295	4.2124	4.1002
6	5.6014	5.5081	5.4172	5.3286	5.2421	5.1579	5.0757	4.9173	4.7665
7	6.4720	6.3494	6.2303	6.1145	6.0021	5.8927	5.7864	5.5824	5.3893
8	7.3255	7.1701	7.0197	6.8740	6.7327	6.5959	6.4632	6.2098	5.9713
9	8.1622	7.9709	7.7861	7.6077	7.4353	7.2688	7.1078	6.8017	6.5152
10	8.9826	8.7521	8.5302	8.3166	8.1109	7.9127	7.7217	7.3601	7.0236
11	9.7868	9.5142	9.2526	9.0016	8.7605	8.5289	8.3064	7.8869	7.4987
12	10.5753	10.2578	9.9540	9.6633	9.3851	9.1186	8.8633	8.3838	7.9427
13	11.3484	10.9832	10.6350	10.3027	9.9856	9.6829	9.3936	8.8527	8.3577
14	12.1062	11.6909	11.2961	10.9205	10.5631	10.2228	9.8986	9.2950	8.7455
15	12.8493	12.3814	11.9379	11.5174	11.1184	10.7395	10.3797	9.7122	9.1079
16	13.5777	13.0550	12.5611	12.0941	11.6523	11.2340	10.8378	10.1059	9.4466
17	14.2919	13.7122	13.1661	12.6513	12.1657	11.7072	11.2741	10.4773	9.7632
18	14.9920	14.3534	13.7535	13.1897	12.6593	12.1600	11.6896	10.8276	10.0591
19	15.6785	14.9789	14.3238	13.7098	13.1339	12.5933	12.0853	11.1581	10.3356
20	16.3514	15.5892	14.8775	14.2124	13.5903	13.0079	12.4622	11.4699	10.5940
21	17.0112	16.1845	15.4150	14.6980	14.0292	13.4047	12.8212	11.7641	10.8355
22	17.6580	16.7654	15.9369	15.1671	14.4511	13.7844	13.1630	12.0416	11.0612
23	18.2922	17.3321	16.4436	15.6204	14.8568	14.1478	13.4886	12.3034	11.2722
24	18.9139	17.8850	16.9355	16.0584	15.2470	14.4955	13.7986	12.5504	11.4693
25	19.5235	18.4244	17.4131	16.4815	15.6221	14.8282	14.0939	12.7834	11.6536
26	20.1210	18.9506	17.8768	16.8904	15.9828	15.1466	14.3752	13.0032	11.8258
27	20.7069	19.4640	18.3270	17.2854	16.3296	15.4513	14.6430	13.2105	11.9867
28	21.2813	19.9649	18.7641	17.6670	16.6631	15.7429	14.8981	13.4062	12.1371
29	21.8444	20.4535	19.1885	18.0358	16.9837	16.0219	15.1411	13.5907	12.2777
30	22.3965	20.9303	19.6004	18.3920	17.2920	16.2889	15.3725	13.7648	12.4090
31	22.9377	21.3954	20.0004	18.7363	17.5885	16.5444	15.5928	13.9291	12.5318
32	23.4683	21.8492	20.3888	19.0689	17.8736	16.7889	15.8027	14.0840	12.6466
33	23.9886	22.2919	20.7658	19.3902	18.1476	17.0229	16.0025	14.2302	12.7538
34	24.4986	22.7238	21.1318	19.7007	18.4112	17.2468	16.1929	14.3681	12.8540
35	24.9986	23.1452	21.4872	20.0007	18.6646	17.4610	16.3742	14.4982	12.9477
36	25.4888	23.5563	21.8323	20.2905	18.9083	17.6660	16.5469	14.6210	13.0352
37	25.9695	23.9573	22.1672	20.5705	19.1426	17.8622	16.7113	14.7368	13.1170
38	26.4406	24.3486	22.4925	20.8411	19.3679	18.0500	16.8679	14.8460	13.1935
39	26.9026	24.7303	22.8082	21.1025	19.5845	18.2297	17.0170	14.9491	13.2649
40	27.3555	25.1028	23.1148	21.3551	19.7928	18.4016	17.1591	15.0463	13.3317
41	27.7995	25.4661	23.4124	21.5991	19.9931	18.5661	17.2944	15.1380	13.3941
42	28.2348	25.8206	23.7014	21.8349	20.1856	18.7236	17.4232	15.2245	13.4524
43	28.6616	26.1664	23.9819	22.0627	20.3708	18.8742	17.5459	15.3062	13.5070
44	29.0800	26.5038	24.2543	22.2828	20.5488	19.0184	17.6628	15.3832	13.5579
45	29.4902	26.8330	24.5187	22.4955	20.7200	19.1563	17.7741	15.4558	13.6055
46	29.8923	27.1542	24.7754	22.7009	20.8847	19.2884	17.8801	15.5244	13.6500
47	30.2866	27.4675	25.0247	22.8994	21.0429	19.4147	17.9810	15.5890	13.6910
48	30.6731	27.7732	25.2667	23.0912	21.1951	19.5356	18.0772	15.6500	13.7305
49	31.0521	28.0714	25.5017	23.2766	21.3415	19.6513	18.1687	15.7076	13.7668
50	31.4236	28.3623	25.7298	23.4556	21.4822	19.7620	18.2559	15.7619	13.8007

1 + r	*log (1 + r)*
1.005	00216 60617 56508
1.010	00432 13737 82643
1.015	00646 60422 49232
1.020	00860 01717 61918
1.025	01072 38653 91773
1.030	01283 72247 05172
1.035	01494 03497 92937
1.040	01703 33392 98780
1.045	01911 62904 47073
1.050	02118 92990 69938

r	*1 + r*	*log (1 + r)*
5½ %	1.055	02325 24596 33711
6 %	1.060	02530 58652 64770
6½ %	1.065	02734 96077 74757
7 %	1.070	02938 37776 85210
7½ %	1.075	03140 84642 51624
8 %	1.080	03342 37554 86950
8½ %	1.085	03542 97381 84548
9 %	1.090	03742 64979 40624
9½ %	1.095	03941 41191 76137
10 %	1.100	04139 26851 58225

For Amount, A, of any principal, P, after n years: $A = P(1 + r)^n$.

For present worth, P, of any amount, A, at the end of n years: $P = A \div (1 + r)^n$.

To find logarithms and antilogarithms of A and P to many significant figures, use Table XI, p. 126, and Table I *a*, p. 20.

Table XII f — American Experience Mortality Table

Based on 100,000 living at age 10

At Age	Number Surviving	Deaths	At Age	Number Surviving	Deaths	At Age	Number Surviving	Deaths	At Age	Number Surviving	Deaths
10	100,000	749	**35**	81,822	732	**60**	57,917	1,546	**85**	5,485	1,292
11	99,251	746	**36**	81,090	737	**61**	56,371	1,628	**86**	4,193	1,114
12	98,505	743	**37**	80,353	742	**62**	54,743	1,713	**87**	3,079	933
13	97,762	740	**38**	79,611	749	**63**	53,030	1,800	**88**	2,146	744
14	97,022	737	**39**	78,862	756	**64**	51,230	1,889	**89**	1,402	555
[illegible]	96,285	735	**40**	78,106	765	**65**	49,341	1,980	**90**	847	385
[illegible]	95,550	732	**41**	77,341	774	**66**	47,361	2,070	**91**	462	246
[illegible]	94,818	729	**42**	76,567	785	**67**	45,291	2,158	**92**	216	137
[illegible]	94,089	727	**43**	75,782	797	**68**	43,133	2,243	**93**	79	58
[illegible]	93,362	725	**44**	74,985	812	**69**	40,890	2,321	**94**	21	18
[illegible]	92,637	723	**45**	74,173	828	**70**	38,569	2,391	**95**	3	3
21	91,914	722	**46**	73,345	848	**71**	36,178	2,448			
22	91,192	721	**47**	72,497	870	**72**	33,730	2,487			
23	90,471	720	**48**	71,627	896	**73**	31,243	2,505			
24	89,751	719	**49**	70,731	927	**74**	28,738	2,501			
25	89,032	718	**50**	69,804	962	**75**	26,237	2,476			
26	88,314	718	**51**	68,842	1,001	**76**	23,761	2,431			
27	87,596	718	**52**	67,841	1,044	**77**	21,330	2,369			
28	86,878	718	**53**	66,797	1,091	**78**	18,961	2,291			
29	86,160	719	**54**	65,706	1,143	**79**	16,670	2,196			
30	85,441	720	**55**	64,563	1,199	**80**	14,474	2,091			
31	84,721	721	**56**	63,364	1,260	**81**	12,383	1,964			
32	84,000	723	**57**	62,104	1,325	**82**	10,419	1,816			
33	83,277	726	**58**	60,779	1,394	**83**	8,603	1,648			
34	82,551	729	**59**	59,385	1,468	**84**	6,955	1,470			

Logarithms of Important Constants

n = number	Value of n	$\log_{10} n$
π	3.14159265	0.49714987
$1 \div \pi$	0.31830989	9.50285013
π^2	9.86960440	0.99429975
$\sqrt{\pi}$	1.77245385	0.24857494
e = Naperian Base	2.71828183	0.43429448
$M = \log_{10} e$	0.43429448	9.63778431
$1 \div M = \log_e 10$	2.30258509	0.36221569
$180 \div \pi$ = degrees in 1 radian	57.2957795	1.75812263
$\pi \div 180$ = radians in 1°	0.01745329	8.24187737
$\pi \div 10800$ = radians in 1′	0.0002908882	6.46372612
$\pi \div 648000$ = radians in 1″	0.000004848136811095	4.68557487
sin 1″	0.000004848136811076	4.68557487
tan 1″	0.000004848136811133	4.68557487
centimeters in 1 ft.	30.480	1.4840158
feet in 1 cm.	0.032808	8.5159842
inches in 1 m.	39.37 (exact legal value)	1.5951654
pounds in 1 kg.	2.20462	0.3433340
kilograms in 1 lb.	0.453593	9.6566660
g (average value)	32.16 ft./sec./sec.	1.5073
	= 981 cm./sec./sec.	2.9916690
weight of 1 cu. ft. of water	62.425 lb. (max. density)	1.7953586
weight of 1 cu. ft. of air	0.0807 lb. (at 32° F.)	8.907
cu. in. in 1 (U. S.) gallon	231 (exact legal value)	2.3636120
ft. lb. per sec. in 1 H. P.	550 (exact legal value)	2.7403627
kg. m. per sec. in 1 H. P.	76.0404	1.8810445
watts in 1 H. P.	745.957	2.8727135

Several Numbers Very Accurately

$$\pi = 3.14159\ 26535\ 89793\ 23846\ 26433\ 83280$$
$$e = 2.71828\ 18284\ 59045\ 23536\ 02874\ 71353$$
$$M = 0.43429\ 44819\ 03251\ 82765\ 11289\ 18917$$
$$1 \div M = 2.30258\ 50929\ 94045\ 68401\ 79914\ 54684$$
$$\log_{10} \pi = 0.49714\ 98726\ 94133\ 85435\ 12682\ 88291$$
$$\log_{10} M = 9.63778\ 43113\ 00536\ 78912$$

Certain Convenient Values for $n = 1$ to $n = 10$

n	$1/n$	$\sqrt{n}$	$\sqrt[3]{n}$	$n!$	$1/n!$	$\log_{10} n$
1	1.000000	1.00000	1.00000	1	1.0000000	0.000000000
2	0.500000	1.41421	1.25992	2	0.5000000	0.301029996
3	0.333333	1.73205	1.44225	6	0.1666667	0.477121255
4	0.250000	2.00000	1.58740	24	0.0416667	0.602059991
5	0.200000	2.23607	1.70998	120	0.0083333	0.698970004
6	0.166667	2.44949	1.81712	720	0.0013889	0.778151250
7	0.142857	2.64575	1.91293	5040	0.0001984	0.845098040
8	0.125000	2.82843	2.00000	40320	0.0000248	0.903089987
9	0.111111	3.00000	2.08008	362880	0.0000028	0.954242509
10	0.100000	3.16228	2.15443	3628800	0.0000003	1.000000000

Table XIV a — Four Place Logarithms

N			2						8		1	2	3	4	5		7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	16	17
	3617	636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5		9	11	12	14	16
	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	4	5		9	10	12	14	16
	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5		8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	12	14
	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
	624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7		10	11	13
	4914	4928	4942	4955	4969	4983	4997	5011	024	5038	1	3	4	5	7	8	10	11	12
	051	065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
	185	5198	211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	7	8	9	11	12
	315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	2	4	5	6	8	9	10	11
	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
	5563	5575	587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
	682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2		5	6	7	8	9	11
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2		5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2		4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6		9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6		8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6		8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6		8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2		4	5	6		8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2		4	5	6		8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2		4	5	6		7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2		4	5	6		7	8
48	681	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2		4	5	6		7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2		4	4	5		7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2		3	4	5		7	8
52	7160	7168	717	7185	7193	720	7210	7218	7226	7235	1	2		3	4	5		7	7
53	7243	7251	7259	7267	727	7284	7292	7300	7308	7316	1	2		3	4	5		6	7
54	7324	7332	7340	7348	7356	7364	737	7380	7388	7396	1	2	2	3	4	5	6	6	7
N									8	9	1	2	3	4	5	6	7	8	9

The proportional parts are stated in full for every tenth at the right-hand side. he logarithm of any number of four significant figures can be read directly by add-

N											1	2	3	4	5	6	7	8	9
	'404	'412	'419	'427	7435	'443	'451	'459	'466	'474	1	2	2	3	4	5	5	6	7
	7482	'490	'497	'505	7513	'520	'528	'536	'543	'551	1	2	2	3	4	5	5	6	7
	'559	'566	'574	'582	'589	'597	'604	'612	'619	'627	1	1	2	3	4	5	5	6	7
	'634	'642	'649	'657	7664	'672	'679	'686	'694	7701	1	1	2	3	4	4	5	6	7
	'709	'716	'723	'731	'738	'745	'752	'760	'767	7774	1	1	2	3	4	4	5	6	7
60	782	789	796	'803	7810	'818	'825	'832	'839		1	1	2	3	4	4	5	6	6
	7853	7860	'868	'875	'882	'889	'896	'903	7910	7917	1	1	2	3	3	4	5	6	6
	'924	'931	'938	'945	'952	'959	'966	'973	'980	'987	1	1	2	3	3	4	5	5	6
	'993	8000	8007	8014	8021	8028	:035	8041	8048	8055	1	1	2	3	3	4	5	5	6
	:062	8069	:075	8082	8089	8096	102	8109	116	122	1	1	2	3	3	4	5	5	6
	129	8136	8142	3149	8156	.162	.169	3176	.182	8189	1	1	2	3	3	4	5	5	6
	8195	8202	8209	8215	8222	8228	8235	:241	8248	8254	1	1	2	3	3	4	5	5	6
	8261	8267	8274	8280	8287	8293	:299	:306	8312	8319	1	1	2	3	3	4	5	5	6
	8325	8331	8338	8344	8351	8357	8363	:370	8376	8382	1	1	2	3	3	4	4	5	6
	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	3	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	3	3	4	4	5	6
	8513	8519	8525	8531	8537	8543	:549	8555	:561	8567	1	1		3	3	4	4	5	6
	8573	8579	8585	8591	8597	:603	:609	:615	:621	8627	1	1		3	3	4	4	5	6
	8633	8639	:645	8651	:657	:663	:669	:675	8681	8686	1	1		2	3	4	4	5	5
	3692		:704	:710	:716	:722	:727	:733	8739	8745	1	1	2	2	3	4	4	5	5
	:751	8756	3762	8768	8774	8779	:785	8791	8797	8802	1	1	2	2	3	3	4	5	5
	8808	8814	8820	:825	8831	8837	8842	8848	:854	8859	1	1	2	2	3	3	4	4	5
	:865	8871	:876	8882	8887	8893	8899	:904	8910	8915	1	1	2	2	3	3	4	4	5
	8921	8927	:932	3938	8943	8949	3954	:960	8965	:971	1	1	2	2	3	3	4	4	5
	8976	8982	8987	:993	8998	9004	:009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	·069	9074	9079	1	1	2	2	3	3	4	4	5
	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1		2	3	3	4	4	5
	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1		2	3	3	4	4	5
	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1		2	3	3	4	4	5
	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1		2	3	3	4	4	5
	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1		2	3	3	4	4	5
	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1		2	3	3	4	4	5
	939!	9400	9405	9410	9415	9420	9425	9430	9435	9440	1	1	2	2	3	3	4	4	5
	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	955:	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3			
	9590	9595	9600	960!	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3		
	9638	9643	964'	965:	965'	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	
	9685	9689	9694	9699	9703	9708	971:	971'	9722	9727	0	1	1	2	2	3	3	4	
	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	
	9777	9782	9786	9791	9795	9800	980:	9809	9814	9818	0	1	1	2	2	3	3	4	4
	9823	9827	983.	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
	9868	987:	9877		9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
	991:	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	3	4
	9956	9961	9965	9969	9974	997:	9983	9987		9996	0	1	1	2	2	3	3	3	4
N							6				1	2	3	4	5	6	7	8	9

ing the proportional part corresponding to the fourth figure to the tabular number corresponding to the first three figures. There may be an error of 1 in the last place.

											1	2	3	4	5	6	7	8	9
.00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
.10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
	1318	1321	1324	1327	.330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
	1349	1352	1355	1358	.361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
	1380	1384	1387	1390	.393	1396	1400	1403	1406	1409		1	1	1	2	2	2	3	3
	1413	1416	1419	1422	.426	1429	1432	1435	1439	1442		1	1	1	2	2	2	3	3
	1445	1449	1452	1455	.459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510		1	1	1	2	2	2	3	3
	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545		1	1	1	2	2	2	3	3
	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	2	3	3
.20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	1	2	2	3	3	3
	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	3
	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	3	3	3	4
	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
	186[illegible]	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
.30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
.34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
.35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	5	5
	2455	2460	2466	247[illegible]	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
.40	2512	2518	2523	2529	253[illegible]	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	6
	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1		2	3	4	4	5	6
	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1		2	3	4	4	5	6
	2754	2761	276[illegible]	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3		5	6	6
	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	3		5	6	6
	3090	309[illegible]	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6

											1	2	3	4	5		7	8	9
.50	162	170	3177	3184	192	3199	206	3214	3221	228	1	1	2	3	4	4	5	6	7
	236	3243	3251	3258	266	3273	3281	3289	3296	3304	1	1	2		4	4	5	6	
	311	3319	3327	3334	342	3350	3357	3365	3373	3381	1	1	2		4	5	5	6	
	388	3396	3404	3412	420	428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	
	3467	3475	3483	3491	3499	3508	3516	524	3532	3540	1	2	2		4	5	6	6	7
	548	3556	3565	3573	581	589	3597	3606	3614	3622	1	2	2		4	5	6	7	7
	631	3639	3648	3656	664	3673	3681	690	3698	707	1	2	2		4	5	6	7	8
	715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
	802	3811	3819	3828	3837	3846	3855	864	3873	3882	1	2	3	3	4	5	6	7	8
	890		3908	3917	3926	3936	3945	954	3963	3972	1	2	3	4	5	5	6	7	8
.60	981	990	999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	7	8	8
.61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
	4169	4178	188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
.65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	5	6	7	8	9	10
	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
.70	5012	5023	5035	5047	058	070	082	5093	5105	5117	1	2	3	5	6	7	8	9	10
	129	5140	5152	5164	176	5188	5200	212	5224	5236	1	2	4		6	7	8	10	11
	248	5260	5272	284	297	5309	5321	5333	5346	5358	1	2	4		6	7	9	10	11
	370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	7	9	10	11
.74	5495	5508	5521	5534	5546	5559	572	5585	5598	5610	1	3	4		6	8	9	10	12
	623	5636	5649	5662	675	689	702	715	728	5741	1	3	4		7	8	9	11	12
	754	5768	5781	5794	808	5821	5834		5861	5875	1	3	4		7	8	9	11	12
	888	590	5916	5929	943	5957	5970	984	5998	6012	1	3	4	5	7	8	10	11	12
	6026	6039	6053	6067	6081	609	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
	6166	6180	6194	6209	6223	6237	625	6266	6281	6295	1	3	4	6	7	9	10	11	13
.80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
	6457	6471	6486	6501	6516	6531	6546	6561	6577	659	2	3	5	6	8	9	11	12	14
	660	6622	6637	6653	6668		6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
	6761	6776	6792	6808	6823	6839	6855	6871	6887	690	2	3	5	6	8	9	11	13	14
84	6918	6934	6950	6966	698	6998	7015	7031	7047	7063	2	3	5	7	8	10	11	13	15
.85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
.86	7244	7261	7278	7295	7311	7328	7345	736	7379	7396	2	3	5	7	8	10	12	14	15
.87	7413	7430	744	7464	7482	7499	7516	7534	7551	7568	2	4	5	7	9	10	12	14	16
.88	7586	760	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
.89	776	7780	7798	7816	7834	785	7870	7889	790	7925	2	4	6	7	9	11	13	15	16
.90	7943	7962	7980	7998	801	8035	8054	807	8091	8110	2	4	6	7	9	11	13	15	17
	8128	814	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
	8318	8337	8356	8375	8395	8414	8433	8453	847	8492	2	4	6	8	10	12	14	15	17
	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
.94	8710	873	8750	8770	8790	8810	8831	8851	887	8892	2	4	6	8	10	12	14	16	18
.95	8913	893	8954	897	899	9016	9036	905	9078	9099	2	4	6	8	10	12	15	17	19
	9120	914	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	9	11	13	15	17	19
	9333	9354	937	939	9419	9441	9462	9484	9506	9528	2	4	6	9	11	13	15	17	19
	955	9572	9594	9616	963	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
	977	979	981	984	986	9886	9908	9931	995	9977	2	5	7	9	11	14	16	18	21

[Characteristics of Logarithms omitted—determine by the usual rule from the value]

Radians	Degrees	Sine Value	Sine Log10	Tangent Value	Tangent Log10	Cotangent Value	Cotangent Log10	Cosine Value	Cosine Log10		
.0000	0° 00′	.0000	——	.0000	——	——	——	1.0000	.0000	90° 00′	1.5708
.0029	10	.0029	.4637	.0029	.4637	343.77	.5363	1.0000	.0000	50	1.5679
.0058	20	.0058	.7648	.0058	.7648	171.89	.2352	1.0000	.0000	40	1.5650
.0087	30	.0087	.9408	.0087	.9409	114.59	.0591	1.0000	.0000	30	1.5621
.0116	40	.0116	.0658	.0116	.0658	85.940	.9342	.9999	.0000	20	1.5592
.0145	50	.0145	.1627	.0145	.1627	68.750	.8373	.9999	.0000	10	1.5563
.0175	1° 00′	.0175	.2419	.0175	.2419	57.290	.7581	.9998	.9999	89° 00′	1.5533
.0204	10	.0204	.3088	.0204	.3089	49.104	.6911	.9998	.9999	50	1.5504
.0233	20	.0233	.3668	.0233	.3669	42.964	.6331	.9997	.9999	40	1.5475
.0262	30	.0262	.4179	.0262	.4181	38.188	.5819	.9997	.9999	30	1.5446
.0291	40	.0291	.4637	.0291	.4638	34.368	.5362	.9996	.9998	20	1.5417
.0320	50	.0320	.5050	.0320	.5053	31.242	.4947	.9995	.9998	10	1.5388
.0349	2° 00′	.0349	.5428	.0349	.5431	28.636	.4569	.9994	.9997	88° 00′	1.5359
.0378	10	.0378	.5776	.0378	.5779	26.432	.4221	.9993	.9997	50	1.5330
.0407	20	.0407	.6097	.0407	.6101	24.542	.3899	.9992	.9996	40	1.5301
.0436	30	.0436	.6397	.0437	.6401	22.904	.3599	.9990	.9996	30	1.5272
.0465	40	.0465	.6677	.0466	.6682	21.470	.3318	.9989	.9995	20	1.5243
.0495	50	.0494	.6940	.0495	.6945	20.206	.3055	.9988	.9995	10	1.5213
.0524	3° 00′	.0523	.7188	.0524	.7194	19.081	.2806	.9986	.9994	87° 00′	1.5184
.0553	10	.0552	.7423	.0553	.7429	18.075	.2571	.9985	.9993	50	1.5155
.0582	20	.0581	.7645	.0582	.7652	17.169	.2348	.9983	.9993	40	1.5126
.0611	30	.0610	.7857	.0612	.7865	16.350	.2135	.9981	.9992	30	1.5097
.0640	40	.0640	.8059	.0641	.8067	15.605	.1933	.9980	.9991	20	1.5068
.0669	50	.0669	.8251	.0670	.8261	14.924	.1739	.9978	.9990	10	1.5039
.0698	4° 00′	.0698	.8436	.0699	.8446	14.301	.1554	.9976	.9989	86° 00′	1.5010
.0727	10	.0727	.8613	.0729	.8624	13.727	.1376	.9974	.9989	50	1.4981
.0756	20	.0756	.8783	.0758	.8795	13.197	.1205	.9971	.9988	40	1.4952
.0785	30	.0785	.8946	.0787	.8960	12.706	.1040	.9969	.9987	30	1.4923
.0814	40	.0814	.9104	.0816	.9118	12.251	.0882	.9967	.9986	20	1.4893
.0844	50	.0843	.9256	.0846	.9272	11.826	.0728	.9964	.9985	10	1.4864
.0873	5° 00′	.0872	.9403	.0875	.9420	11.430	.0580	.9962	.9983	85° 00′	1.4835
.0902	10	.0901	.9545	.0904	.9563	11.059	.0437	.9959	.9982	50	1.4806
.0931	20	.0929	.9682	.0934	.9701	10.712	.0299	.9957	.9981	40	1.4777
.0960	30	.0958	.9816	.0963	.9836	10.385	.0164	.9954	.9980	30	1.4748
.0989	40	.0987	.9945	.0992	.9966	10.078	.0034	.9951	.9979	20	1.4719
.1018	50	.1016	.0070	.1022	.0093	9.7882	.9907	.9948	.9977	10	1.4690
.1047	6° 00′	.1045	.0192	.1051	.0216	9.5144	.9784	.9945	.9976	84° 00′	1.4661
.1076	10	.1074	.0311	.1080	.0336	9.2553	.9664	.9942	.9975	50	1.4632
.1105	20	.1103	.0426	.1110	.0453	9.0098	.9547	.9939	.9973	40	1.4603
.1134	30	.1132	.0539	.1139	.0567	8.7769	.9433	.9936	.9972	30	1.4573
.1164	40	.1161	.0648	.1169	.0678	8.5555	.9322	.9932	.9971	20	1.4544
.1193	50	.1190	.0755	.1198	.0786	8.3450	.9214	.9929	.9969	10	1.4515
.1222	7° 00′	.1219	.0859	.1228	.0891	8.1443	.9109	.9925	.9968	83° 00′	1.4486
.1251	10	.1248	.0961	.1257	.0995	7.9530	.9005	.9922	.9966	50	1.4457
.1280	20	.1276	.1060	.1287	.1096	7.7704	.8904	.9918	.9964	40	1.4428
.1309	30	.1305	.1157	.1317	.1194	7.5958	.8806	.9914	.9963	30	1.4399
.1338	40	.1334	.1252	.1346	.1291	7.4287	.8709	.9911	.9961	20	1.4370
.1367	50	.1363	.1345	.1376	.1385	7.2687	.8615	.9907	.9959	10	1.4341
.1396	8° 00′	.1392	.1436	.1405	.1478	7.1154	.8522	.9903	.9958	82° 00′	1.4312
.1425	10	.1421	.1525	.1435	.1569	6.9682	.8431	.9899	.9956	50	1.4283
.1454	20	.1449	.1612	.1465	.1658	6.8269	.8342	.9894	.9954	40	1.4254
.1484	30	.1478	.1697	.1495	.1745	6.6912	.8255	.9890	.9952	30	1.4224
.1513	40	.1507	.1781	.1524	.1831	6.5606	.8169	.9886	.9950	20	1.4195
.1542	50	.1536	.1863	.1554	.1915	6.4348	.8085	.9881	.9948	10	1.4166
.1571	9° 00′	.1564	.1943	.1584	.1997	6.3138	.8003	.9877	.9946	81° 00′	1.4137
		Value Cosine	Log10 Cosine	Value Cotangent	Log10 Cotangent	Value Tangent	Log10 Tangent	Value Sine	Log10 Sine	Degrees	Radians

[Characteristics of Logarithms omitted—determine by the usual rule from the value]

Radians	Degrees	Sine Value	Sine Log10	Tangent Value	Tangent Log10	Cotangent Value	Cotangent Log10	Cosine Value	Cosine Log10		
.1571	**9° 00′**	.1564	.1943	.1584	.1997	6.3138	.8003	.9877	.9946	**81° 00′**	1.4137
.1600	10	.1593	.2022	.1614	.2078	6.1970	.7922	.9872	.9944	50	1.4108
.1629	20	.1622	.2100	.1644	.2158	6.0844	.7842	.9868	.9942	40	1.4079
.1658	30	.1650	.2176	.1673	.2236	5.9758	.7764	.9863	.9940	30	1.4050
.1687	40	.1679	.2251	.1703	.2313	5.8708	.7687	.9858	.9938	20	1.4021
.1716	50	.1708	.2324	.1733	.2389	5.7694	.7611	.9853	.9936	10	1.3992
.1745	**10° 00′**	.1736	.2397	.1763	.2463	5.6713	.7537	.9848	.9934	**80° 00′**	1.3963
.1774	10	.1765	.2468	.1793	.2536	5.5764	.7464	.9843	.9931	50	1.3934
.1804	20	.1794	.2538	.1823	.2609	5.4845	.7391	.9838	.9929	40	1.3904
.1833	30	.1822	.2606	.1853	.2680	5.3955	.7320	.9833	.9927	30	1.3875
.1862	40	.1851	.2674	.1883	.2750	5.3093	.7250	.9827	.9924	20	1.3846
.1891	50	.1880	.2740	.1914	.2819	5.2257	.7181	.9822	.9922	10	1.3817
.1920	**11° 00′**	.1908	.2806	.1944	.2887	5.1446	.7113	.9816	.9919	**79° 00′**	1.3788
.1949	10	.1937	.2870	.1974	.2953	5.0658	.7047	.9811	.9917	50	1.3759
.1978	20	.1965	.2934	.2004	.3020	4.9894	.6980	.9805	.9914	40	1.3730
.2007	30	.1994	.2997	.2035	.3085	4.9152	.6915	.9799	.9912	30	1.3701
.2036	40	.2022	.3058	.2065	.3149	4.8430	.6851	.9793	.9909	20	1.3672
.2065	50	.2051	.3119	.2095	.3212	4.7729	.6788	.9787	.9907	10	1.3643
.2094	**12° 00′**	.2079	.3179	.2126	.3275	4.7046	.6725	.9781	.9904	**78° 00′**	1.3614
.2123	10	.2108	.3238	.2156	.3336	4.6382	.6664	.9775	.9901	50	1.3584
.2153	20	.2136	.3296	.2186	.3397	4.5736	.6603	.9769	.9899	40	1.3555
.2182	30	.2164	.3353	.2217	.3458	4.5107	.6542	.9763	.9896	30	1.3526
.2211	40	.2193	.3410	.2247	.3517	4.4494	.6483	.9757	.9893	20	1.3497
.2240	50	.2221	.3466	.2278	.3576	4.3897	.6424	.9750	.9890	10	1.3468
.2269	**13° 00′**	.2250	.3521	.2309	.3634	4.3315	.6366	.9744	.9887	**77° 00′**	1.3439
.2298	10	.2278	.3575	.2339	.3691	4.2747	.6309	.9737	.9884	50	1.3410
.2327	20	.2306	.3629	.2370	.3748	4.2193	.6252	.9730	.9881	40	1.3381
.2356	30	.2334	.3682	.2401	.3804	4.1653	.6196	.9724	.9878	30	1.3352
.2385	40	.2363	.3734	.2432	.3859	4.1126	.6141	.9717	.9875	20	1.3323
.2414	50	.2391	.3786	.2462	.3914	4.0611	.6086	.9710	.9872	10	1.3294
.2443	**14° 00′**	.2419	.3837	.2493	.3968	4.0108	.6032	.9703	.9869	**76° 00′**	1.3265
.2473	10	.2447	.3887	.2524	.4021	3.9617	.5979	.9696	.9866	50	1.3235
.2502	20	.2476	.3937	.2555	.4074	3.9136	.5926	.9689	.9863	40	1.3206
.2531	30	.2504	.3986	.2586	.4127	3.8667	.5873	.9681	.9859	30	1.3177
.2560	40	.2532	.4035	.2617	.4178	3.8208	.5822	.9674	.9856	20	1.3148
.2589	50	.2560	.4083	.2648	.4230	3.7760	.5770	.9667	.9853	10	1.3119
.2618	**15° 00′**	.2588	.4130	.2679	.4281	3.7321	.5719	.9659	.9849	**75° 00′**	1.3090
.2647	10	.2616	.4177	.2711	.4331	3.6891	.5669	.9652	.9846	50	1.3061
.2676	20	.2644	.4223	.2742	.4381	3.6470	.5619	.9644	.9843	40	1.3032
.2705	30	.2672	.4269	.2773	.4430	3.6059	.5570	.9636	.9839	30	1.3003
.2734	40	.2700	.4314	.2805	.4479	3.5656	.5521	.9628	.9836	20	1.2974
.2763	50	.2728	.4359	.2836	.4527	3.5261	.5473	.9621	.9832	10	1.2945
.2793	**16° 00′**	.2756	.4403	.2867	.4575	3.4874	.5425	.9613	.9828	**74° 00′**	1.2915
.2822	10	.2784	.4447	.2899	.4622	3.4495	.5378	.9605	.9825	50	1.2886
.2851	20	.2812	.4491	.2931	.4669	3.4124	.5331	.9596	.9821	40	1.2857
.2880	30	.2840	.4533	.2962	.4716	3.3759	.5284	.9588	.9817	30	1.2828
.2909	40	.2868	.4576	.2994	.4762	3.3402	.5238	.9580	.9814	20	1.2799
.2938	50	.2896	.4618	.3026	.4808	3.3052	.5192	.9572	.9810	10	1.2770
.2967	**17° 00′**	.2924	.4659	.3057	.4853	3.2709	.5147	.9563	.9806	**73° 00′**	1.2741
.2996	10	.2952	.4700	.3089	.4898	3.2371	.5102	.9555	.9802	50	1.2712
.3025	20	.2979	.4741	.3121	.4943	3.2041	.5057	.9546	.9798	40	1.2683
.3054	30	.3007	.4781	.3153	.4987	3.1716	.5013	.9537	.9794	30	1.2654
.3083	40	.3035	.4821	.3185	.5031	3.1397	.4969	.9528	.9790	20	1.2625
.3113	50	.3062	.4861	.3217	.5075	3.1084	.4925	.9520	.9786	10	1.2595
.3142	**18° 00′**	.3090	.4900	.3249	.5118	3.0777	.4882	.9511	.9782	**72° 00′**	1.2566
		Value Cosine	Log10 Cosine	Value Cotangent	Log10 Cotangent	Value Tangent	Log10 Tangent	Value Sine	Log10 Sine	Degrees	Radians

[Characteristics of Logarithms omitted—determine by the usual rule from the value]

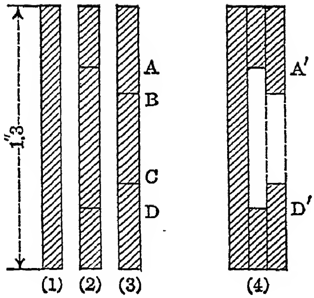
	Degrees	Sine Value	Sine L(	Tangent Value	Tangent Log$_{10}$	Cotangent Value	Cotangent Log$_{10}$	Cosine Value	Cosine Log$_{10}$		
.3142	**18° 00′**	.3090	.4900	.3249	.5118	3.0777	.4882	.9511	.9782	**72° 00′**	1.2566
.3171	10	.3118	.4939	.3281	.5161	3.0475	.4839	.9502	.9778	50	1.2537
.3200	20	.3145	.4977	.3314	.5203	3.0178	.4797	.9492	.9774	40	1.2508
.3229	30	.3173	.5015	.3346	.5245	2.9887	.4755	.9483	.9770	30	1.2479
.3258	40	.3201	.5052	.3378	.5287	2.9600	.4713	.9474	.9765	20	1.2450
.3287	50	.3228	.5090	.3411	.5329	2.9319	.4671	.9465	.9761	10	1.2421
.3316	**19° 00′**	.3256	.5126	.3443	.5370	2.9042	.4630	.9455	.9757	**71° 00′**	1.2392
.3345	10	.3283	.5163	.3476	.5411	2.8770	.4589	.9446	.9752	50	1.2363
.3374	20	.3311	.5199	.3508	.5451	2.8502	.4549	.9436	.9748	40	1.2334
.3403	30	.3338	.5235	.3541	.5491	2.8239	.4509	.9426	.9743	30	1.2305
.3432	40	.3365	.5270	.3574	.5531	2.7980	.4469	.9417	.9739	20	1.2275
.3462	50	.3393	.5306	.3607	.5571	2.7725	.4429	.9407	.9734	10	1.2246
.3491	**20° 00′**	.3420	.5341	.3640	.5611	2.7475	.4389	.9397	.9730	**70° 00′**	1.2217
.3520	10	.3448	.5375	.3673	.5650	2.7228	.4350	.9387	.9725	50	1.2188
.3549	20	.3475	.5409	.3706	.5689	2.6985	.4311	.9377	.9721	40	1.2159
.3578	30	.3502	.5443	.3739	.5727	2.6746	.4273	.9367	.9716	30	1.2130
.3607	40	.3529	.5477	.3772	.5766	2.6511	.4234	.9356	.9711	20	1.2101
.3636	50	.3557	.5510	.3805	.5804	2.6279	.4196	.9346	.9706	10	1.2072
.3665	**21° 00′**	.3584	.5543	.3839	.5842	2.6051	.4158	.9336	.9702	**69° 00′**	1.2043
.3694	10	.3611	.5576	.3872	.5879	2.5826	.4121	.9325	.9697	50	1.2014
.3723	20	.3638	.5609	.3906	.5917	2.5605	.4083	.9315	.9692	40	1.1985
.3752	30	.3665	.5641	.3939	.5954	2.5386	.4046	.9304	.9687	30	1.1956
.3782	40	.3692	.5673	.3973	.5991	2.5172	.4009	.9293	.9682	20	1.1926
.3811	50	.3719	.5704	.4006	.6028	2.4960	.3972	.9283	.9677	10	1.1897
.3840	**22° 00′**	.3746	.5736	.4040	.6064	2.4751	.3936	.9272	.9672	**68° 00′**	1.1868
.3869	10	.3773	.5767	.4074	.6100	2.4545	.3900	.9261	.9667	50	1.1839
.3898	20	.3800	.5798	.4108	.6136	2.4342	.3864	.9250	.9661	40	1.1810
.3927	30	.3827	.5828	.4142	.6172	2.4142	.3828	.9239	.9656	30	1.1781
.3956	40	.3854	.5859	.4176	.6208	2.3945	.3792	.9228	.9651	20	1.1752
.3985	50	.3881	.5889	.4210	.6243	2.3750	.3757	.9216	.9646	10	1.1723
.4014	**23° 00′**	.3907	.5919	.4245	.6279	2.3559	.3721	.9205	.9640	**67° 00′**	1.1694
.4043	10	.3934	.5948	.4279	.6314	2.3369	.3686	.9194	.9635	50	1.1665
.4072	20	.3961	.5978	.4314	.6348	2.3183	.3652	.9182	.9629	40	1.1636
.4102	30	.3987	.6007	.4348	.6383	2.2998	.3617	.9171	.9624	30	1.1606
.4131	40	.4014	.6036	.4383	.6417	2.2817	.3583	.9159	.9618	20	1.1577
.4160	50	.4041	.6065	.4417	.6452	2.2637	.3548	.9147	.9613	10	1.1548
.4189	**24° 00′**	.4067	.6093	.4452	.6486	2.2460	.3514	.9135	.9607	**66° 00′**	1.1519
.4218	10	.4094	.6121	.4487	.6520	2.2286	.3480	.9124	.9602	50	1.1490
.4247	20	.4120	.6149	.4522	.6553	2.2113	.3447	.9112	.9596	40	1.1461
.4276	30	.4147	.6177	.4557	.6587	2.1943	.3413	.9100	.9590	30	1.1432
.4305	40	.4173	.6205	.4592	.6620	2.1775	.3380	.9088	.9584	20	1.1403
.4334	50	.4200	.6232	.4628	.6654	2.1609	.3346	.9075	.9579	10	1.1374
.4363	**25° 00′**	.4226	.6259	.4663	.6687	2.1445	.3313	.9063	.9573	**65° 00′**	1.1345
.4392	10	.4253	.6286	.4699	.6720	2.1283	.3280	.9051	.9567	50	1.1316
.4422	20	.4279	.6313	.4734	.6752	2.1123	.3248	.9038	.9561	40	1.1286
.4451	30	.4305	.6340	.4770	.6785	2.0965	.3215	.9026	.9555	30	1.1257
.4480	40	.4331	.6366	.4806	.6817	2.0809	.3183	.9013	.9549	20	1.1228
.4509	50	.4358	.6392	.4841	.6850	2.0655	.3150	.9001	.9543	10	1.1199
.4538	**26° 00′**	.4384	.6418	.4877	.6882	2.0503	.3118		.9537	**64° 00′**	1.1170
.4567	10	.4410	.6444	.4913	.6914	2.0353	.3086	.8975	.9530	50	1.1141
.4596	20	.4436	.6470	.4950	.6946	2.0204	.3054	.8962	.9524	40	1.1112
.4625	30	.4462	.6495	.4986	.6977	2.0057	.3023	.8949	.9518	30	1.1083
.4654	40	.4488	.6521	.5022	.7009	1.9912	.2991	.8936	.9512	20	1.1054
.4683	50	.4514	.6546	.5059	.7040	1.9768	.2960	.8923	.9505	10	1.1025
.4712	**27° 00′**	.4540	.6570	.5095	.707:	1.9626	.2928	.8910	.9499	**63° 00′**	1.0996
		Value	Log$_{10}$	Value	Log$_{10}$	Value	Log$_{10}$	Value	Log$_{10}$		Radians
		Cosine		Cotangent		Tangent		Sine			

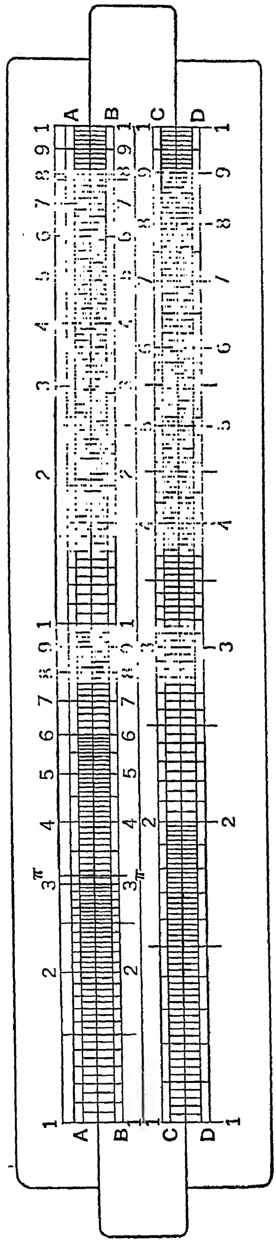
[Characteristics of Logarithms omitted—determine by the usual rule from the value]

Radians	Degrees	Sine Value	Sine Log_{10}	Tangent Value	Tangent Log_{10}	Cotangent Value	Cotangent Log_{10}	Cosine Value	Cosine Log_{10}		
.4712	**27° 00′**	.4540	.6570	.5095	.7072	1.9626	.2928	.8910	.9499	**63° 00′**	1.0996
.4741	10	.4566	.6595	.5132	.7103	1.9486	.2897	.8897	.9492	50	1.0966
.4771	20	.4592	.6620	.5169	.7134	1.9347	.2866	.8884	.9486	40	1.0937
.4800	30	.4617	.6644	.5206	.7165	1.9210	.2835	.8870	.9479	30	1.0908
.4829	40	.4643	.6668	.5243	.7196	1.9074	.2804	.8857	.9473	20	1.0879
.4858	50	.4669	.6692	.5280	.7226	1.8940	.2774	.8843	.9466	10	1.0850
.4887	**28° 00′**	.4695	.6716	.5317	.7257	1.8807	.2743	.8829	.9459	**62° 00′**	1.0821
.4916	10	.4720	.6740	.5354	.7287	1.8676	.2713	.8816	.9453	50	1.0792
.4945	20	.4746	.6763	.5392	.7317	1.8546	.2683	.8802	.9446	40	1.0763
.4974	30	.4772	.6787	.5430	.7348	1.8418	.2652	.8788	.9439	30	1.0734
.5003	40	.4797	.6810	.5467	.7378	1.8291	.2622	.8774	.9432	20	1.0705
.5032	50	.4823	.6833	.5505	.7408	1.8165	.2592	.8760	.9425	10	1.0676
.5061	**29° 00′**	.4848	.6856	.5543	.7438	1.8040	.2562	.8746	.9418	**61° 00′**	1.0647
.5091	10	.4874	.6878	.5581	.7467	1.7917	.2533	.8732	.9411	50	1.0617
.5120	20	.4899	.6901	.5619	.7497	1.7796	.2503	.8718	.9404	40	1.0588
.5149	30	.4924	.6923	.5658	.7526	1.7675	.2474	.8704	.9397	30	1.0559
.5178	40	.4950	.6946	.5696	.7556	1.7556	.2444	.8689	.9390	20	1.0530
.5207	50	.4975	.6968	.5735	.7585	1.7437	.2415	.8675	.9383	10	1.0501
.5236	**30° 00′**	.5000	.6990	.5774	.7614	1.7321	.2386	.8660	.9375	**60° 00′**	1.0472
.5265	10	.5025	.7012	.5812	.7644	1.7205	.2356	.8646	.9368	50	1.0443
.5294	20	.5050	.7033	.5851	.7673	1.7090	.2327	.8631	.9361	40	1.0414
.5323	30	.5075	.7055	.5890	.7701	1.6977	.2299	.8616	.9353	30	1.0385
.5352	40	.5100	.7076	.5930	.7730	1.6864	.2270	.8601	.9346	20	1.0356
.5381	50	.5125	.7097	.5969	.7759	1.6753	.2241	.8587	.9338	10	1.0327
.5411	**31° 00′**	.5150	.7118	.6009	.7788	1.6643	.2212	.8572	.9331	**59° 00′**	1.0297
.5440	10	.5175	.7139	.6048	.7816	1.6534	.2184	.8557	.9323	50	1.0268
.5469	20	.5200	.7160	.6088	.7845	1.6426	.2155	.8542	.9315	40	1.0239
.5498	30	.5225	.7181	.6128	.7873	1.6319	.2127	.8526	.9308	30	1.0210
.5527	40	.5250	.7201	.6168	.7902	1.6212	.2098	.8511	.9300	20	1.0181
.5556	50	.5275	.7222	.6208	.7930	1.6107	.2070	.8496	.9292	10	1.0152
.5585	**32° 00′**	.5299	.7242	.6249	.7958	1.6003	.2042	.8480	.9284	**58° 00′**	1.0123
.5614	10	.5324	.7262	.6289	.7986	1.5900	.2014	.8465	.9276	50	1.0094
.5643	20	.5348	.7282	.6330	.8014	1.5798	.1986	.8450	.9268	40	1.0065
.5672	30	.5373	.7302	.6371	.8042	1.5697	.1958	.8434	.9260	30	1.0036
.5701	40	.5398	.7322	.6412	.8070	1.5597	.1930	.8418	.9252	20	1.0007
.5730	50	.5422	.7342	.6453	.8097	1.5497	.1903	.8403	.9244	10	.9977
.5760	**33° 00′**	.5446	.7361	.6494	.8125	1.5399	.1875	.8387	.9236	**57° 00′**	.9948
.5789	10	.5471	.7380	.6536	.8153	1.5301	.1847	.8371	.9228	50	.9919
.5818	20	.5495	.7400	.6577	.8180	1.5204	.1820	.8355	.9219	40	.9890
.5847	30	.5519	.7419	.6619	.8208	1.5108	.1792	.8339	.9211	30	.9861
.5876	40	.5544	.7438	.6661	.8235	1.5013	.1765	.8323	.9203	20	.9832
.5905	50	.5568	.7457	.6703	.8263	1.4919	.1737	.8307	.9194	10	.9803
.5934	**34° 00′**	.5592	.7476	.6745	.8290	1.4826	.1710	.8290	.9186	**56° 00′**	.9774
.5963	10	.5616	.7494	.6787	.8317	1.4733	.1683	.8274	.9177	50	.9745
.5992	20	.5640	.7513	.6830	.8344	1.4641	.1656	.8258	.9169	40	.9716
.6021	30	.5664	.7531	.6873	.8371	1.4550	.1629	.8241	.9160	30	.9687
.6050	40	.5688	.7550	.6916	.8398	1.4460	.1602	.8225	.9151	20	.9657
.6080	50	.5712	.7568	.6959	.8425	1.4370	.1575	.8208	.9142	10	.9628
.6109	**35° 00′**	.5736	.7586	.7002	.8452	1.4281	.1548	.8192	.9134	**55° 00′**	.9599
.6138	10	.5760	.7604	.7046	.8479	1.4193	.1521	.8175	.9125	50	.9570
.6167	20	.5783	.7622	.7089	.8506	1.4106	.1494	.8158	.9116	40	.9541
.6196	30	.5807	.7640	.7133	.8533	1.4019	.1467	.8141	.9107	30	.9512
.6225	40	.5831	.7657	.7177	.8559	1.3934	.1441	.8124	.9098	20	.9483
.6254	50	.5854	.7675	.7221	.8586	1.3848	.1414	.8107	.9089	10	.9454
.6283	**36° 00′**	.5878	.7692	.7265	.8613	1.3764	.1387	.8090	.9080	**54° 00′**	.9425
		Value	Log_{10}	Value	Log_{10}	Value	Log_{10}	Value	Log_{10}	Degrees	Radians
		Cosine		Cotangent		Tangent		Sine			

[Characteristics of Logarithms omitted—determine by the usual rule from the value]

Radians	Degrees	Sine Value	Sine Log10	Tangent Value	Tangent Log10	Cotangent Value	Cotangent Log10	Cosine Value	Cosine Log10		
.6283	**36° 00′**	.5878	.7692	.7265	.8613	1.3764	.1387	.8090	.9080	**54° 00′**	.9425
.6312	10	.5901	.7710	.7310	.8639	1.3680	.1361	.8073	.9070	50	.9396
.6341	20	.5925	.7727	.7355	.8666	1.3597	.1334	.8056	.9061	40	.9367
.6370	30	.5948	.7744	.7400	.8692	1.3514	.1308	.8039	.9052	30	.9338
.6400	40	.5972	.7761	.7445	.8718	1.3432	.1282	.8021	.9042	20	.9308
.6429	50	.5995	.7778	.7490	.8745	1.3351	.1255	.8004	.9033	10	.9279
.6458	**37° 00′**	.6018	.7795	.7536	.8771	1.3270	.1229	.7986	.9023	**53° 00′**	.9250
.6487	10	.6041	.7811	.7581	.8797	1.3190	.1203	.7969	.9014	50	.9221
.6516	20	.6065	.7828	.7627	.8824	1.3111	.1176	.7951	.9004	40	.9192
.6545	30	.6088	.7844	.7673	.8850	1.3032	.1150	.7934	.8995	30	.9163
.6574	40	.6111	.7861	.7720	.8876	1.2954	.1124	.7916	.8985	20	.9134
.6603	50	.6134	.7877	.7766	.8902	1.2876	.1098	.7898	.8975	10	.9105
.6632	**38° 00′**	.6157	.7893	.7813	.8928	1.2799	.1072	.7880	.8965	**52° 00′**	.9076
6661	10	.6180	.7910	.7860	.8954	1.2723	.1046	.7862	.8955	50	.9047
.6690	20	.6202	.7926	.7907	.8980	1.2647	.1020	.7844	.8945	40	.9018
.6720	30	.6225	.7941	.7954	.9006	1.2572	.0994	.7826	.8935	30	.8988
.6749	40	.6248	.7957	.8002	.9032	1.2497	.0968	.7808	.8925	20	.8959
.6778	50	.6271	.7973	.8050	.9058	1.2423	.0942	.7790	.8915	10	.8930
.6807	**39° 00′**	.6293	.7989	.8098	.9084	1.2349	.0916	.7771	.8905	**51° 00′**	.8901
.6836	10	.6316	.8004	.8146	.9110	1.2276	.0890	.7753	.889	50	.8872
.6865	20	.6338	.8020	.8195	.9135	1.2203	.0865	.7735	.8884	40	.8843
.6894	30	.6361	.8035	.8243	.9161	1.2131	.0839	.7716	.8874	30	.8814
.6923	40	.6383	.8050	.8292	.9187	1.2059	.0813	.7698	.8864	20	.8785
.6952	50	.6406	.8066	.8342	.9212	1.1988	.0788	.7679	.885	10	.8756
.6981	**40° 00′**	.6428	.8081	.8391	.9238	1.1918	.0762	.7660	.8843	**50° 00**	.8727
.7010	10	.6450	.8096	.8441	.9264	1.1847	.0736	.7642	.883	50	.8698
.7039	20	.6472	.8111	.8491	.9289	1.1778	.0711	.7623	.8821	40	.8668
.7069	30	.6494	.8125	.8541	.9315	1.1708	.0685	.7604	.8810	30	.8639
.7098	40	.6517	.8140	.8591	.9341	1.1640	.0659	.7585	.8800	20	.8610
.7127	50	.6539	.8155	.8642	.9366	1.1571	.0634	.7566	.8789	10	.8581
.7156	**41° 00′**	.6561	.8169		.9392	1.1504	.0608	.7547	.8778	**49° 00**	.8552
.7185	10	.6583	.8184	.8744	.9417	1.1436	.0583	.7528	.876	50	.8523
.7214	20	.6604	.8198	.8796	.9443	1.1369	.0557	.7509	.8756	40	.8494
.7243	30	.6626	.8213	.8847	.9468	1.1303	.0532	.7490	.8745	30	.8465
.7272	40	.6648	.8227	.8899	.9494	1.1237	.0506	.7470	.873	20	.8436
.7301	50	.6670	.8241	.8952	.9519	1.1171	.0481	.7451	.8722	10	.8407
.7330	**42° 00′**	.6691	.8255	.9004	.9544	1.1106	.0456	.7431	.8711	**48° 00**	.8378
.7359	10	.6713	.8269	.9057	.9570	1.1041	.0430	.7412	.8699	50	.8348
.7389	20	.6734	.8283	.9110	.9595	1.0977	.040	.7392	.8688	40	.8319
.7418	30	.6756	.8297	.9163	.9621	1.0913	.0379	.7373	.8676	30	.8290
.7447	40	.6777	.8311	.9217	.9646	1.0850	.0354	.7353	.866	20	.8261
.7476	50	.6799	.8324	.9271	.9671	1.0786	.0329	.7333	.8653	10	.8232
.7505	**43° 00′**	.6820	.8338	.9325	.9697	1.0724	.0303	.7314	.8641	**47° 00′**	.8203
.7534	10	.6841	.8351	.9380	.9722	1.0661	.0278	.7294	.8629	50	.8174
.7563	20	.6862	.8365	.9435	.9747	1.0599	.0253	.7274	.8618	40	.8145
.7592	30	.6884	.8378	.9490	.9772	1.0538	.0228	.7254	.8606	30	.8116
.7621	40	.6905	.8391	.9545	.9798	1.0477	.0202	.7234	.8594	20	.8087
.7650	50	.6926	.8405	.9601	.9823	1.0416	.0177	.7214	.8582	10	.8058
.7679	**44° 00′**	.6947	.8418	.9657	.9848	1.0355	.0152	.7193	.8569	**46° 00**	.8029
.7709	10	.6967	.8431	.9713	.9874	1.0295	.0126	.7173	.8557	50	.7999
.7738	20	.6988	.8444	.9770	.9899	1.0235	.0101	.7153	.8545	40	.7970
.7767	30	.7009	.8457	.9827	.9924	1.0176	.0076	.7133	.853	30	.7941
.7796	40	.7030	.8469	.9884	.9949	1.0117	.0051	.7112	.8520	20	.7912
.7825	50	.7050	.848	.9942	.9975	1.0058	.0025	.7092	.850	10	.7883
.7854	**45° 00′**	.7071	.8495	1.0000	.0000	1.0000	.0000	.7071	.849	**45° 00**	.7854
		Value	Log10	Value	Log10	Value	Log10	Value	Log10	Degrees	Radians
		Cosine		Cotangent		Tangent		Sine			





Directions

A reasonably accurate slide-rule may be made by the student, for temporary practice, as follows. Take three strips of heavy stiff cardboard 1″.3 wide by 6″ long; these are shown in cross-section in (1), (2), (3) above. On (3) paste or glue the adjoining cut of the slide rule. Then cut strips (2) and (3) accurately along the lines marked. Paste or glue the pieces together as shown in (4) and (5). Then (5) forms the slide of the slide-rule, and it will fit in the groove in (4) if the work has been carefully done. Trim off the ends as shown in the large cut.